Centre for Development Economics

WORKING PAPER SERIES

Sustaining Inter-Generational Altruism When Social Memory Is Bounded

V. Bhaskar

Delhi School of Economics

Working Paper No: 10

Centre for Development Economics
Delhi School of Economics
Delhi 110 007 INDIA

SUSTAINING INTER-GENERATIONAL ALTRUISM WHEN SOCIAL MEMORY IS BOUNDED

V. Bhaskar
Delhi School of Economics
University of Delhi
Delhi 110007, India

Abstract
We consider an infinite horizon overlapping generations economy where agents are endowed with a perishable and finitely divisible good when young, and are unendowed when old. Dynamic efficiency requires some transfer of the good from the young to the old. However, such transfers cannot be supported by pure-strategy sequential equilibria when social memory is bounded, so that an agent only observes the transfers of a finite number of previous agents. Mixed strategies allow transfers to be sustained; however, these equilibria are not robust. If each agent's utility function is subjected to a small random perturbation, these mixed strategy equilibria unravel, and only the zero-transfer equilibrium survives. These results extend when we allow the commodity to be perfectly divisible. We also suggest that money may play an informational role in this context, as a device for overcoming the boundedness of social memory.

Keywords: overlapping generations, dynamic games, monetary theory, purification of mixed strategies.

* I am grateful to seminar participants at the Delhi School of Economics and the Indian Statistical Institute, and to Dilip Mookherjee in particular, for their comments.

1. INTRODUCTION

Consider a simple version of Samuelson's (1958) infinite horizon overlapping generations economy. In each period, a single agent is born and lives for two periods. The young agent is endowed with two units of an indivisible and perishable consumption good - fish for example. The old agent is without any endowment. The young agent may consume both fish or she may give one to her mother. The old agent is passive, and has no choices to make. Agents are selfish, and prefer more consumption to less, but they would rather have the same total consumption spread out so as to not starve when old. There is one equilibrium of this economy, where every agent *behaves* selfishly; she consumes two fish when young, giving nothing to her mother, and in turn receives nothing when old. This equilibrium is inefficient. There is another (Nash) equilibrium where every agent *behaves* altruistically, and gives one fish to her mother. An agent who deviates and from this norm and behaves selfishly is punished by receiving no transfer from her daughter. The altruistic outcome can also be supported by a subgame perfect equilibrium if each agent is able to observe the entire past history, as Hammond (1975) observed. There are in fact a large number of pure strategies which support altruistic behavior. One strategy is the "grim" strategy where if any agent behaves selfishly, and transfers no fish to her mother, every succeeding agent retaliates by being selfish. A more attractive strategy requires an agent to punsish her mother if and only if she is a "deviant", where a deviant is one who has been selfish when she should have been altruistic. It is easy to show that there are in fact an infinite number of subgame perfect equilibria which support altruistic behavior.

This overlapping generations model and its variants have been extra-ordinarily influential in economic theory. Much of the macro-economic literature on dynamic efficiency has such a model as its underpinning. This is most clear in monetary theory. Reinterpret the model, so that the old agent at the initial date can issue money, and exchange this money for the consumption good. Each young agent will accept money in exchange for the good if she expects her daugther to accept money. Money has value only because it is expected to have value in the future. However, the question arises, what is so sacrosant about the money issued at the intial date? Why does'nt a young agent simply issue her own money when old? The answer of course is that in an equilibrium, this money will not be acceptable to future generations. It is clear that this model of money is formally equivalent to the altruistic equilibrium described earlier. Money has value if and only if there is an equilibrium with altruistic behavior.

This paper takes a closer look at the informational basis for such an altruistic equilibrium. Specifically, we relax the assumption that agents are able to observe the entire history of past actions, an assumption which has attracted some flak (see Esteban and Sakovics (1993) for example). We assume instead that social memory is bounded, so that any agent has information only about the last m actions, where m is some natural number? This assumption seems natural to us - agents today have little direct information about the past, and what information they have is filtered through past generations. With bounded memory the overlapping generations economy becomes a game of imperfect information. The subgame perfection condition must be replaced by the requirement that equilibrium strategies be sequentially rational, i.e. Nash equilibria at every information set. Our basic finding is that with bounded memory, no matter how large the bound, altruistic behavior cannot be sustained.

Section 2 presents a simple example which illustrates the difficulty in supporting altruistic behavior. Section 3 presents a general analysis of pure strategy equilibria, and shows that altruistic behavior cannot be supported in such an equilibrium if social memory is bounded. This result applies generically if the perishable commodity is finitely divisible, so that the action set of each agent is finite. However, if if randomized punishments are possible, section 4 shows that altruism can be supported provided that there is some memory, i.e. an agent can observe at least the actions of her mother. These mixed strategies however turn out to be fragile. In section 5 we perturb the overlapping generations economy in the manner of Harsanyi (1973). All agents are ex ante identical, but each agent's utility function is subject to a small random shock, the realization of which is private information. We show that the randomized punishments which support altruism unravel, and the only equilibrium which survives is the one where every agent behaves selfishly every period. Section 6 allows the commodity to be perfectly divisible, so that each agent's action set is a compact interval. We see that there are pure strategy equilibria which support altruistic behavior, but once again these equilibria do not survive when we perturb the agent's utility function. Section 7 discusses a possible informational role for money in this context, as a device for overcoming the boundedness of social memory. The final section concludes.

