ABSTRACT
In this paper a model of monopolistic competition is used to show that debt-policy could be welfare-improving for all generations. Debt crowds out capital but that is welfare-improving because capital accumulation is immiserizing.

An earlier version of this paper was presented at seminars at EPRU, Copenhagen Business School and Jadavpur University. I am grateful to Sanjay Banerji, Sugata Marjit, Ajitabha Roy Choudhry and Abhirup Sarkar for comments. I am especially grateful to the Acting Director of EPRU, Peter Birch Sorensen for his hospitality and going over an earlier written version with a fine tooth comb.
1. INTRODUCTION.

In overlapping generations models, it is well known that an increase in government debt crowds out capital in the long run. This, of course, requires that Ricardian equivalence does not hold and thus the current generations are not altruistically linked to future generations. If convergence to the steady state is monotonic the slowdown in capital accumulation begins immediately. If the increased debt was a result of a reduced lump-sum tax then the generation receiving the handout gains and (in a closed economy) some future generation onwards there is an unambiguous loss of welfare. The new steady-state level of utility is lower. All this presupposes that the economy is dynamically efficient. The only case for Pareto-superior debt policy arises when the economy is dynamically inefficient and an increase in debt brings it closer to the golden rule capital-labour ratio. (see Buiter(1988a), Blanchard (1985), Fried and Howitt (1988), Persson (1985) and Obstfeld (1989)).

The endogenous growth literature also addresses these issues. Here (at least in some versions e.g. Buiter (1991), Grossman and Yanagawa (1993)) the market economy accumulates too little capital because it ignores the spillover effects of investment on labour productivity, and the economy is always dynamically efficient. In such a set up debt policy cannot but accentuate the prevailing under-accumulation of capital (see also Saint-Paul(1992)).

In this paper we set up a one sector model where the economy produces a differentiated goods under increasing returns to scale. In this model the economy is dynamically efficient in that the interest rate is greater than the population growth rate (which is set at zero). And yet an increase in welfare for all results from an increase in debt
if the fixed cost component is not small (more precisely it requires the share of the labour force in the fixed cost sector not to be too small). The reason is not far to seek. The presence of the monopolistically competitive sector gives rise to a distortion and in the example of this paper the social return to capital is negative i.e. capital accumulation is "immiserizing". Thus an increase in debt which crowds out capital is a Pareto improvement. Debt policy is, of course, not the best way to tackle this distortion—a capital tax is.

The rest of the paper is organized as follows: Section 2 sets out the model, while section 3 analyzes the effects of debt policy. Section 4 offers some conclusions and discusses the results in the context of the existing literature.

2. THE MODEL.

The model consists of households, who are identical in every respect except the time of their births and deaths. They are born without any financial wealth i.e., they are not linked altruistically to any other household alive at the time of their birth. Each household sells one unit of labour in each period of its life. All of them also face an identical, birth-independent probability of death (denoted by \( \pi \)). In the aggregate their is no uncertainty and a proportion \( \pi \) of the population dies each period. The birth rate is also assumed to be \( \pi \), so that there is no net growth in the population. Each agent buys insurance from competitive insurance firms, who supply these at acturially fair rates, and get a return (make a payment) \( \pi \) on their financial wealth if it is positive (if it is negative). The insurance company inherits the household's financial wealth or liabilities on its death.

There is only one good in this economy—a differentiated one. Each brand of this good requires a fixed cost of production. Barring this
output is produced under constant returns to scale. But the fixed cost component makes overall production subject to increasing returns to scale. Both the fixed cost component and the variable cost component use inputs in fixed proportions i.e., technology is Leontief. Firms are assumed to be monopolistically competitive. All agents possess perfect foresight.

There are two factors of production. Labour is supplied inelastically by each household while capital can be accumulated. The factor markets, unlike the goods market, behave competitively.

The government levies lump-sum taxes and pays interest at the market rate on its accumulated debt. Because of deaths and births (more accurately only because of the latter—see Buiiter(1988b)) Ricardian equivalence does not hold in this model.

