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Debt Financing with Limited Liability and Quantity Competition

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INTRODUCTION

In this paper we will analyse the limited liability[2] effect of a debt financed firm for its output strategies. In a very simplified framework we will first examine the implications of the Brander and Lewis (1986) proposition, that in a homogeneous product duopoly in which financial and output decisions follow in a sequence, the limited liability nature of debt may commit a debt financed firm to be more aggressive in quantity competition. Then we will see that the introduction of debt may change the nature of the product (from strategic substitutes to strategic complements, at least for a range). This also raises the possibility of excess capacity being used as an entry deterring strategy (Dastidar and Sengupta, 1993). The basic set up is given below.

A two stage sequential duopoly game in a homogeneous product market will be considered. In the first stage the two firms decide upon financial structure (i.e. the debt - equity ratio). In the second stage they select output levels taking as given the financial composition determined in the first stage. The output decisions of firms are made before the realisation of a random variable reflecting variations in demand or costs. Once profits

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Limited liability is the restriction of an owner's loss in a business to the amount of capital that the owner has contributed to the company. This limitation allows the people to invest in a corporation without fear of loosing all their personal assets should the corporation become unprofitable. Limited liability was an important factor in the rise of large corporations.

are determined, firms are liable to pay debt claims out of operating profits, if possible. If profits are insufficient to meet all debt obligations, the firm goes bankrupt and its assets are turned over to the bondholders.

We will now provide the technicalities of our argument through a model. We will show that the aggressive behaviour may be alternatively interpreted in terms of the nature of reaction functions. This would allow us to show that when firms have limited liability, some standard conclusions need not follow. Before going into the model let us briefly review the concepts of strategic substitutes and complements.

In the literature conventional substitutes and complements can be distinguished by whether a more "aggressive" strategy by firm A (example lower price in price competition, greater quantity in quantity competition, increased advertising etc.) lowers or raises B's total profits. Strategic substitutes and complements are analogously defined by whether a more aggressive strategy by A lowers or raises B's marginal profits (see Bulow, Klemperer and Geanakoplos, 1985b). One can say that for output competition reaction functions of the firms are downward sloping if the products are strategic substitutes while it is upward sloping for the strategic complements. The shapes of the reaction function functions, in general will depend on the primitives of the of the models including the nature of competition (that is price or quantity competition), the form of the demand curves, the cost structures and others. Our analysis will suggest that such classifications may no longer remain valid when one considers the possibility of debt financing by the firms. The reaction functions of an equity financed firm may look very different (as we have seen) from that of a debt financed firm. Moreover the shape and position of the reaction functions may also depend on the extent of debt financing. We will provide an example where the reaction function in quantity competition is downward sloping for an equity financed firm, but is upward sloping (at least for a range) for a debt financed one. This means that the inclusion of debt changes the product from strategic substitutes to strategic complements. This can create problem for the analysts since the extent of debt financing typically cannot be taken as a primitive of any model but has to be solved for in the context of a particular model.

THE MODEL

Consider a homogeneous product duopoly with symmetric costs given by $C_i(Q_i) = wQ_i$, where Q_i is the output of the ith firm, and w is the constant marginal cost.

Let $R_i(Q_1, Q_2, \theta)$ be the revenue function of the ith firm, where θ is a random variable with distribution function $F(\theta)$.

The following assumptions are made :

(a) $R_i(Q_1, Q_2, \theta)$ is concave in Q_i given Q_j and θ, for $\forall i$ and $j \neq i$.

(b) $R_i(Q_1,Q_2,\theta)$ and $MR_i(Q_1,Q_2,\theta)$ are decreasing in Q_j and increasing in θ, $\forall i$ and $j \neq i$. The uncertainty is resolved only after the output decisions have been made by the firms.

Let D_{fi} and D_{mi} be the face value and market value (respectively) of the debt raised by firm i. In other words D_{mi} is the amount which the bondholders give to the firm which in turn promises to pay back D_{fi}.