2. AN EXAMPLE

We present a simple example which illustrates the problem in supporting altruistic behavior. As in the introduction each young agent is endowed with two fish. She may give one to her mother or none, so that the set of possible transfers she could make is $A = \{0,1\}$ (we assume that preferences are such that transferring 2 and consuming 0 is strictly dominated and may be ruled out). All agents have identical preferences, and the utility $u(x,y)$ where x is the transfer made by the agent when young and y is the transfer received by her when old, is:

$$u(0,1) > u(1,1) > u(0,0) > u(1,0) \tag{2.1}$$

Let $m = 2$, so that any agent only observes the last two actions taken. Let the first agent transfer 1 and the second agent simply match the action of the first agent. This implies that the first two agents will behave altruistically. After t=3, every agent observes the actions of the two previous agents. Hence for $t>2$, the agent's strategy s_t, specifies the action to be taken for every possible pair of actions last observed. We restrict attention to pure strategies, and to strategy profiles where $s_t = s_{t+1} = s$ for $t>1$, i.e. all agents after period 2 adopt the same strategy. Since m=2, there are 4 possible observed histories.

Since we are interested in the possibility of supporting altruistic behavior, the strategy must choose 1 after observing (1,1). To sustain this, we must punish a deviator; hence we must choose 0 after (1,0). With these determined, we can fill in the choices after (0,0) and (0,1) in four different ways. These allow four possible strategies, which we label I, II, III and IV. Table 1 shows what happens to a player after any of the four possible observed histories if every agent adopts the same strategy. Given any observed history, the strategy determines the action taken by the agent at date t, and thereby also the information of the agent at date t+1, which we call the the "induced history". The induced history and the strategy determine the "next-period action, i.e.

the action taken at t+1. The actions at t and t+1 determine the utility of the agent at t. Table 1 shows why each of these four strategies fails to be sequentially rational, since there is one observed history at which the agent at t can deviate profitably, given that the agent at t+1 is following the strategy. Consider strategy I which is "nice", and chooses zero only after observing (1,0). This is not optimal if the observed history is (0,0), since the agent still gets 1 the next period if she chooses 0 rather than 1. II on the other hand is "grim", and chooses 0 at every state except (1,1). This is too grim; after (0,1), the agent prefers to choose 1 rather than 0. By choosing 1, she ensures that the history next period is (1,1), thereby ensuring a transfer to herself. III and IV are intermediate; they choose 1 after two of the four histories. They too fail, and interestingly, both fail to be optimal after the history (0,0). III calls the player to choose 0, but it is preferable to deviate to 1, since this ensures a transfer of 1 in the next period. IV chooses 1 after (0,0), but the player can deviate to 0 without being punished.

It might be conjectured that the problem arises because we have required every agent to choose the same strategy. However, this is not the case, and removing this restriction does not improve matters. Nor is the case of m=2 particularly special - the point generalizes to m = two million. The problem arises since each agent has better information about the past than her daughter. To support altruistic behavior, we must reward altruism and punish selfish behavior. This requires that the agent at t+1 must vary her behavior in a non-trivial way depending upon the information she observes. However, the agent at t can manipulate the information that her daughter receives. Any pure strategy profile aimed at supporting altruistic behavior either turns out to be too grim or too nice, and any attempt to rectify one problem only brings in the other problem.

We turn now to a formal analysis of the model.

3. PURE STRATEGY EQUILIBRIA

We consider an economy over periods 1,2,... The t-th agent is born in period t, with an endowment of e. The agent has an action finite action set A, where $a \in A$ represents the amount the agent transfers to agent t-1. Given a, agent t's consumption at date t is (e-a). The finiteness of A can be justified on two grounds. First, it is physically impossible to have an infinitely divisble commodity. The second reason is informational: subsequent generations may not be able to observe t's transfer as finely as t can.

The agent's utility u, is a function, $u: A \times A \rightarrow R$, where $u(x,y)$ is the agent's utility when she transfers x units to her mother and recieves y units from her daughter, i.e. it is the utility from consuming (e-x) units when young and y units when old. If A has k elements, the agent's utility function can also be identified with a point in R^{2k}. We make the following assumption regarding u:

Assumption A1. u is strictly decreasing in its first argument and strictly increasing in its second argument.

A1 implies that any agent t will transfer zero if the transfer of the succeeding agent is fixed and does not depend upon t's transfer.

Assumption A2: Let $\underline{w}, \underline{z} \in A^2$. If $u(\underline{w}) = u(\underline{z})$, then $\underline{w} = \underline{z}$.

A2 says that if $\underline{w}$, $\underline{z}$ are distinct vectors, then they yield different utility levels. Since A is finite, this will be the case almost always, i.e. the set of points in R^{2k} such that A2 is violated is a closed set of Lebesgue measure zero in R^{2k}.

The history at period t, h^t, is the sequence of preceding actions, $(a_1, a_2, \ldots, a_{t-1})$. The history at period 1 is the null history, h^1. H^t is the set of all possible histories at t, i.e. $H^t = A^{t-1}$.