THE HOUSEHOLDS

A representative household of vintage \( v \) faces a constant probability of death (\( \pi \)), at each instant. It maximizes its lifetime expected utility i.e.,

\[
\int_t^\infty \left( \log X(t,v) \exp \left( - (\beta + \pi)(t-t) \right) \right) \, dt
\]

subject to

\[
A(t,v) = (r(t)+\pi)A(t,v) + w(t) = T(t) - P(t)X(t,v)
\]

where \( X(t,v) \) is the (aggregate of) consumption of the differentiated good (\( P \) is the associated price index (defined below)) and \( A(t,v) \) is the financial wealth at time \( t \) of a person born in period \( v \). \( \beta \) is the rate of time preference. \( T \) is the lump-sum tax paid by the household, \( r \) the return on wealth and \( w \) the wage rate (these are all independent of the date of birth).

In addition the household has an initial condition on financial wealth.
\[ A(t,v) = \bar{A}(t,v) \quad \text{for } t > v \]
\[ = 0 \quad \text{for } t = v \]

and a transversality condition
\[ \lim_{T \to \infty} \exp \left( -\int_0^T [r(t) + \pi(t)] \, dt \right) A(T) \geq 0 \]

This gives rise to the following path of consumption
\[ C(t,v) = (\pi + \beta)[A(t,v) + H(t)] \quad \text{(5)} \]
and
\[ C(t,v) = C(t,v)(r(t) - \beta) \quad \text{(6)} \]

where
\[ H(t) = \int_t^\infty [w(\tau) - T(\tau)] \exp \left( -\int_t^\tau [r(s) + \pi(s)] \, ds \right) \, d\tau \quad \text{(7)} \]

and
\[ C(t,v) = P(t)X(t,v) \quad \text{(8)} \]

Equation (5) is the consumption function, equation (6) is the Euler equation and equations (7) and (8) the definitions of human wealth and nominal expenditure respectively.

Now given \( X \) from (8), the consumer allocates this over the various brands of the differentiated good i.e., to maximize (suppressing the time indices)
\[ X = \left[ \sum_i y_i^b \right]^{b^{-1}} \quad \text{subject to} \]
where \( b = 1 - 1/\sigma \quad \sigma > 1 \)

\[ \sum_i p_i y_i = P.X \quad \text{where} \]
where \( p_i^{(1-\sigma)} = \left[ \sum_i p_i^{1-\sigma} \right] \)

where \( y_i \) is the amount of the \( i^{th} \) brand consumed whose price is \( p_i \). There are \( n \) brands. \( \sigma \) is the elasticity of substitution between the various brands of \( X \).
This gives rise to the following demand function

\[ y_i = C_i \left( \frac{p_i^{-\sigma}}{P^{1-\sigma}} \right) \quad i = 1, \ldots, n \quad (9) \]

Since we shall be concerned with a symmetric equilibrium where all \( p_i \)'s and \( y_i \)'s are the same we shall drop the subscripts.

Finally, financial wealth consists of two assets—government debt (D) and capital (K)

\[ A(t,v) = K(t,v) + D(t,v) \quad (10) \]

Aggregating over all the households of different vintages we get

\[ C(t) = (\pi+\beta).\left( H(t)+K(t)+D(t) \right) \quad (11) \]

\[ C(t) = C(t).\left( r(t)-\beta(t) \right) - \pi(\pi+\beta).\left( K(t)+D(t) \right) \quad (12) \]

where a variable without the vintage index \( v \) indicates its aggregate. In (11) we have normalized the size of the population to unity.

The last term on the right-hand side of (12) is by now very familiar from these models. It arises from the fact that the new-born are born without any financial wealth. There are \( \pi \) of them and from (11) they would have consume a proportion \( (\pi+\beta) \) of financial wealth if they had any.

We shall take the price of brands \( p \) to be the numeraire i.e.,

\[ p = 1 \quad (13) \]

**THE FIRMS**

There is only one good in the economy— the differentiated good. There are two types of costs that a firm has to incur in production. The first is the variable cost and the other the fixed cost. We can think of these being produced in different "sectors"— the x-sector producing the variable cost component and the F-sector producing the fixed cost component. The output of a brand is given by \( x \).