(c) Our formulation assumes that the firm has a limited liability contract vis-a-vis the debtholders, that is the debtholders receive whatever is left over from the revenue, net of the costs incurred in the production stage upto a maximum of D_{fi}. Here it may be noted that for simplicity we have assumed that the asset value of the firm is zero, as if assets are completely used up in the production of output. Creditors can therefore, collect only current operating profits if the firm becomes insolvent.

Let us denote the profit of the ith firm by $\pi_i(Q_1,Q_2,\theta)$. Then
$\pi_i(Q_1,Q_2,\theta) = R_i(Q_1,Q_2,\theta) - wQ_i + D_{mi}$

The above is what the firm receives for output pair (Q_1,Q_2) and state of the world θ, in the second stage. D_{mi} is the amount it has received from the debtholders in the first stage. If the firm is completely equity financed then $D_{mi} = 0$. In any case when the firm is choosing output in the second stage D_{mi} is given.

For any output pair (Q_1,Q_2) the expected payoff to the firm i is given by $\int \max\ [\pi_1(Q_1,Q_2,\theta) - D_{f1},\ 0]dF(\theta)$

One may note that the above is really the payoff to the shareholders (i.e. the owners) of the firms who receive profits net of the face value of debt. If profits are insufficient to cover debts, the debt holders are paid all there is and the shareholders get zero.

<u>The Example</u>

Let the demand be given by $P = [\ A - (Q_1+Q_2)] + \theta^2$, where θ is a random variable with $a \le \theta \le b$, and θ follows an uniform density function given by $f(\theta) = \frac{1}{b-a}$. The cost functions are given by $C_1(Q_1) = wQ_1$.

We will concentrate on 1's reaction function. Now for each output pair (Q_1,Q_2) the profit of firm 1 is given by,

$$\pi_1(Q_1,Q_2,\theta) = R_1(Q_1,Q_2,\theta) - C_1(Q_1) + D_{m1}$$
$$= (AQ_1 - Q_1^2 - Q_1Q_2 + \theta^2 Q_1) - wQ_1 + D_{m1}$$

Now $MR_1(Q_1,Q_2,\theta) = (A - 2Q_1 - Q_2 + \theta^2)$

and $MC_1(Q_1) = w$

One may note that both revenue and marginal revenue are increasing in the random variable θ.

Define $\psi(Q_2) = \arg_{Q_1 \ge 0} \max \int_a^b \pi_1(Q_1,Q_2,\theta) f(\theta)\ d\theta$

That is $\psi(Q_2)$ is the solution in Q_1 of the following :

$$\int_a^b [\ MR_1(Q_1,Q_2,\theta) - MC_1(Q_1)\ ]\frac{1}{b-a}\, d\theta = 0$$

$$\Rightarrow \int_a^b [\ (A - 2Q_1 - Q_2 + \theta^2) - w\ ]\frac{1}{b-a}\, d\theta = 0$$

$$\Rightarrow [\ (A - 2Q_1 - Q_2 - w)\theta + \tfrac{1}{3}\theta^3\ ]_a^b = 0$$

$$\Rightarrow (A - 2Q_1 - Q_2 - w)(b-a) + \tfrac{1}{3}(b^3 - a^3) = 0$$

$$\Rightarrow (A - 2Q_1 - Q_2 - w)\ (b - a) + \tfrac{1}{3}(b - a)\ (b^2 + a^2 + ab) = 0$$

$$\Rightarrow (A - 2Q_1 - Q_2 - w) + \tfrac{1}{3}(b^2 + a^2 + ab) = 0$$

Therefore we can write that,

$$\psi(Q_2) = \tfrac{1}{2}\ [A - Q_2 - w + \tfrac{1}{3}(b^2+a^2+ab), \text{ if } Q_2 < A - w + \tfrac{1}{3}(b^2 + a^2+ab)$$

$$= 0, \text{ otherwise.}$$

One may note that $\psi(Q_2)$ is firm 1's reaction function when it is completely equity financed. (This is because $\psi(Q_2)$ gives those choices of firm 1's output which maximises it's exepected profits given Q_2.) It may be noted that $\psi'(Q_2) < 0$. Therefore the equity financed firm's reaction function is downward sloping and so the products are strategic substitutes.

