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Working Paper No: 22

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COMPARING COURNOT AND BERTRAND IN A HOMOGENEOUS PRODUCT MARKET[1]

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[This paper reexamines some general notions regarding the comparison of Cournot and Bertrand equilibrium outcomes. It recasts the Vives (1985) result in a homogeneous product framework and it is shown that the prevailing notion that Bertrand equilibrium involves lower prices and profits than a Cournot equilibrium is not always true, especially when costs are asymmetric.]

[1] I am indebted to Anjan Mukherji for :is paper. This paper was revised when I was a "visiting fellow" at the Centre for Development, Delhi School of Economics. I gratefully acknowledge the excellent research opportunities being offered there.

INTRODUCTION

In this paper we will try to reexamine some general notions regarding the comparison of Cournot and Bertrand equilibrium outcomes in a homogeneous product market. It is a fairly well known idea that Bertrand (price) competition is more competitive than Cournot (quantity) competition. In fact with a homogeneous product and constant marginal costs the Bertrand outcomes involves pricing at marginal costs. This is not the case with differentiated products where margins over marginal costs are positive even in Bertrand competition.

However even in a differentiated product set up Singh and Vives (1984) provide a thorough comparison of Bertrand and Cournot equilibria for the special case of constant marginal costs and the linear system of equations. So long as goods are substitutes the Bertrand equilibrium is more competitive. Cheng (1985) provides a geometric proof of this same result that applies to a more general class of cost and demand functions. Vives (1985) gives fairly general conditions under which the Cournot equilibria involve higher prices and profits (and lower welfare) than do Bertrand equilibria. He shows that if the demand structure is symmetric then (and Bertrand and Cournot equiibria are unique) then prices and profits are larger and quantities smaller in Cournot than in Bertrand competition (Proposition 1). If Bertrand reaction functions are upward sloping (and continuous) then even with an asymmetric demand structure) given any Cournot equilibrium price vector one can find a Bertrand equilibrium with lower prices. In particular, if the Bertrand equilibrium is unique then it has lower prices than any Cournot equilibrium (Proposition 2). Okuguchi (1987) also compares the equilibrium prices for the Bertrand and Cournot oligopolies with product differentiation. If all firms have linear demand and cost functions, and if, in addition, the Jacobian matrix of the demand functions has a dominant negative diagonal, the Cournot equilibrium prices are not lower than the Bertrand ones. Okuguchi also derives the general condition for the comparison of the Bertrand and Cournot equilibrium prices when nonlinearities are involved in the cost and/or demand functions. The condition ensuring the equilibrium prices in the Cournot oligopoly to be not lower than those of the Bertrand oligopoly is shown to be closely related to the global stability condition for the Bertrand equilibrium prices.

We will now reexamine the Vives (1985) results in a homogeneous product market. Consider a n firm oligopoly satisfying the following assumptions

(a) F(P) is continuous, twice continuously differentiable, concave and ∃ positive numbers P^{max} and Q^{max} such that $F(P^{max}) = 0$ and $F(0) = Q^{max}$ and $F'(P) < 0$, for $0 \leq P \leq P^{max}$.

(b) $C(Q_i)$ is continuous, twice continuosly differentiable and convex with $C(0) \geq 0$ and $C'(Q_i) \geq 0$

(c) In pure price competition typically firm i is viewed as facing the following demand curve

$$D_i(P_i, P_j) = 0 \quad \text{if } P_i > P_j \text{ for some } j.$$

$$= \tfrac{1}{m}F(P_i) \quad \text{if } P_i \le P_j \ \forall\, j \text{ and } P_{ik} = P_i$$

$$\text{where } k = (1,2 \ldots .m)$$

$$= F(P_i) \quad \text{if } P_i < P_j \ \forall\, j.$$

The above means that if i's price is below j's price it gets the whole market whereas if they charge the same price they share the market equally.

(d) We assume that in price competition a firm always supplies the demand it faces.

Define the following:

$$\pi_i(P) = PF(P) - C(F(P)),$$

$$\hat{\pi}_i(P) = \tfrac{1}{m}PF(P) - C(\tfrac{1}{m}F(P)) \text{ where } 2 \le m \le n$$

$$\hat{P}_i(m) \text{ s.t. } \hat{\pi}_i(\hat{P}_i(m), m) = -C(0),$$

$$\bar{P}_i(m) \text{ s.t. } \hat{\pi}_i(\bar{P}_i(m), m) = \pi_i(\bar{P}_i(m), m)$$

We know from Dastidar (Forthcoming) that $\hat{P}_i$ and $\bar{P}_i$ exist and they are unique. Also $\hat{P}_i < \bar{P}_i$. Also define the following,

$$P_i^m = \arg_{P \ge 0} \max\, [PF(P) - C(F(P)]$$

$$\hat{P}_i^m = \arg_{P \ge 0} \max\, [\tfrac{1}{2}PF(P) - C(\tfrac{1}{2}F(P)].$$

(e) Assume $\pi_i(P_i^m)$, $\hat{\pi}_i(\hat{P}_i^m) > -C(0)$

Price Competition

Suppose that both firms employ price as their strategic variable.