It is worth emphasizing a point at this stage namely that there are no intertemporal decisions involved in production. The firms in question solve a static problem at each moment in time. The fixed cost
is like an (recurring) overhead cost and not a sunk cost.

The marginal cost component is produced by a constant returns to scale technology using $K$ and $L$.

$$a_{lx}w + a_{Kx}r = m$$  \hspace{1cm} (14)

where $a_{ij}$ is the fixed amount of the input $i$ used in the "production" of "sector" $j$ ($i=k,l$ and $j=x,F$), $w$ is the wage rate, $r$ is the rental rate and $m$ is the marginal cost of production.

The industry is monopolistically competitive and therefore price is a mark-up on variable costs

$$p = \sigma/(\sigma-1) m$$  \hspace{1cm} (15)

We assume that entry drives profits down to zero—the large group case. This implies that $1/\sigma$ of total revenue would go towards covering fixed costs, $F$ (since $(1-(1/\sigma))$ goes to cover marginal cost)

$$\sigma^{-1}(p,x) = F$$  \hspace{1cm} (16)

$F$ is also produced by the two factors by a linear homogeneous technology

$$a_{1F}w + a_{KF}r = F$$  \hspace{1cm} (17)

Note that both for $m$ and $F$ we have assumed that the elasticity of substitutions are zero (i.e., the $a_{ij}$'s are fixed).

In terms of rates of change the price equations (from (13) to (17) can be written as

$$\hat{\theta}_{lx}w + \hat{\theta}_{Kx}r = 0$$  \hspace{1cm} (18)

$$\hat{\theta}_{1F}w + \hat{\theta}_{KF}r = x$$  \hspace{1cm} (19)

where $\hat{\theta}_{ij}$ is the share of the $i^{th}$ input in the relevant cost equation.

We can solve these two equations for two variables in terms of the third. In particular $\hat{w}$ and $\hat{r}$ can be solved as functions of $\hat{x}$.

$$\hat{w} / \hat{x} = -\hat{\theta}_{Kx}/\Omega$$  \hspace{1cm} (20)

$$\hat{r} / \hat{x} = \hat{\theta}_{1x}/\Omega$$  \hspace{1cm} (21)

where $\Omega = \hat{\theta}_{lx} - \hat{\theta}_{1F}$.
We shall assume, and it seems reasonable, that variable cost is relatively labour-intensive than the fixed cost component i.e., $a_{lx}/a_{Kx} > a_{lf}/a_{Kf}$. This makes $\hat{w} / \hat{x} < 0$ and $\hat{r} / \hat{x} > 0$. This is in spite the fact that $x$ is relatively labour-intensive. This happens because $x$ acts as a shift parameter in the fixed cost sector pricing equation.

**THE GOVERNMENT**

The government in this model levies lump-sum taxes to pay for the interest payment on its outstanding debt

$$r(t), D(t) = T(t)$$  \hspace{1cm} (22)

**MARKET-CLEARING**

Equations (23) and (24) give the factor market clearing condition for the two factor markets

$$a_{lx}nx + a_{lf}n = 1$$ \hspace{1cm} (23)

$$a_{kx}nx + k_{Kx}n = K$$ \hspace{1cm} (24)

Equation (23) is the labour market clearing condition. $nx$ is the output of the differentiated good (we have normalized the total employment to unity). Equation (24) is the market-clearing condition for capital.

In terms of rates of change we have

$$\delta_{lx} \hat{x} + \hat{n} = 0$$ \hspace{1cm} (25)

$$\delta_{kx} \hat{x} + \hat{n} = \hat{K}$$ \hspace{1cm} (26)

where $\delta_{ij}$ is the share of the $j^{th}$ sector in the total employment of the $i^{th}$ factor.