Now suppose that firm 1 is debt financed with limited liability. Let the face value of its debt be D.

Define $\beta(Q_2,\theta) = \arg_{Q_1 \geq 0} \max \pi_1(Q_1,Q_2,\theta)$

Here it may be noted that $\beta(Q_2,\theta)$ is firm 1's best response to firm 2's output level Q_2 when a particular state of the world θ prevails.

Now $\beta(Q_2,\theta)$ is the solution in Q_1 of the following :

$MR_1(Q_1,Q_2,\theta) - MC_1(Q_1) = 0$

$\Rightarrow A - 2Q_1 - Q_2 + \theta^2 - w = 0$

$\Rightarrow \beta(Q_2,\theta) = \frac{1}{2}(A - Q_2 - w + \theta^2)$, if $Q_2 < A - w + \theta^2$

$= 0$, otherwise.

Assume that $\pi_1(\beta(0,a),\ 0,\ a) < D < \pi_1(\beta(0,b),\ 0,\ b)$ -----(d)

Now it may be noted that,

$\pi_1(\beta(Q_2,\theta),\ Q_2,\ \theta) = \beta(Q_2,\theta)\ [(A - \beta(Q_2,\theta) - Q_2 + \theta^2 - w\ ]$

$= \frac{1}{2}(A - Q_2 - w + \theta^2)\ [A - \frac{1}{2}(A - Q_2 - w + \theta^2) - Q_2 - w + \theta^2]$

Therefore $\pi_1(\beta(Q_2,\theta),\ Q_2,\ \theta) = \frac{1}{4}(\ A - Q_2 - w + \theta^2\ )^2$ -----(1)

From (1) it is clear that $\pi_1(\beta(Q_2,\theta),\ Q_2,\ \theta)$ is decreasing in Q_2 and increasing in θ.

Lemma 1 $\exists\ \bar{Q}_2 > 0$ and $\hat{\theta}(Q_2)$ s.t. $\pi_1(\beta(Q_2,\hat{\theta}(Q_2)),\ Q_2,\ \hat{\theta}(Q_2)) = D$, where $Q_2 \in [0,\ \bar{Q}_2]$.

Proof From (1) we know that $\pi_1(\ \beta(Q_2,\theta),\ Q_2,\ \theta)$ is increasing in θ and decreasing in Q_2.

And from (d) we have, $\pi_1(\beta(0,a),\ 0,\ a) < D < \pi_1(\beta(0,b),\ 0,\ b)$

Consider $Q_2 = 0$. Since (f) holds we have some $\hat{\theta}(0)$, where $a < \hat{\theta}(0)$

$< b$, for which $\pi_1(\beta(0,\hat{\theta}(0),\ 0,\ \hat{\theta}(0)) = D$ is true.

Now consider some $Q_2 > 0$, where Q_2 is small enough. Since $\hat{\theta}(0) < b$, there exists some $\hat{\theta}(Q_2) \leq b$, s.t. $\pi_1(\beta(Q_2,\hat{\theta}(Q_2)),\ Q_2,\ \hat{\theta}(Q_2)) = D$. That is $\exists\ \bar{Q}_2 > 0$, s.t. $\pi_1(\beta(\bar{Q}_2,\hat{\theta}(\bar{Q}_2)),\ \bar{Q}_2,\ \hat{\theta}(\bar{Q}_2)) = D$. ■

Define $\alpha(Q_2) = \arg_{Q_1 \geq 0} \max \int_a^b \max\ [\pi_1(Q_1,Q_2,\theta) - D,\ 0] f(\theta)\ d\theta$

It may be noted that $\alpha(Q_2)$ is firm 1's reaction function when it is debt financed with limited liability. We will now analyse the nature of $\alpha(Q_2)$.