Social memory is bounded, so that agents observe only the last m actions, where m is a natural number. Given any history, h^t, the *observed history*, b^t, equals h^t if $t-1 \leq m$, and equals the last m components of h^t if $t-1 > m$. B^t is the set of observable histories at t, i.e.:

$$B^t = H^t \text{ if } t \leq m$$
$$= A^m \text{ if } t > m$$

A pure strategy for agent t is a function $s_t: B^t \longrightarrow A$. Agent t's strategy set, S^t, is the set of all such functions. If $m < \infty$, then for all $t > m$, $s_t: A^m \longrightarrow A$. Hence, if $t' t > m$, $S^t = S^{t'}$. Hence let $S^t = S$ if $t > m$.

Any pure strategy also defines a function from one set of observable histories to another. Write γ_t for this function, where $\gamma_t: S^t \times B^t \longrightarrow B^{t+1}$. If $t > m$, $\gamma_t: S \times A^m \longrightarrow A^m$, i.e. with any s in S is associated a map from the set A^m to itself. The structure of γ_t is simple. If $t \leq m$, any $b^t \in B^t$ has less than m components so that $\gamma_t(s_t, b^t) = (b^t, s^t(b^t))$, i.e. the b^t is augmented by the action $s_t(b^t)$. If $t > m$, b^t has m components, and the first component (i.e. the action taken in period t-m) is deleted and the latest action is added as the m-th component. We also write $\gamma_t(s_t, b_t)$ as $(b_t / s_t(b^t))$ in this case.

The case when $t > m$, so that the bounded memory constraint bites, plays an important role in our analysis. Recall that $B^t = A^m$ in this case, and $\gamma_t(s_t)$: $A^m \longrightarrow A^m$. Consider the set A^m with typical element b = $(a_m, a_{m-1}, \ldots, a_1)$. a_j is the action taken j periods earlier, i.e. in period t-j. We define on A^m a series of equivalence relations, $\sim_0$, $\sim_1, \ldots, \sim_m$.

Definition: $\sim_i$ is an equivalence relation on A^m such that:

$b \sim_i b$ iff $a_j = a'_j$, $j = 1,2,\ldots,i$.

b and b' are i-equivalent if their last i components are equal, i.e. if the last i observed actions are the same. Each $\sim_i$ defines a partition of A^m. If $i<k$, $\sim_i$ is a coarsening of $\sim_k$. $\sim_0$ is the coarsest partition, since every element of A^m is 0-equivalent to every other element. $\sim_m$ is the finest partition, since no two distinct elements of A^m are m-equivalent.

A strategy profile, $\langle s \rangle$, is a infinite sequence $\langle s_t \rangle$ where:

$s_t \in S_t$ if $t \leq m$

$s_t \in S$ if $t > m$

Given a strategy profile $\langle s \rangle$, the *realized history at t,* $b^t(\langle s \rangle)$, is the element of B^t which is induced when $\langle s_t \rangle$ is played. Similarly, given $\langle s_t \rangle$, $t>\tau$, and an observed history b^τ, the *realized history at t given* b^τ, $b^t(\langle s \rangle, b^\tau)$, is the element of B^t which is induced when $\langle s \rangle$ is played after b^τ.

Observe that agent t's utility is affected directly only by her own action and the action of agent t+1. Agent t's utility is affected indirectly by the actions of agents t-i, i=1,2,...t-1, since these actions determine the observed history. Agent t's utility is unaffected by the actions of agents at dates after t+1. Given an observed history b^t and the strategy $\langle s_t \rangle$, agent t's utility u is:

$$u(\langle s \rangle / b^t) = u(s_t, s_{t+1} / b^t) = u\,[s_t(b^t),\ s_{t+1}(\gamma_t(s_t, b^t)] \qquad (3.1)$$

Note that this is well defined even if b^t is not observed given the strategy profile $\langle s \rangle$.

A strategy profile $\langle s \rangle$ is a sequentially rational equilibrium (abbreviated to equilibrium henceforth) if $\forall t$, $\forall b^t \in B^t$,

$$u(s_t, s_{t+1} / b^t) \geq u(s'_t, s_{t+1} / b^t) \ \forall\ s'_t \in S_t \qquad (3.2)$$

Remark: Our equilibrium definition is remarkably simple. We do not have to invoke any beliefs regarding past actions, as is usual in games of imperfect information, since past actions do not directly affect current or future utility. Further, at any date t, the information partition of agent t regarding the past, is always finer than the information partitions of agents at future dates t+k, k>0.

We now state the main result of this section:

Theorem 1. Let social memory be bounded. There is a unique pure strategy equilibrium where each agent transfers zero and consumes her entire endowment, after every observed history.

We prove the theorem via two lemmata. The first lemma is straight-forward and says that an agent will make a positive transfer only if she expects to get a positive transfer in return when old.

Lemma 1. Let $\langle s\rangle$ be a sequential equilibrium. Let $k<t$. If $s_t(b^t(\langle s\rangle,b^k)$ $= 0$, then $s_r(b^r(\langle s\rangle,b^k) = 0$ for all r, $k<r<t$.

Proof: By backward induction from t. Since agent t-1 receives a zero transfer under $\langle s\rangle$ after b^k, by A1 she must optimally choose 0, and so must any agent $r<t$. □

Lemma 2 is more substantial:

Lemma 2. Let $t>m$. If $\langle s\rangle$ is a equilibrium, s_t is a constant function on A^m.

