Immiserizing Growth in a Model of Trade with Monopolistic Competition

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IMMISERIZING GROWTH IN A MODEL OF TRADE

WITH MONOPOLISTIC COMPETITION

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ABSTRACT

In models of trade with monopolistic competition an externality is present because of mark-up pricing. An example is provided in this paper where because of this a growth in stock of a factor of production lowers welfare.

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1. **INTRODUCTION**

Ever since Bhagwati's seminal paper on immiserizing growth (see Bhagwati (1958)), international trade theory has thrown up examples of welfare-reducing growth. This literature has shown that in the presence of a distortion, growth in the stocks of factors of production may be welfare-reducing.

The Brecher-Diaz-Alejandro (1977) model, for instance, gives an example of this phenomenon. In the presence of tariffs, an increase in the inflow of foreign capital whose profits are going to be repatriated leaves the domestic economy worse-off than in the absence of such inflows.

In the last fifteen years, since Krugman (1979), there has been a considerable output on non-competitive trade theory, especially models with monopolistic competition. This literature has forced us to rethink some of the policy prescriptions of the earlier trade-theoretic models because of the presence of externalities in them (see Helpman (1990) and Helpman and Krugman (1989) for surveys of these models).

More recently some of the non-competitive models have been applied to macroeconomic issues as well. This literature, called the New-Keynesian macroeconomics sees a role for government intervention because of the presence of a (demand) externality.

In models of monopolistic competition what would be the welfare consequences of a growth in the stocks of factors of production? Because of pricing above marginal cost in at least one sector, this raises the possibility of immiserizing growth. In this paper we give an example of this possibility.

In our model an increase in the labour force reduces welfare per capita.
This happens because it causes a reduction in the number of brands produced domestically, a rise in the the prices of the domestically produced brands and a decline in per capita income. Per capita income falls while the gross domestic product rises.

In the existing literature on growth under monopolistic competition the consequences for welfare have always been favourable (as in Krugman (1979), and Venables (1987) as also the endogenous growth literature e.g., Grossman and Helpman (1991)). This is probably due to the fact that there is very little emphasis on factor markets in these models.

In the next section we set out the model In section 3 we look an increase in the labour force, while section 4 offers some concluding comments.

2. THE MODEL

The model consists of identical households (L in number), firms who produce the two goods. At the aggregate level we shall assume that this economy buys the n* units of the foreign brands of the differentiated goods in exchange for the homogeneous good. That is we are assuming that the home produced brands of the differentiated good are non-traded. This is an assumption made elsewhere in the literature in a different context e.g., Venables (1982). We shall refer to this as inter-industry trade. The exercise in this paper could then thought of as a developing economy (because it exports a "primary commodity") experiencing an increase in its labour force. Note however that this interindustry-trade specification is for expositional convenience only. It can be shown that in the other extreme case where all the trade is of the intra-industry trade variety (i.e., imports are paid for by the export of domestic brands of the differentiated good) also this result goes through.
The representative consumer maximizes a Cobb-Douglas utility function with the homogeneous good $y$ and an aggregate $X$ of the differentiated good as arguments
\[ U = y^{1-\alpha} X^\alpha \] (1)
subject to the following budget constraint
\[ y + PX = z \] (2)
where $P$ is the price index corresponding to $X$ (defined below), the price of the homogeneous good is unity and $z$ is the gross national product per capita.

This maximization exercise yields the following demand functions
\[ PX = \alpha . z \] (3)
\[ y = (1-\alpha).z \] (4)
The indirect utility function for this problem can be written as
\[ V = h.P^{-\alpha}.z \] (5)
where $h$ is a constant.

Now given $X$ from (4), the consumer allocates this over the differentiated goods i.e., to maximize
\[ \left[ \Sigma x_i^b + \Sigma x_j^* b \right]^{-1} \]
\[ i=1,...n. \quad j=1,...n^* \]
where $b = 1 - 1/\sigma \quad \sigma > 1$
subject to
\[ \Sigma p_i x_i + \Sigma p_j x_j^* = PX \]
\[ i=1,...n. \quad j=1,...n^* \]
where
\[ P = \left[ \Sigma p_i^{1-\sigma} + \Sigma p_j^{1-\sigma} (1-\sigma)^{-1}\right] \]
\[ i=1,...n. \quad j=1,...n^* \]
where $x_i(x_j^*)$ is the amount of the $i^\text{th}$ ($j^\text{th}$) brand consumed whose price is $p_i(p_j^*)$. There are $n(n^*)$ of domestic (foreign) brands. $\sigma$ is the elasticity of
substitution between the various brands of $X$.