Lemma 1 $\Rightarrow$ for $Q_2 \in [0,\ \bar{Q}_2]$, we have $\pi_1(\beta(Q_2,\hat{\theta}(Q_2)),\ Q_2,\ \hat{\theta}(Q_2)) = D$. The above means that if $\theta = \hat{\theta}$, then at that state (of demand), the maximum possible π_1 given Q_2 is equal to D. Therefore when $\theta < \hat{\theta}$ (i.e. in states lower than $\hat{\theta}$), the maximum possible π_1 is less than D. This is because in lower states, maximum profits for firm 1 given Q_2 are smaller. Therefore we can write that,

$$\int_a^{\hat{\theta}} [\pi_1(Q_1,Q_2,\theta) - D]\ f(\theta)\ d\theta < 0$$

Now $\alpha(Q_2) = \arg_{Q_1 \geq 0} \max \int_a^b \max\ [\pi_1(Q_1,Q_2,\theta) - D,\ 0] f(\theta)\ d\theta$

$$= \arg_{Q_1 \geq 0} \max \left[\int_a^{\hat{\theta}} \max\ \{\pi_1(Q_1,Q_2,\theta) - D,\ 0\} f(\theta)\ d\theta + \int_{\hat{\theta}}^b \max\ \{\pi_1(Q_1,Q_2,\theta) - D,\ 0\} f(\theta)\ d\theta \right]$$

Since $\int_{a}^{\hat{\theta}} \max\{\pi_1(Q_1,Q_2,\theta) - D, 0\} f(\theta)\, d\theta = 0, \ \forall \ Q_1, Q_2$

Therefore $\alpha(Q_2) = \arg_{Q_1 \geq 0} \max \int_{\hat{\theta}}^{b} \max[\pi_1(Q_1,Q_2,\theta) - D, 0] f(\theta)\, d\theta$

The above means that the lower states of the world are irrelevant for the equityholders (or the owners) as profits fail to cover debt. One may recall that due to limited liability owners prefer large (positive) profits net of debt to smaller ones, but are indifferent between small and large losses. As debtholders are residual claimants in the bad states of the world, the equityholders go in for profit maximisation in the higher states.

Now $\alpha(Q_2)$ is the solution in Q_1 of the following,

$$\int_{\hat{\theta}}^{b} [\ MR_1(Q_1,Q_2,\theta) - MC_1(Q_1)\ ]\ f(\theta) d\theta = 0$$

$$\Rightarrow \int_{\hat{\theta}}^{b} [\ (A - 2Q_1 - Q_2 + \theta^2 - w\ ]\ \frac{1}{b-a}\ d\theta = 0$$

$$\Rightarrow [(A - 2Q_1 - Q_2 - w)\theta + \tfrac{1}{3}\theta^3\ ]_{\hat{\theta}}^{b} = 0$$

$$\Rightarrow (A - 2Q_1 - Q_2 - w)\ (b - \hat{\theta}) + \tfrac{1}{3}\ (b^3 - \hat{\theta}^3) = 0$$

$$\Rightarrow (A - 2Q_1 - Q_2 - w)\ (b - \hat{\theta}) + \tfrac{1}{3}\ (b - \hat{\theta})\ (b^2 + \hat{\theta}^2 + b\hat{\theta}) = 0$$

$$\Rightarrow (A - 2Q_1 - Q_2 - w) + \tfrac{1}{3}\ (b^2 + \hat{\theta}^2 + b\hat{\theta}) = 0 \ (\text{since } b > \hat{\theta})$$

$$\Rightarrow \alpha(Q_2) = \tfrac{1}{2}\ [\ A - Q_2 - w + \tfrac{1}{3}\ (b^2 + \hat{\theta}^2 + b\hat{\theta})\ ]$$

We know that $\alpha(Q_2)$ is the reaction function of firm 1 when it is debt financed with limited liability. Comparing it with $\psi(Q_2)$, it's reaction function when it is completey equity financed we get that $\alpha(Q_2) > \psi(Q_2)$, since $\hat{\theta} > a$. *That is a debt financed firm with limited liability is more aggressive in quantity competition* (see Brander and Lewis, 1986).