We thus have

$$\hat{x} / \hat{K} = -1 / \Delta < 0$$ \hspace{1cm} (27)

$$\hat{n} / \hat{K} = \delta_{lx} / \Delta > 0$$ \hspace{1cm} (28)

$$(\hat{n} + \hat{x})/ \hat{K} = -\delta_{kx} / \Delta < 0$$ \hspace{1cm} (29)

where $\Delta = (\delta_{lx} - \delta_{kx}) > 0$

Note, and this is crucial, that $(\hat{n} + \hat{x})/ \hat{K} < 0$ i.e., GDP actually falls
when the capital stock expands! This is the equivalent to saying, in this model, that the social marginal product is negative (while the private marginal product, $r$, is positive and greater than the population growth rate which is zero).

Equations (27) to (29) are nothing but Rybczynski effects of an exogenous change in the capital stock. An increase in capital increases the output of the capital-intensive sector (more than proportionately) and reduces the output of the other sector (again more than proportionately). The interesting point here is that the value of output falls when capital increases.

The other market clearing condition is the goods market clearing condition. Due to the balanced-budget policy of the government this can be written as

$$n(t).x(t) = C + K$$  \hspace{1cm} (30)

In (30) it is implicit that capital does not depreciate.

**DYNAMICS**

We can express the dynamics of the model in terms of two differential equations in $C$ and $K$

$$\dot{C} = (r(K) - \beta)C - \pi(\pi + \beta)(K + D)$$  \hspace{1cm} (31)

$$\dot{K} = n(K).x(K) - C$$  \hspace{1cm} (32)

Linearizing these differential equations around the steady state values ($C^*$ and $K^*$) we have

$$\begin{bmatrix} \dot{C} \\ \dot{K} \end{bmatrix} = \begin{bmatrix} r - \beta & C.r_{K} - \pi(\pi + \beta) \\ -1 & C.(n_{K} + x_{K}) \end{bmatrix} \begin{bmatrix} C - C^* \\ K - K^* \end{bmatrix} + \begin{bmatrix} -\pi(\pi + \delta) \\ 0 \end{bmatrix} \ dD$$  \hspace{1cm} (33)

The determinant of the coefficient matrix is negative if $r > \beta$ which is required for the steady-state value of assets to be positive (see equation (36) below). This is a necessary and sufficient condition for the steady state to be a saddle-point. We have the state variable $K$, which is predetermined (associated with the stable root) and the control
variable $C$, which is a "jump" variable (associated with the unstable root).

Barring expected future shocks and temporary shocks the economy is always on the saddle-path. Along this path the two variables evolve according to

$$K(t) = K + (K_0 - K^*).\exp(\lambda t)$$  \hspace{1cm} (34)
$$C(t) = C + z.(K_0 - K^*).\exp(\lambda t)$$  \hspace{1cm} (35)

where $\lambda$ is the stable root, $[1, z]^\top$ is the (column) eigen-vector associated with it and $z = -(\pi - \delta - \lambda)/(\pi r + K^*) > 0$. This implies that along the saddle-path $C$ and $K$ move together (see figure 1).

**THE STEADY STATE**

We obtain the steady state by imposing $\dot{C} = \dot{K} = 0$ in (31) and (32).

$$\begin{align*}
(r - \beta).C &= \pi(\pi + \delta)(K + D) \hspace{1cm} (36) \\
n \cdot x &= C \hspace{1cm} (37) \\
(a_{lx}(n \cdot x) + a_{lf}n) &= 1 \hspace{1cm} (38) \\
a_{kx}(n \cdot x) + a_{kf}n &= K \hspace{1cm} (39) \\
(a_{lx}w + a_{kx}r)(\sigma / \sigma - 1) &= 1 \hspace{1cm} (40) \\
(a_{lf}w + a_{kf}r) &= (1 / \sigma). x \hspace{1cm} (41)
\end{align*}$$

These six equations determine six unknowns $r, w, x, n, K$ and $C$.

**3. THE EFFECT OF AN INCREASE IN D.**

Consider a "helicopter" drop of $D$. This is only a thought-experiment whereas in actual economies the increase in $D$ is achieved over time by the government running deficits. But in the debt policy literature a stock-shift in debt is routinely done and I follow that tradition here (see Blanchard (1985) and Obstfeld (1989)).