This gives rise to the following demand function

$$x_i = PX(p_{i-\sigma}/p^{1-\sigma})$$

$$j = 1, \ldots, n$$

$$x^*_j = PX(p^*_{j-\sigma}/p^{1-\sigma})$$

$$j = 1, \ldots, n^*$$

Since we shall be concerned with a symmetric equilibrium where all $p_i$'s and $x_i$'s are the same and so are all the $p_j$'s and $x_j$'s we shall drop the subscripts.

Each producer in the domestic differentiated goods industry takes the market equivalent of (6) as the demand curve. The number of brands produced domestically $- n -$ is large enough for each producer to take $P$ as given and for us to treat $n$ as a continuous variable. This implies that the elasticity of demand facing each firm is $\sigma$.

The economy is a small one which takes as given the price and the number of brands of the differentiated good produced abroad. This is a natural assumption to make for a small open economy (see Venables (1982)).

The domestic firms produce the homogeneous good, using a constant returns to scale technology, and the differentiated good, the production of which exhibits increasing returns to scale due to the presence of fixed costs. There are two factors of production--capital ($k$) and labour ($l$). The homogeneous good, which is taken as the numeraire, is produced using labour and capital. This good is relatively labour-intensive. The unit cost equal to price in this industry is given below

$$a_{ly}w + a_{ky}r = 1$$

where $a_{ij}$ is the amount of the input $i$ used in the production of good $j$ ($i=k, l$ and $j=y, x$), $w$ is the wage rate and $r$ is the rental rate.
In the differentiated goods industry there is a fixed cost of production, \( F \). Only capital is used as a fixed input. Production requires \( k_F \) units of overhead capital. We have

\[
\begin{align*}
K_F \cdot r &= F \\
(9)
\end{align*}
\]

The marginal cost component of the differentiated goods sector is also produced by a constant returns to scale technology using \( k \) and \( l \).

\[
\begin{align*}
a_{lx} w + a_{kx} r &= m \\
(10)
\end{align*}
\]

where \( m \) is the marginal cost of producing \( x \).

Note in equations (8) and (10) the coefficients \( a_{ij} \)'s themselves depend on the factor prices. In this paper to keep things manageable we shall assume that the elasticities of substitution in (8) and (10) are equal to unity i.e., we assume that the two production functions are Cobb-Douglas. Further, as mentioned above, we shall assume that the homogeneous sector is labor-intensive i.e., \( a_{ly}/a_{ky} > a_{lx}/a_{kx} \).

Now \( \sigma \) is the elasticity of demand facing an individual producer in the \( x \) industry and in equilibrium price of a brand will be a mark-up over marginal cost i.e.,

\[
p = (\sigma/\sigma - 1).m \\
(11)
\]

We assume that entry drives profits down to zero—the large group case. This implies that \( 1/\sigma \) of total revenue would go towards covering fixed costs (since \( (1-(1/\sigma)) \) goes to cover marginal cost)

\[
\sigma^{-1} \cdot (p \cdot v) = F \\
(12)
\]

where \( v \) is the output per brand in the differentiated goods industry.

There are two factor market clearing conditions

\[
\begin{align*}
a_{ly} Y + a_{lx} n_v &= L \\
(13) \\
a_{ky} Y + a_{kx} n_v + k_F n &= K \\
(14)
\end{align*}
\]

Equation (13) is the labour market clearing condition, \( Y \) and \( n_v \) are
respectively the domestic output of the homogeneous good and the
differentiated good. Equation (14) is the market-clearing condition for
capital. The total availability of the two factors is given by Land K.

In terms of rates of change the price equations may be written as
\[ \theta_{ly} \dot{w} + \theta_{ky} \dot{r} = 0 \]  \hspace{1cm} (15)
\[ \theta_{lx} \dot{w} + \theta_{kx} \dot{r} = p \]  \hspace{1cm} (16)
\[ \dot{r} = p + \nu \]  \hspace{1cm} (17)

where \( \theta_{ij} \) is the share of the \( i \)th input in the relevant cost equation.

We can solve these three equations for three variables in terms of the
fourth. In particular \( \dot{w}, \dot{p} \) and \( \dot{r} \) can be solved as functions of \( \nu \) (see the
Appendix).