Proof: The proof is by backward induction. Given any $t>m$, we show that s_{t+i} is measurable with respect to $_{\sim i}$, $i=0,1,2,\ldots,m$.

i) s_{t+m} is measurable w.r.t. $_{\sim m}$: by definition, since $_{\sim m}$ is the finest partition, where each set in the partition is singleton.

ii)If s_{t+i+1} is measurable w.r.t. $_{\sim i+1}$, then s_{t+i} is measurable w.r.t. $_{\sim i}$, $i= 0,1,\ldots,m-1$.

Note first that if $b \sim_i b'$, then $(b/a) \sim_{i+1} (b'/a)$, i.e. if the same action a is taken at b and b', the resulting observed histories are i+1-equivalent.

We claim that if s_{t+i+1} is measurable w.r.t. $\sim_{i+1}$, and $b \sim_i b'$, then:

$$u_{t+i}(s_{t+i}, s_{t+i+1}/b) = u_{t+i}(s_{t+i}, s_{t+i+1}/b') \tag{3.3}$$

Suppose not. Let $u_{t+i}(s_{t+i}, s_{t+i+1}/b) > u_{t+i}(s_{t+i}, s_{t+i+1}/b')$. Then $s_{t+i}(b')$ is not optimal, since by choosing the action $s_{t+i}(b)$, agent t+i ensures the history $(b'/s_{t+i}(b))$. Since $b' \sim_i b$,

$$(b'/s_{t+i}(b)) \sim_{i+1} (b/s_{t+i}(b)) \tag{3.4}$$

Since s_{t+i+1} is measurable w.r.t. $\sim_{i+1}$, agent t+i ensures that t+i+1 takes the same action, and hence the payoff $u_{t+i}(s_{t+i}, s_{t+i+1}/b)$. Hence if $\langle s \rangle$ is a equilibrium, (3.3) must hold.

If (3.3) applies, Assumption A2 implies that $s_{t+i}(b)$ cannot be distinct from $s_{t+i}(b')$. Hence s_{t+i} is measurable w.r.t. $\sim_{i+1}$.

(i) and (ii) together imply lemma 2. □

The proof of the theorem is now simple. t can be chosen to be arbitrarily large, with s_t a constant function. By backward induction, the transfers made by agents at dates t-k, k=1,2,...t-1, do not affect the transfer they receive. Hence by A1 and lemma 1, the agent must choose 0 after every possible history. □

Remark: Note that the proof consistently uses backward induction even though we have an infinite horizon model.

We offer the following intuition for Theorem 1. If altruistic behavior is to be supported, agents must vary their behavior depending upon the observed history. Since the strategy profile is pure, this implies that the agent's utility under the strategy profile *differs* depending upon the history they have observed. However, each agent has better information about the past than the succeeding agent, and this allows her to manipulate the information that is transmitted. The only way in which this informational advantage can be nullified is if the strategy profile does not condition upon information at all.

This intuition suggests that mixed strategies may be able to overcome the problem, and we turn to these.

4. MIXED STRATEGIES

Theorem 1 applies to pure strategies. In this section we consider mixed strategies. Specifically, we ask, is it possible to support the altruistic outcome by the use of randomized punishments?

Definition: $a^* = \underset{a\in A}{\operatorname{argmax}}\ u(a,a)$

a^* is the optimal transfer that an agent is willing to make, given that her daughter makes a matching transfer. It is clear that the outcome where every agent transfers a^* is Pareto-efficient.

Assumption A3: $a^*>0$. If $a'<a^*$,

$$u(a',a^*) > u(a^*,a^*) > u(a',a') > u(a^*,a') \qquad (4.1)$$

(4.1) obviously applies when $a'=0$, i.e. it implies (4.2)

$$u(0,a^*) > u(a^*,a^*) > u(0,0) > u(a^*,0) \qquad (4.2)$$

Theorem 2. If social memory is non-zero, i.e. $m>0$, there exists an efficient equilibrium where every agent transfers a^*, which is enforced by the use of randomized punishments.

Proof: We first present a constructive proof for $m\geq 2$, mainly because this is more interesting than the proof for $m\geq 1$. Consider $t\geq m$. Agent t's strategy is now a function from A^m to the set of probability measures over A, i.e $s_t : A^m \rightarrow \Delta A$. We construct a strategy which is measurable w.r.t. $_{\sim 2}$, i.e. it conditions only on the last two actions. Let a' be any action different from a*. The strategy as a function of the last two actions, is defined as follows:

$(a^*,a^*) \rightarrow a^*$

$(a',a^*) \rightarrow a^*$

$(a^*,a') \rightarrow 0$

$(a',a') \rightarrow$ a* with probability p, 0 with probability (1-p)

$p = [u(a^*,a^*) - u(0,0)] / [u(0,a^*) - u(0,0)]$

We now verify that the strategy is an equilibrium. If the last observed action is a*, agent t is required to play a*, which gives her utility u(a*,a*). A deviation is punished by the transfer of 0, and hence gives at most u(0,0), which is strictly less, by (4.2).