Now from Dastidar (Forthcoming) we know that under assumptions (a) to (e) $P \in [\hat{P}_i, \bar{P}_i]$ is a pure strategy Nash equilibrium in price competition. If a firm i quotes a price in that range then it is best for firm j to quote that price and not undercut it or charge more. To see that consider a firm i quoting a price $P^* \in [\hat{P}_i, \bar{P}_i]$. Quoting the same P^* yields firm j a payoff equal to $\hat{\pi}_i(P^*)$, quoting more j nets $-C(0)$ and quoting less it gets $\pi_i(P^* - \epsilon)$. Now from lemmas 1 to 8 of Dastidar we know that for any $P \in [\hat{P}_i, \bar{P}_i]$, the following two conditions hold simultaneously

$$\hat{\pi}_i(P) \geq \pi_i(P) > \pi_i(P-\epsilon) \ \forall \ \epsilon > 0, \text{ and } \hat{\pi}_i(P) \geq -C(0)$$

Since the above two conditions hold for P^* also our claim follows.

<u>Quantity Competition</u>

If both firms employ quantity as its strategic variables then we have the Cournot equilibrium.

(f) We assume that $QH''(Q) + H'(Q) \leq 0$. where $H(Q) = F^{-1}(Q) = P$. and $Q = \sum_i Q_i$

The above guarantees the existence of Cournot equilibrium (see Novshek (1985) and Shapiro (1989)). Let the Cournot price, quantities and profits be denoted by P^c, Q_i^c and π_i^c respectively. If firms have symmetric costs then the Vives results (Proposition 1 and 2 in Vives (1985)) follows. The reason is as follows :

Suppose all firms have same constant marginal costs w. Then the Bertrand equilibrium is unique and $P^B = w$. At a Cournot equilibrium the first order conditions are given by,

$Q_i H'(Q) + H(Q) - C'(Q_i) = 0, \; i = 1, 2 \ldots n.$

$\Rightarrow Q_i H'(Q) + H(Q) - w = 0,$

$\Rightarrow P^c = H(Q^c) = w - Q_i^c H'(Q^c) > w$ since $H'(Q) < 0$

Therefore the Cournot price $P^c > w = P^B$.

If all firms have symmetric costs $C_i(.)$ which are strictly convex then the Bertrand equilibrium is necessarily non-unique. In fact any $P^* \in [\hat{P}_i(n), \bar{P}_i(n)]$ is a Bertrand equilibrium.

Now consider the Bertrand equilibrium price of $\hat{P}_i(n)$. We know from Dastidar (forthcoming) that,

$$\hat{\pi}_i(\hat{P}_i(n), n) = - C(0)$$

and $\hat{\pi}_i(P, n) \le - C(0), \; \forall \; P \le \hat{P}_i(n)$

and $\hat{\pi}_i(P, n) > - C(0), \; \forall \; \hat{P}_i(n) \le P < P^{max}$

Again the first order conditions for Cournot equilibrium are as follows :

$Q_i H'(Q) + H(Q) - C'(Q_i) = 0, \; i = 1, 2 \ldots n.$

Therefore $Q_i^C = [C'(Q_i^C) - H(Q^C)]/H'(Q^C)$

$Q_i^C > 0 \Rightarrow H(Q^C) > C'(Q_i^C)$, since $H'(.) < 0$ -----(1)

We claim that $Q_i^C > 0 \Rightarrow \pi_i^C > -C(0)$

Suppose on the contrary that $\pi_i^C \le -C(0)$

$\Rightarrow Q_i^C H(Q^c) - C(Q_i^C) \le -C(0)$

$\Rightarrow C(0) - C(Q_i^C) \le -Q_i^c \, C'(Q_i^c)$

Now $C(0) - C(Q_i^c) > Q_i^c \, C'(Q_i^c)$, since $C(.)$ is strictly convex.

Therefore $-Q_i^c \, C'(Q_i^c) < -Q_i^c \, H(Q^c)$

$\Rightarrow C'(Q_i^c) > H(Q^c)$ which contradicts (1)

Hence our claim follows.

Since $\pi_i^c > -C(0)$, $P^c > \hat{P}_i(n)$

Note that any $P^* \in [\hat{P}_i(n), \bar{P}_i(n)]$ is a Bertrand equilibrium. Since $P^c > \hat{P}_i(n)$ there is always a Bertrand equilibrium price which is lower than the Cournot price. Therefore the Vives (1985, Proposition 1) is valid in a homogeneous product market where firms have symmetric costs.

However proposition 2 is not always valid in an asymmetric market. We produce below a simple, but rather extreme example to illustrate this point. Consider a homogeneous product duopoly where the demand is given by the following :

$P = 10 - (Q_1 + Q_2)$,

and costs are given by $C_1(Q_1) = \frac{1}{2}Q_1^2$ and $C_2(Q_2) = 5Q_2^2$

Routine calculations lead to the following,

$P^c = 44/7$, $P_1^m = 20/3$, $P_2^m = 80/9$, $\hat{P}_1 = 2$, $\bar{P}_1 = 30/7$, $\hat{P}_2 = 50/7$ and $\bar{P}_2 = 150/17$

It should be noted that P_i^m is the monopoly price of the ith firm. From the analysis in Dastidar (forthcoming) it is evident that the Bertrand equilibrium is unique and is given by $P^B = P_1^m = 20/3$. Firm 1 undercuts firm 2 and since its monopoly price is less than $\hat{P}_2$ (the least that 2 will charge) it remains the only operating firm in the market. We may note that $P^c < P^B$. That is we have shown that even when Bertrand and Cournot equilibrium are unique it is not necessary that Bertrand equilibrium is more competitive than Cournot equilibrium especially if costs are very asymmetric. In other words Vives results are not always valid in a homogeneous product market where firms have asymmetric costs.

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