Turning now to the factor market equilibrium conditions for K and L, we
have
\[ \delta_{ly} \dot{y} + \delta_{lx} \dot{x} + \delta_{lx} \dot{n} = L - \{ \delta_{ly} \theta_{ky} + \delta_{lx} \theta_{kx} \}(\dot{w} - \dot{r}) \]  \hspace{1cm} (18)
\[ \delta_{ly} \dot{y} + \delta_{kx} \dot{x} + (1 - \delta_{ky}) \dot{n} = K + \{ \delta_{ky} \theta_{ly} + \delta_{kx} \theta_{lx} \}(\dot{w} - \dot{r}) \]  \hspace{1cm} (19)

Since the upper-tier utility function is assumed to be Cobb-Douglas and
the differentiated good is non-traded i.e., \( L \cdot x = \nu \) (remember \( x \) is the demand
per domestic brand per capita and \( L \) is the size of the labour force), we have
from equations (3) and (7),
\[ \nu = -\sigma \dot{p} + (\sigma - 1) \dot{P} + (L \cdot z) \]  \hspace{1cm} (20)

In equation (20) the last term is the percentage change in GDP (\( L \cdot z \)), where
\( z \) is per capita income.

Now consider an increase in the labour force. As shown in the Appendix we
have \( \dot{y}/L > 0, \dot{v}/L > 0 \) and \( \dot{n}/L < 0 \). These results make sense intuitively. A
growth in the labour force causes those sectors to expand which use labour
(\( y \) is the most labour-intensive followed by the variable component of the
differentiated goods sector) while \( n \) contracts as capital is drawn out of that sector. A rise in \( v \) raises \( p \). This in turn implies that the wage rate would fall and the rental rate would rise. Since there is an excess supply of labour and capital becomes relatively scarce, we would expect the price of the former to fall while the price of the latter rises. Going back to the indirect utility function (equation (5) above) and the definition of \( P \), we find that both the rise in \( p \) and the decline in \( n \) raise \( P \) and are therefore welfare reducing. The third determinant of welfare is per capita income to which we turn to next.

First we note, as shown in the Appendix, that the gross domestic product rises following the increase in \( L \). The magnitude of the increase in \( Y \) outweighs the that of a decline in \( n \) on the GDP, while \( p \cdot v \) (the value of output per brand of the differentiated good) increases. However, per capita income (i.e., \( GDP/L \)) falls. Loosely speaking, there are diminishing returns at the aggregate level to labour.

To sum up then we have

\[
\hat{P} = \beta (\hat{p} + (1-\sigma)^{-1} \cdot \hat{n}) = \beta \cdot \hat{v} \cdot (\theta_{xy} - \theta_{yx}) / (\theta_{1x}) + \beta \cdot \hat{n} / (1-\sigma) > 0 \tag{21}
\]

\( \beta \) is the share of the domestic differentiated goods in \( P_X \). The sign in (21) follows since \( y \) is the labour-intensive good.

And the change in per capita welfare is given by

\[
\hat{V} = -\alpha \cdot \hat{P} + \hat{z} < 0 \tag{22}
\]

So we see that in this example an increase in the labour force unambiguously reduces per capita welfare. The reasons are fairly obvious. From a social point of view there is an underproduction of the differentiated good and the growth in the labour force exacerbates this. It also reduces per capita income. Note in a one factor model with \( w \) given (as in Venables (1987)) per capita income does not change.
Before leaving this section note that this result would hold in a closed economy as well. For a closed economy the only difference would be that $\beta=1$.

4 CONCLUSIONS

What are the effects of growth in the supply of a factor of production in a model with an imperfectly competitive sector? From the literature on immiserizing growth, the possibility of growth in a model with a monopolistically competitive sector reducing the welfare of a representative individual exists. However in the literature where growth has been considered it has always been found to be welfare increasing. This includes the papers by Krugman (1979) Venables (1987) and the endogenous growth literature e.g., Grossman and Helpman (1991).

In this paper we provided an example of welfare reducing growth in a trade model with monopolistic competition. The example provided looked at an exogenous increase in the labour force. We found that per capita income and the number of brands fall, while price per brand increased thus unambiguously reducing welfare.

In models of monopolistic competition there is an underproduction, from the social point of view, in the differentiated goods industry. This because of pricing above marginal cost. In the Dixit-Stiglitz type utility function (among others) there is a love for variety. In the example provided in this paper, following an increase in the labour force, the number of brands produced domestically falls (with the number of foreign brands given exogenously). Output per brand increases but this raises the price of each brand and thus is welfare reducing. Income per capita falls which reinforces the immiserizing tendency.
FOOTNOTE

1. This is shown in an earlier version of this paper available from the author on request.
REFERENCES


APPENDIX

From equations (15), (16) and (17) we have,

\[ \hat{w} / \hat{v} = -\theta_{x} / \theta_{1x} \quad (A1) \]
\[ \hat{r} / \hat{v} = \theta_{y} / \theta_{1x} \quad (A2) \]
\[ \hat{p} / \hat{v} = (\theta_{y} - \theta_{1x}) / \theta_{1x} \quad (A3) \]