Let the last observed action pairs be (a,a'), where either a = a* or a = a'. Consider the utility of agent t from two alternative actions, a* and 0.

$$u(a^*,s_{t+1}/(a,a')) = u(a^*,a^*) \qquad (4.3)$$

$$u(0,s_{t+1}/(a,a')) = p\ u(0,a^*) + (1-p)\ u(0,0)$$
$$= u(a^*,a^*) \qquad (4.4)$$

(4.3) and (4.4) show that if a' is the last observed action, the agent's utility from the action a* equals her utility from the action 0. Any other action yields strictly lower utility. Hence this verifies that it is optimal to chose 0 after observing (a*,a'), and to play the mixed strategy p after (a',a').

Similarly we can construct a strategy which is measurable w.r.t. $_{\sim 1}$, which covers the case for m≥1. The strategy transfer's a* if a* is the last transfer, and randomizes as in "p" if the last transfer differs from a*.Since the expected utility from this strategy is u(a*,a*) no matter what the observed history, this is an equilibrium. □

The mixed strategy equilibrium solves the problem which arose in the case of pure strategies, which was that a strategy was either too nice or too grim. Randomized punishments can be fine tuned to be just right. Consequently, it is possible to have an agent take different actions at different information sets, since the use of mixed strategies allows us to equalize the payoffs to these actions. Nevertheless, this knife-edge balance is unstable, as we shall see in the next section.

5. EQUILIBRIA OF THE PERTURBED GAME

Are the mixed strategy equilibrium which support altruistic behavior robust? In this section we ask whether these equilibria survive when each player's payoff function is perturbed, and this perturbation is private information, in the manner of Harsanyi (1973). We adapt the framework of van Damme (1991, chapter 5) to our set up, which is of an extensive form game.

Index agents by t as before. Recall that each agent's action set, A, has k elements. Let X_t be a random vector with values in a set Z in R^{2k}. Let $c > 0$.

$$Z = \{x \in R^{2k}: -c \leq x^i \leq c,\ i = 1,2,..,2k\}$$

Let μ be a probability measure on Z.

The *disturbed overlapping generations game* is as follows:

i) Nature chooses an outcome x_t of X_t for each agent t, independently, and by the probability measure μ.

ii) Agent t, t=1,2,..., gets to know the outcome x_t, and nothing else.

iii) Agents 1 chooses an element of ΔA, having observed x_1. Each succeeding agent observes x_t, and the observed history b^t, and chooses an element of ΔA.

iv) If a_t and a_{t+1} are chosen, the payoff to the t-th agent is given by:

$$u_t(a_t,a_{t+1}) = u(a_t,a_{t+1}) + x_t(a_t, a_{t+1}) \tag{5.1}$$

(5.1) shows that the payoff to agent t from any action pair depends upon two components. The first is common to all agents, whereas the second, x_t, is private information. We make the following assumptions regarding this private information.

Assumption 4. c is sufficiently small that assumption A1 holds for all realizations of x_t, i.e. $u_t(.)$ is decreasing in its first argument and decreasing in its second argument.

Assumption 5. μ is atomless, absolutely continuous with respect to Lebsegue measure, and the associated density f is continuous.

A behavior strategy for agent t is now a Borel measurable function, $\sigma_t{:}B^t \times Z \rangle \Delta A$. Two behavior strategies of agent t are *equivalent* if, for every b^t in B^t, they differ on a subset of Z of μ-measure zero. Let $s_t{:}B^t \rangle \Delta A$, and let S_t be the set of all such functions s_t. If σ_t is a behavior strategy, σ_t induces an element s_t of S_t, defined by $s_t = \int \sigma_t d\mu$. Call s_t the aggregate of σ_t. If player t plays σ_t, to an outside observer, and to all players $\tau < t$, it seems as though t plays the aggregate s_t of σ_t.

A behavior strategy profile, $\langle\sigma\rangle$, is a sequence of behavior strategies, $\langle\sigma_t\rangle$. Associated with this is the sequence of aggregates, $\langle s_t\rangle$. $\langle\sigma_t\rangle$ is a sequential equilibrium, if for every t, and for every realization of x_t,

$$u_t(\sigma_t,s_{t+1},/b^t,x_t) \geq u_t(a_t,s_{t+1}/b^t,x_t) \tag{5.2}$$

The following theorem says that altruistic behavior is impossible with these small perturbations:

Theorem 3. If $\langle\sigma\rangle$ is an equilibrium of the disturbed game, then under $\langle\sigma\rangle$, every player transfers zero after every observed history, and for every realization of x_t.

Before proving the theorem, we provide some intuition by considering why the mixed strategy equilibrium of the previous section cannot be approximated in the disturbed game. We simplify the game by allowing only two actions, 0 and a*. Recall that the mixed strategy's prescription after the observed histories (a*,0) and (0,0):

$(a^*,0) \longrightarrow 0$

$(0,0) \longrightarrow$ a* with probability p, 0 with probability (1−p)

To keep things simple, we perturb only one payoff, the payoff $u(0,a^*)$, so that the payoffs of agent t are:

$$u_t(0,a^*) = u(0,a^*) + x_t$$

$$u_t(x,y) = u(x,y) \text{ for all other } (x,y) \text{ in } A^2 \qquad (5.3)$$

where x_t is i.i.d. on [−c,c] with a uniform density.