From Lemma 1 we know that $\pi_1(\beta(Q_2,\hat{\theta}(Q_2), \; Q_2, \; \hat{\theta}(Q_2)) = D$. And using (1) we get,

$$\tfrac{1}{4}(A - Q_2 - w + \hat{\theta}^2)^2 = D$$
$$\Rightarrow (A - Q_2 - w + \hat{\theta}^2)^2 - 4D = 0$$

Let $g(Q_2, \hat{\theta}) = (A - Q_2 - w + \hat{\theta}^2)^2 - 4D$

That is $g(Q_2, \hat{\theta}) = 0$ is the implicit function relating $\hat{\theta}$ and Q_2.

Now $\partial g(Q_2, \hat{\theta})/\partial Q_2 = -2 \; (A - Q_2 - w + \hat{\theta}^2)$ and

$\partial g(Q_2, \hat{\theta})/\partial\hat{\theta} = 4\hat{\theta} \; (A - Q_2 - w + \hat{\theta}^2)$

Now $d\hat{\theta}/dQ_2 = (-\partial g(Q_2, \hat{\theta})/\partial Q_2) \; / \; (\partial g(Q_2, \hat{\theta})/\partial\hat{\theta})$

$$= 2 \; (A - Q_2 - w + \hat{\theta}^2) \; / \; 4\hat{\theta} \; (A - Q_2 - w + \hat{\theta}^2)$$

Therefore $d\hat{\theta}/dQ_2 = 1/2\hat{\theta}$

We know that $\alpha(Q_2) = \frac{1}{2} \; [\; A - Q_2 - w + \frac{1}{3} \; (b^2 + \hat{\theta}^2 + b\hat{\theta}) \;]$

Therefore $\alpha'(Q_2) = \frac{1}{2} \; [\; -1 + \frac{1}{3} \; (2\hat{\theta} + b) \; d\hat{\theta}/dQ_2]$

$$\Rightarrow \alpha'(Q_2) = \tfrac{1}{2} \; [\; -1 + \tfrac{1}{3} \; (2\hat{\theta} + b) \; 1/2\hat{\theta} \;]$$
$$= \tfrac{1}{2} \; [\; -1 + \tfrac{1}{3} \; (1 + b/2\hat{\theta}) \;]$$

Now we will provide a specific example to show that $\alpha(Q_2)$ can be upward sloping (at least for a range).