Using these in equations (18), (19) and (20), we get the following system

\[
\begin{bmatrix}
\delta_{1y} & (\delta_{1x} + \delta_{ly} \cdot \theta_{ky}) / \theta_{1x} & \delta_{1x} \\
\delta_{ky} & -\delta_{ky} \cdot \theta_{ly} / \theta_{1x} & 1 - \delta_{ky} \\
(1-\alpha \beta) & a_{32} & \beta(1-\alpha)
\end{bmatrix}
\begin{bmatrix}
\hat{Y} \\
\hat{v} \\
\hat{n}
\end{bmatrix}
= \begin{bmatrix}
\hat{L} \\
0 \\
0
\end{bmatrix}
(A4)
\]

where \( a_{32} = 1 + \sigma - (\sigma - 1) \beta (\theta_{ly} - \theta_{1x}) / \theta_{1x} - \alpha \beta \cdot \theta_{ly} / \theta_{1x} = (\sigma - 1) \). (1-\beta).

\((\theta_{y} - \theta_{1x}) / \theta_{1x} \cdot (\alpha \beta) \cdot (\theta_{y} / \theta_{1x})\) is positive. \( \beta \) is the share of the domestically produced brands in PX. The term \( 1-\alpha \beta \) is the share of \( Y \) in GDP (because \( \alpha \beta \) is the share of \( \nu \)).

The determinant of the coefficient matrix in (A4) is given by

\[
D = (1-\alpha \beta) ((\delta_{ky} / \theta_{1x}) (\delta_{1y} + \delta_{ly} \cdot \theta_{ky}) + \delta_{ky} \cdot \theta_{ly} / \theta_{1x}) / \theta_{1x} - a_{32} (\delta_{ly} - \delta_{ky}) - \beta(1-\alpha) \cdot \delta_{ky} / \theta_{1x}
\]

\[
= -(\delta_{ky} / \theta_{1x}) (\sigma - 1) (1-\beta) (\theta_{y} - \theta_{1x}) / \theta_{1x} - (1-\alpha \beta - \delta_{ky}) (1-\beta) / \theta_{1x} < 0
\]

Solving for \( \hat{Y}, \hat{v} \) and \( \hat{n} \) from (A4), we get equations (A5) to (A7)

\[ \hat{Y} / \hat{L} = (-1/D)\delta_{ky} \theta_{ly} \beta(1-\alpha) / \theta_{1x} + (1-\delta_{ky}) a_{32} > 0 \quad (A5) \]
\[ \hat{v} / \hat{L} = (-1/D)\delta_{ky} \beta(1-\alpha) + (1-\alpha \beta) \] >0 \quad (A6) \]
\[ \hat{n} / \hat{L} = (1/D)\delta_{ky} (a_{32} - (1-\alpha \beta) \theta_{ly} / \theta_{1x}) \]
\[ = -\delta_{ky} \sigma - 1 \cdot (1-\beta) \cdot (\theta_{ly} - \theta_{1x}) / \theta_{1x} < 0 \quad (A7) \]

11
Now
\[
\frac{\text{GDP}}{\hat{L}} = (1-\alpha \beta) \hat{Y}/\hat{L} + \alpha \beta \cdot (\hat{n} + \hat{p} + \hat{V})
\]
\[-(1/D, \theta_{1x}) \left\{ (1-\alpha \beta - \delta_{ky})(\sigma-1)(1-\beta)(\theta_{1y}-\theta_{1x}) \left\{ (1-\alpha \beta - \delta_{ky})(1-\beta) \right\} \theta_{1y} > 0 \right\}
\]

and
\[
\hat{z} = \text{GDP} - \hat{L} = -(1/D, \theta_{1x}) \left\{ (1-\alpha \beta - \delta_{1y})(\sigma-1)(1-\beta)(\theta_{1y}-\theta_{1x}) \left\{ (1-\alpha \beta - \delta_{ky})(1-\beta) \right\} \theta_{ky} < 0 \right\}
\]

In equations (A8) and (A9) we have used the assumption that \( y \) is the labour-intensive good so that
\[
1-\alpha \beta \equiv Y / (n.p.v + Y) \equiv (w.a_{1y} + r_.a_{ky}) / (r.K + w.L)
\]

Dividing and multiplying the first term in the numerator by \( w.L \) and the last term in the numerator by \( r.K \), we get
\[
1-\alpha \beta = \delta_{1y} \cdot \gamma + \delta_{ky} \cdot (1-\gamma)
\]

where \( \gamma \) is the share of wages in National Income. Because \( \delta_{ky} \leq \delta_{1y} \), we have \( \delta_{1y} \geq 1-\alpha \beta \geq \delta_{ky} \).
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