Let the last observed history be (a,0), where a is either a* or 0. Consider agent t's payoff from the two actions, a* and 0:

$$u_t(a^*,s_{t+1}/(a,0) = u(a^*,a^*) \qquad (5.4)$$

$$u_t(0,s_{t+1}/(a,0) = pu_t(0,a^*) + (1-p)\, u(a^*,a^*) \qquad (5.5)$$

The difference in payoff between the two actions, a* and 0, is:

$$u_t(a^*,s_{t+1}/(a,0) - u_t(0,s_{t+1}/(a,0) = x_t\, p \qquad (5.6)$$

(5.6) shows that agent t has a unique best response unless $x_t = 0$, i.e. for almost all realizations of x_t. Further, t will choose a* with probability one if $x_t<0$, and 0 with probability one if $x_t>0$. Hence the aggregate of this strategy conditional on (a,0) is :

$$s_t(a,0) = a^* \text{ with probability } 1/2,\ 0 \text{ with probability } 1/2 \qquad (5.7)$$

In other words, the aggregate strategy, $s_t(0,0) = s_t(a^*,0)$, since $s_t(a,0)$ is uniquely determined by (5.7), no matter whether $a = a^*$ or $a = 0$. However, the strategy requires t to take *different* actions at (0,0) and $(a^*,0)$. Hence, the strategy cannot be an equilibrium.

We now turn to proving the theorem, but first we need the following lemma which is straight-forward since μ is atomless.

Lemma 3. Let $\langle\sigma\rangle$ be a equilibrium and let b^t be any observed history. For almost every realization of x_t, player t has a unique optimal action in A, and hence chooses a pure action.

We extend the definition of section 2 regarding the measurability of strategies in the following manner.

Definition: σ_t is measurable w.r.t. $\sim_i$ if for any $b \sim_i b'$, the set of x_t such that (5.8) does not apply has μ-measure zero:

$$\sigma_t(x_t,b) = \sigma_t(x_t,b') \qquad (5.8)$$

If σ_t is measurable w.r.t $\sim_i$, it follows that the associated aggregate s_t is likewise measurable w.r.t. $\sim_i$, i.e. if $b \sim_i b'$, $s_t(b) = s_t(b')$

Proof of the theorem: The proof is again by backward induction. Given any t>m, we show that σ_{t+i} is measurable with respect to $\sim_i$, $i=0,1,2,\ldots,m$.

i) σ_{t+m} is measurable w.r.t. $\sim_m$: by definition, since $\sim_m$ is the finest partition, where each set in the partition is singleton.

ii)If σ_{t+i+1} is measurable w.r.t. $\sim_{i+1}$, then σ_{t+i} is measurable w.r.t. $\sim_i$, $i= 0,1,..,m-1$.

To see this, note that if σ_{t+i+1} is measurable w.r.t $\sim_{i+1}$, then s_{t+i+1} is measurable w.r.t. $\sim_{i+1}$. Let $b \sim_i b'$, and let x_{t+i} be any realization of X_{t+i}. We claim that :

$$u_{t+i}(\sigma_{t+i}, s_{t+i+1} /b, x_{t+i}) = u_{t+i}(\sigma_{t+i}, s_{t+i+1} /b', x_{t+i}) \qquad (5.9)$$

The proof of this claim mimics the proof of (2.4) in theorem 2, and is hence omitted.

However, by lemma 3, for almost all realizations of x_{t+i}, agent t+i has a unique pure optimal action. Hence (5.9) and lemma 3 imply that for almost all realizations of x_{t+i}:

$$\sigma_{t+i}(b,x_{t+i}) = \sigma_{t+i}(b',x_{t+i}) \qquad (5.10)$$

Hence σ_{t+i} is measurable w.r.t. $_{\sim i}$.

By induction, for all t, σ_t and s_t are measurable on $_{\sim 0}$, and can hence be written as functions from Z to ΔA. By Assumption A1 and backward induction to earlier dates, each agent must choose to transfer zero irrespective of the realization of x_t. □

The basic problem with the mixed strategy equilibrium is that agent t is required to take different (probability distributions over) actions at different information sets. Since future agents cannot distinguish these information sets, agent t must be induced to be indifferent between these actions. Once payoffs are perturbed, these indifferences cannot persist, since for almost all realizations of the private information, the agent has a unique best action. Consequently, the actions of the agent must depend only upon the private information, and not upon the observed history.

6. PERFECT DIVISIBILITY OF THE TRANSFERABLE COMMODITY

We briefly examine the implications of allowing the transferable commodity to be perfectly divisible. In our view, perfect divisibility is an unreasonable assumption. The analysis here is mainly in order to demonstrate that the difficulties with sustaining altruistic outcomes do not stem from this assumption.

Let $A = [0,e]$ and let $u: A \times A \rightarrow R$ be the payoff function, which satisfies A6.

Assumption A6. u(.) is continuous and satisfies A1, i.e. is strictly decreasing in its first argument, strictly increasing in its second argument.

As an example of such a function, let $A = [0,2]$, with u(.) given by:

$$u(x,y) = (2-x)(y+k) \qquad (6.1)$$

where $0.1 \geq k > 0$

Proposition 1: If social memory is positive, there exists an efficient pure strategy equilibrium where every agent transfers a*.