Let $A = 6$, $w = 1$, $a = 1$ and $b = 15$ and $D = 16$

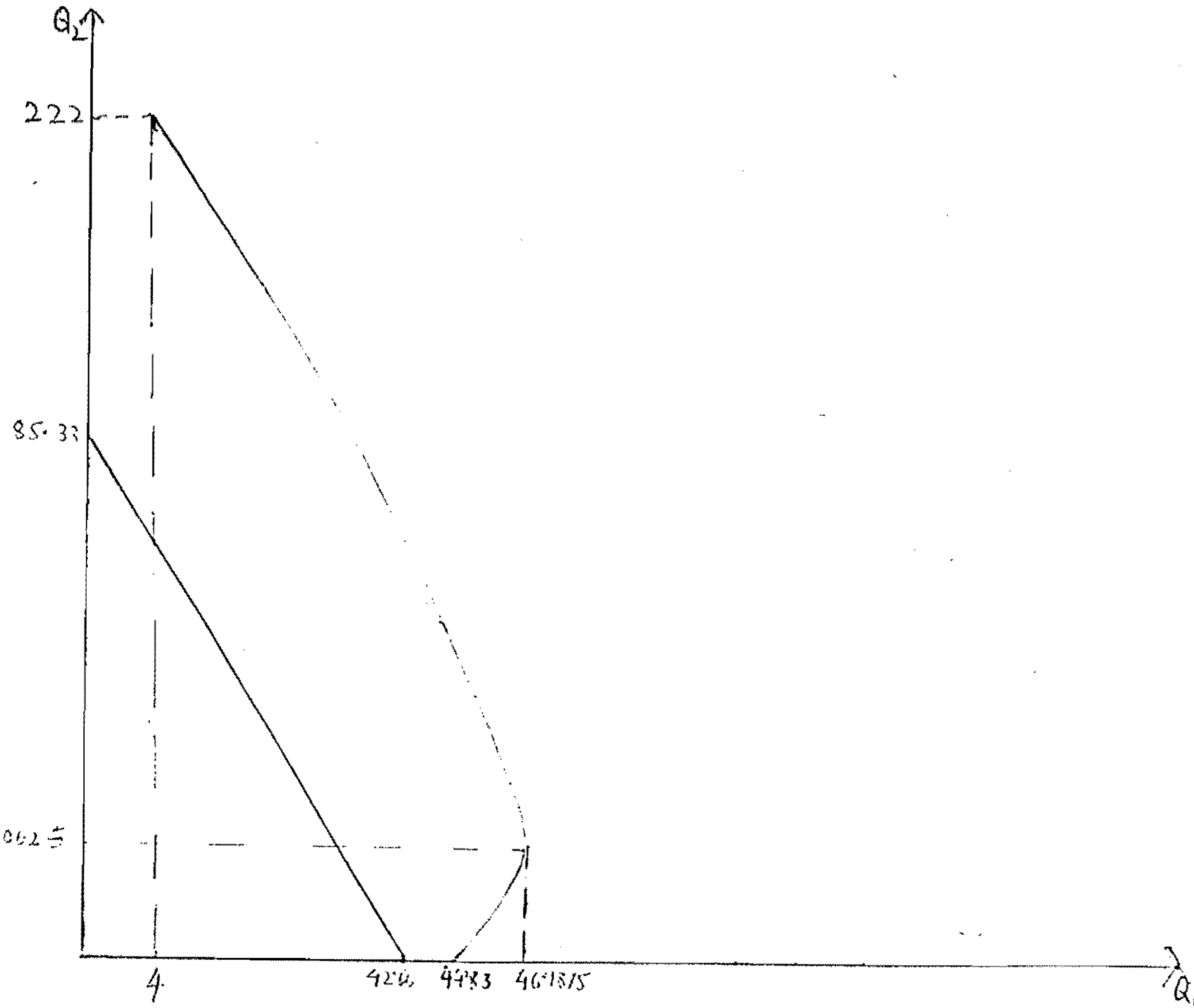
It may be noted that $\hat{\theta}(0) = 1.732$, $\hat{\theta}(11.0625) = 3.75$ and $\hat{\theta}(222) =$

15. Routine calculations lead to the following conclusions.

For $0 \leq Q_2 < 11.0625$, $\alpha'(Q_2) > 0$ and

for $11.0625 \leq Q_2 < 222$, $\alpha'(Q_2) < 0$ and $\bar{Q}_2 = 222$.

Therefore for the range $Q_2 \in [0, 11.0625)$, firm 1's reaction function is upward sloping. Figure (1) below shows firm 1's reaction function both when it is completely equity financed and when it is debt financed with limited liability.



That is firm 1's reaction function is upward sloping (at least for a range) when it is debt financed with limited liability, whereas it is downward sloping throughout if it is completely equity financed. The example proves our point that the inclusion of debt may change the nature of product (from strategic substitutes to strategic complements).

One may also note the following,

(i) The presence of upward sloping segment in the reaction function is a necessary condition for excess capacity being used as an entry deterring strategy (see Bulow, Klemperer and Geanakoplos, 1985a)

(ii) In addition this example is also a counter example to the Brander and Lewis (1986) contention that linear demand curves and a random variable having uniform distribution function will lead to downward sloping reaction function for a debt financed firm with limited liability.

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