In this model Ricardian equivalence does not hold and the new bonds constitute net wealth for those who receive them.
LONG RUN EFFECTS

We have the long run changes in the endogenous variables from the following equations determining the values of the six unknowns $w^*, x^*, n^*, c^*, r^*$ and $\hat{K}$ in terms of $\hat{D}$

The (percentage) change in the variables of interest to us is given by

$$\frac{\hat{c}^*}{\hat{D}} = \delta_{1F}(1-k)/\Delta \Gamma \quad (42)$$

$$\frac{\hat{k}^*}{\hat{D}} = (1-k)/\Gamma \quad (43)$$

where $\Gamma = \delta_{1F}(\delta_{kX} - \delta_{lX}) + [-k- \theta_{lx}\Delta^{-1}\Omega^{-1}(r/(r-\beta))] < 0$, and $k$ is the share of capital in financial wealth.

So steady-state expenditure rises and the capital stock falls. The number of brands falls i.e., exit occurs. It is immediately obvious that the steady-state instantaneous welfare effect is ambiguous and is given by

$$\frac{\hat{V}}{\hat{D}} = (1-k)((\delta_{lx} - b)/b)\Delta^{-1}\hat{c}^*/\hat{D} \quad (44)$$

We shall assume that this expression is positive i.e., welfare rises in the steady state. This requires a high $\sigma$ so that $b = (\sigma-1)/\sigma$ is close to one and/or the share of employment in the fixed cost sector to be large ($\delta_{lx}$ to be small).

DYNAMICS EFFECTS

On impact there is a jump in $C$ (from $E_0$ to $E_{01}$ in figure 1) since at time 0 (when the unanticipated permanent increase in $D$ is effected) $K$ is given. An increase in government debt increases consumption since Ricardian equivalence does not hold. This can be seen from the following equation

$$dC(O^+) = dC + z(-dK^*) < dC > 0 \quad (45)$$

Along the adjustment path $C$ falls (having overshot its long run value), while $K$ moves continuously from $K_0$ to its new long run value (from $E_{01}$ to $E_1$ in figure 1). The path of (average) welfare looks
similar to the consumption path. The adjustment path is of first order, so welfare is always above the initial long run value.

4. CONCLUSIONS.

In an overlapping generations model with monopolistic competition we showed debt policy is twice blessed. It, as always, blesses the receiver but in this model it also blesses the giver. Two factors help generate this result. First, there is a pre-existing distortion in the uncertain lifetime models in that the discount factor in the human wealth component is $r + \pi$ while in the (aggregate) non-human (financial) wealth it is $r$ (see Engel and Kletzer (1989)). Second, there is the well-known underproduction from a social perspective of the output of the differentiated goods sector, in static macro models.

In the literature on overlapping generations models, debt policy is welfare-improving for all generations when the economy is dynamically inefficient. Dynamic inefficiency is also required for the existence of bubbles in asset prices (see Tirole (1985), Well (1987) and Grossman and Yanagawa (1993). It can be readily checked that in our model while debt policy is welfare improving for all generations bubbles cannot exist. Debt policy is welfare-improving because it crowds out capital whose social marginal product is negative, but the bubble must grow at the market rate of interest which is positive and greater than the growth rate of the economy and hence are not sustainable in this economy.
REFERENCES


FOOTNOTES

1. A dot over a variable denotes its time derivative, a subscript a partial derivative and a hat over a variable its percentage change.

2. These necessary conditions also are sufficient because \( \log X \) is concave in \( C \) and the state variable \( K \). Using equations (13) and (28) we have

\[
\begin{align*}
\frac{\partial \log X}{\partial C} &= C^{-1} > 0, \\
\frac{\partial \log X}{\partial K} &= a_{lx} (\sigma - 1)^{-1} (K_{lx} - a_{xx}) > 0, \\
\frac{\partial^2 \log X}{\partial C^2} &= -C^{-2} < 0, \\
\frac{\partial^2 \log X}{\partial K^2} &= -a_{lx}^2 (\sigma - 1)^{-2} (K_{lx} - a_{xx})^{-2} < 0, \\
\frac{\partial^2 \log X}{\partial C \partial K} &= 0.
\end{align*}
\]

3. This normalization is common in the literature on monopolistic competition models. See e.g., Chaterjee and Cooper (1993).