Proof: Define the function $\phi:[0,a^*] \longrightarrow [0,a^*]$ by the equation:

$$u(a,\phi(a)) = u(a^*,a^*) \qquad (6.2)$$

We first show that ϕ is well defined. Let $a \in [0,a^*]$. By the definition of a*:

$$u(a,a) \leq u(a^*,a^*) \qquad (6.3)$$

Further, since u(.) is strictly decreasing in its first argument:

$$u(a,a^*) \geq u(a^*,a^*) \qquad (6.4)$$

Since u(.) is continuous, the intermediate value theorem implies that there exists a $\phi(a)$, $a^* \geq \phi(a) \geq a$, satisfying (6.2). Since u is strictly increasing in its second argument, this solution is unique, so that the function ϕ is well defined.

We construct a pure strategy supporting a* which conditions only on the last observed action as follows:

$s_1 = a^*$

If $s_{t-1} \geq a^*$, $s_t = a^*$

If $s_{t-1} < a^*$, $s_t = \phi(s_{t-1})$

It may be verified that this strategy profile constitutes an equilibrium. No matter what the observed history, the strategy ensures a payoff of $u(a^*,a^*)$. If the agent deviates by choosing any other transfer in $[0,a^*]$, she still gets only $u(a^*,a^*)$. If she deviates by choosing a transfer greater than a^*, she only gets a^* in the next period and hence her utility is less than $u(a^*,a^*)$. □

Nevertheless, this strategy also fails to survive if we perturb the payoff function. Let u_t be given by:

$$u_t(x,y) = u(x,y) + \alpha_t x \qquad (6.5)$$

where α_t is i.i.d with an uniform density on $[-c,c]$, where c is small. Assume further that u_t is strictly concave. We do not present a formal proof of the claim that this strategy fails to be an equilibrium of the disturbed game. However, the argument is clear.For every realization of α_t, and given agent t+1's strategy, there is a unique optimal choice for agent t. This is independent of the observed action of agent t-1. Hence t will not condition her action on the actions of t-1, which in turn makes it optimal for t-1 to choose to transfer zero.

7. THE INFORMATIONAL ROLE OF MONEY

Will a modification of our assumptions help in sustaining altruism? In this section we suggest that money may play an informational role in this context. Re-interpret the basic model in the following way. Money is issued in period one by the old agent. In each period, the young agent may either trade the consumption good for the old agent's money, or she may decline to do so, and issue her own money in the next period. Accepting money corresponds to transferring one fish in our simple example of section 2, i.e. the action 1. Refusing to accepts the money offered and issuing your own money,corresponds to selfish behavior, i.e. the action 0. .In each period, the young agent has no knowledge of the preceding actions. However, she may discern the date of issue of the money that is offered to her. This model may also allow for many agents in each period, where young agents collectively decide whether to honour the money or not.

This is an alternative informational constraint from the one that we have considered so far. Agent t's observed history, b^t, is now simply a date, τ with $\tau < t$. The set of possible observed histories, equals $\{1,2,,\ldots,t-1\}$, and we denote this set by $\underline{t}$. A pure strategy is a function, s_t: $\underline{t} \rightarrow A$.

The information that any agent has is quite restricted. Given that agent t observes τ belonging to $\underline{t}$, she can infer that:

i) agent τ has behaved selfishly and transferred zero.

ii) every agent after τ has accepted the old's money, and therefore made a positive transfer.

iii) no inference can be made regarding the behavior of agents at dates before τ.

Although information is limited, memory is not bounded in this case. If every agent accepts the money issued at date 1, future agents will have information about all the actions taken by all agents. The unboundedness of memory allows us to support altruistic behavior.

Theorem 4. There exists an altruistic equilibrium in this economy, where money issued at date t* (t* is arbitrary) is accepted by every subsequent agent. In any such equilibrium, if any agent deviates and refuses to accept the money issued at t*, money is never accepted subsequently. Proof: Define $\langle s_t \rangle$ as follows:

$$s_t = 0 \text{ if } t \leq t^*$$

$$= 1 \text{ if } t > t^* \text{ and } b^t = t^*$$

$= 0$ if $t > t^*$ and $b^t > t^*$

Since $u(0,0) > u(1,0)$ every agent who is required to choose 0 is choosing optimally. Further since $u(1,1) > u(0,0)$, every agent after t^* who observes t^* is choosing optimally. This verifies that the strategy profile ⟨s⟩ is an equilibrium.

We now show that the date t^* is unique in any equilibrium. Let $\langle s_t \rangle$ be an equilibrium. Suppose that there exists $t^{**} > t^*$ such that $s_{t^{**}+1}(t^{**}) = 1$. We now show that agent t^{**} will not accept the money issued by agent t^*. By not accepting this money, agent t^{**} gets utility $u(0,1)$, since her money will be accepted, whereas by accepting t^*, she gets $u(1,1)$ which is strictly less.This contradicts the assumption that t^* is accepted in equilibrium. □

Remark: The proof can be generalized so that we can show that t^* is not accepted, then money cannot be accepted at any future date no matter what the history.

The equilibrium exists, and moreover is robust to small perturbations in the utility functions of agents. However, the monetary equilibrium is fragile in another sense, since it is vulnerable to a "crazy" behavior by any one generation. If any generation were to be foolish enough to deviate, the money never regains its value. In other words, the loss in confidence is permanent. This fragility seems necessary for the original equilibrium to be self-enforcing. Given informational constraints, crises of confidence must be devastating, and long lasting.

8. CONCLUSIONS

The message of our paper runs counter to much of the recent literature on dynamic games played by overlapping generations of players. Recent papers by Cremer (1986), Salant (1991), Kandori (1992) and Smith (1992) analyze such games and find that cooperation, and versions of the Folk theorem apply. We find that informational constraints, incorporated as an upper bound on social memory in the classical model of Sameulson, make inter-generational altruism impossible, and allow only the inefficient zero-transfer equilibrium. The equilibria which support altruistic behavior fail because they require agents to take different actions at different information sets. This information does not directly affect the agent's utility. Further, each agent has superior information about the past than any of her successors. This strategic advantage makes it impossible to sustain this behavior. Although this paper suggests that money may be an informational device for overcoming the boundedness of social memory, these monetary equilibria are also not robust in a different sense. Once money loses its value, it never recovers it.

TABLE 1

	OBSERVED HISTORY	ACTION THIS PERIOD	INDUCED HISTORY	NEXT-PERIOD ACTION	PAYOFF
	(1,1)	1	(1,1)	1	u(1,1)
	(1,0)	0	(0,0)	1	u(0,1)
I	(0,1)	1	(1,1)	1	u(1,1)
	(0,0)	1*	(0,1)	1	u(1,1)

Action after (0,0) not optimal; choosing 0 induces (0,0) and 1 next period, giving u(0,1)

	OBSERVED HISTORY	ACTION THIS PERIOD	INDUCED HISTORY	NEXT-PERIOD ACTION	PAYOFF
	(1,1)	1	(1,1)	1	u(1,1)
	(1,0)	0	(0,0)	0	u(0,0)
II	(0,1)	0*	(1,0)	0	u(0,0)
	(0,0)	0	(0,0)	0	u(0,0)

Action after (0,1) not optimal; choosing 1 induces (1,1) and 1 next period, giving u(1,1)

	OBSERVED HISTORY	ACTION THIS PERIOD	INDUCED HISTORY	NEXT-PERIOD ACTION	PAYOFF
	(1,1)	1	(1,1)	1	u(1,1)
	(1,0)	0	(0,0)	0	u(0,0)
III	(0,1)	1	(1,1)	1	u(1,1)
	(0,0)	0*	(0,0)	0	u(0,0)

Action after (0,0) not optimal; choosing 1 induces (0,1) and 1 next period, giving u(1,1)

	OBSERVED HISTORY	ACTION THIS PERIOD	INDUCED HISTORY	NEXT-PERIOD ACTION	PAYOFF
	(1,1)	1	(1,1)	1	u(1,1)
	(1,0)	0	(0,0)	1	u(0,1)
IV	(0,1)	0	(1,0)	1	u(0,1)
	(0,0)	1*	(0,1)	0	u(1,0)

Action after (0,0) not optimal; choosing 0 induces (0,0) and 1 next period, giving u(0,1)

* shows sub-optimal action

REFERENCES

Cremer, J., 1986, Cooperation in ongoing organizations, *Quarterly Journal of Economics* 33-49.

Esteban, J., and J. Sakovics, 1993, Intertemporal transfer institutions, *Journal of Economic Theory.*

Hammond, P. , 1975, Charity: Altruism or cooperative egoism?, in E. S. Phelps (ed) *Altruism, Morality and Economic Theory,* New York: Russell Sage Foundation.

Harsanyi, J., 1973, Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points, *International Journal of Game Theory* 2, 1-23.

Kandori, M., 1992, Repeated games played by overlapping generations of players, *Review of Economic Studies*, 59, 81-92.

Salant, D., 1991, A repeated game with finitely lived overlapping generations of players, *Games and Economic Behavior* 3, 244-259.

Samuelson, P., 1958, An exact consumption loan model of interest with or without the social contrivance of money, *Journal of Political Economy* 66, 467-82.

Smith, L., 1992, Folk theorems in overlapping generations games, *Games and Economic Behavior*, 4, 426-449.

van Damme, E., 1991, *Stability and Perfection of Nash Equilibria*, Berlin: Springer Verlag.

CENTRE FOR DEVELOPMENT ECONOMICS WORKING PAPER SERIES

Number	Author(s)	Title
1	Kaushik Basu Arghya Ghosh Tridip Ray	The Babu and The Boxwallah : Managerial Incentives and Government Intervention
2	M. N. Murty Ranjan Ray	Optimal Taxation and Resource Transfers in a Federal Nation
3	V. Bhaskar Mushtaq Khan	Privatization and Employment : A Study of The Jute Industry in Bangladesh
4	V. Bhaskar	Distributive Justice and The Control of Global Warming
5	Bishnupriya Gupta	The Great Depression and Brazil's Capital Goods Sector : A Re-examination
6.	Kaushik Basu	Where There Is No Economist : Some Institutional and Legal Prerequisites of Economic Reform in India
7.	Partha Sen	An Example of Welfare Reducing Tariff Under Monopolistic Competition
8.	Partha Sen	Environmental Policies and North-South Trade : A Selected Survey of The Issues
9.	Partha Sen Arghya Ghosh Abheek Barman	The Possibility of Welfare Gains with Capital Inflows in A Small Tariff-Ridden Economy
10.	V. Bhaskar	Sustaining Inter-Generational Altruism when Social Memory is Bounded