4. The values of \( r_{K}, n_{K} \) and \( x_{K} \) are obtained from (21), (27) and (28).

5. Once we know the values of \( K \), we can derive the values of the other four variables from equations (20), (21), (27) and (28). We see that in the new long run equilibrium \( w \) and \( n \) are lower and \( r \) and \( x \) are higher (as we would expect).

6. We however do not want \( \sigma \) to become very large because that makes the share of fixed cost in total cost very small.

7. One could take \( C \cdot dV \) (i.e., \( dV \), marginal utility of spending) as a measure of welfare.

8. It is easy to do a detailed welfare analysis for each generation.
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<table>
<thead>
<tr>
<th>No.</th>
<th>Author(s)</th>
<th>Title</th>
</tr>
</thead>
<tbody>
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<td>Kaushik Basu, Arghya Ghosh, Tridip Ray</td>
<td>The Babu and The Boxwallah: Managerial Incentives and Government Intervention (Jan 1994)</td>
</tr>
<tr>
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<td>M.N. Murty, Ranjan Ray</td>
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</tr>
<tr>
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<td>V. Bhaskar</td>
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<td>Bishnupriya Gupta</td>
<td>The Great Depression and Brazil's Capital Goods Sector: A Re-examination (Apr 1994)</td>
</tr>
<tr>
<td>6</td>
<td>Kaushik Basu</td>
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</tr>
<tr>
<td>7</td>
<td>Partha Sen</td>
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<tr>
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</tr>
<tr>
<td>9</td>
<td>Partha Sen, Arghya Ghosh, Abheek Barman</td>
<td>The Possibility of Welfare Gains with Capital Inflows in A Small Tariff-Ridden Economy (June 1994)</td>
</tr>
<tr>
<td>10</td>
<td>V. Bhaskar</td>
<td>Sustaining Inter-Generational Altruism when Social Memory is Bounded (June 1994)</td>
</tr>
<tr>
<td>11</td>
<td>V. Bhaskar</td>
<td>Repeated Games with Almost Perfect Monitoring by Privately Observed Signals (June 1994)</td>
</tr>
<tr>
<td>12</td>
<td>S. Nandeibam</td>
<td>Coalitional Power Structure in Stochastic Social Choice Functions with An Unrestricted Preference Domain (June 1994)</td>
</tr>
<tr>
<td>13</td>
<td>Kaushik Basu</td>
<td>The Axiomatic Structure of Knowledge And Perception (July 1994)</td>
</tr>
<tr>
<td>14</td>
<td>Kaushik Basu</td>
<td>Bargaining with Set-Valued Disagreement (July 1994)</td>
</tr>
<tr>
<td>No.</td>
<td>Author(s)</td>
<td>Title</td>
</tr>
<tr>
<td>-----</td>
<td>------------------------------</td>
<td>-----------------------------------------------------------------------</td>
</tr>
<tr>
<td>15</td>
<td>S. Nandeibam</td>
<td>A Note on Randomized Social Choice and Random Dictatorships</td>
</tr>
<tr>
<td>16</td>
<td>Mrinal Datta</td>
<td>Labour Markets As Social Chaudhuri Institutions in India</td>
</tr>
<tr>
<td>17</td>
<td>S. Nandeibam</td>
<td>Moral Hazard in a Principal-Agent(s) Team</td>
</tr>
<tr>
<td>19</td>
<td>K. Ghosh, Dastidar</td>
<td>Debt Financing with Limited Liability and Quantity Competition</td>
</tr>
<tr>
<td>20</td>
<td>Kaushik Basu</td>
<td>Industrial Organization Theory and Developing Economies</td>
</tr>
<tr>
<td>21</td>
<td>Partha Sen</td>
<td>Immiserizing Growth in a Model of Trade with Monopolistic Competition</td>
</tr>
<tr>
<td>23</td>
<td>K. Sundaram, S.D. Tendulkar</td>
<td>On Measuring Shelter Deprivation in India</td>
</tr>
</tbody>
</table>