CDE

Centre for Development Economics

WORKING PAPER SERIES

Preservation of the Commons by Pooling Resources, Modelled as a Repeated Game

Wietze Lise

Delhi School of Economics

Working Paper No: 27

Centre for Development Economics
Delhi School of Economics
Delhi 110 007 INDIA

is no commons dilemma in a single period in the sense of Wade, but the dilemma shows up in the long run. They argue that the commons problem should be rather modelled as a hawk/dove game, where "dove" is preserving the common and "hawk" is exploiting the common. Playing "hawk" too many times will bring the common over the brink of disaster.

Runge (1981, 1984 and 1986) pleads that the commons problem, as contrary to the common belief, is rather akin to the assurance game, where the one prefers to do what the other does, while mutual pooling has the highest preference. This leads to a self-enforceable solution and it gives a better explanation for the finding that the tragedy of the commons has not yet emerged on a bigger scale.

Hence, strong arguments to support the modelling of the commons game as the prisoners' dilemma (Hardin (1982), Hardin and Baden (1977), Taylor (1987)), an assurance game and a hawk-dove game or, what is the same, a chickens game (Taylor, 1987) can be found in the literature. These three games constitute *collective dilemmas* (Bardhan (1993) and Bates (1988)). This debate has not yet brought an answer of the same tenor. It may be that this chapter has been based mainly on the case of the prisoners' dilemma, but it can as well be applied to the hawk/dove game, while the assurance game does not need such an approach, because the solution is self enforcing. Bardhan (1993) goes even further to argue that the commons problem can change from one dilemma to an other or even can be resolved as time goes on. He gives an informal discussion on this transition process. This chapter gives a first attempt to a formal approach of such a transition process.

For the analyses of this paper I assume that there can exist Common Pool Resources (CPRs). CPRs can be either open access or common property of the community. In the latter case, these resources are owned by the (village) community or leased by the government to the village community. The community members have the right to appropriate, while non-community members have the duty to refrain from appropriation (see for example Bromley and Cernea (1989)). CPRs can be extended by pooling labour and private property resources. These poolers will finally divide the marketable yield of the common pool among themselves, but intermediary yields like fodder, fuelwood and fruits can be consumed during the process. This share in the yield is an important incentive which will move the people towards participation. Participation of the people means that they *contribute to the development effort*, *share in the benefits* and *are involved in decision making as well as in evaluation* (see for instance Lise (1995, forthcoming)).

This pooling process is formalized in section 2 as a noncooperative two-person three-strategy repeated normal-form game. *The payoffs are taken as functions of the amount and quality of resources. The aggregate of these two factors will be referred to as the level of resources.*

Section 3 focuses on all possible payoff orderings in two strategy games, in order to derive minimum required discount factors under which cooperation is an equilibrium outcome. The game is repeated an infinite number of times. The payoffs are fixed to sum up all relevant cases under different payoff orderings. The prisoners' dilemma, hawk/dove or chicken game, assurance game, Pareto game, coordination or battle of the sexes game, and the reverse battle of the sexes game are six possibilities out of which the first two will get major attention, because they have non-trivial discount factors. The effect of different trigger strategies is demonstrated for the case of the hawk/dove game.

In section 4 the case of finite punishments in the infinitely repeated prisoners' dilemma is analyzed. A relation between the permissible discount factor and the number of periods of punishment is derived. It will be proved that the critical discount factor is strictly decreasing in the number of punishments for a deviation from pooling.

In section 5 comparative statics is used to find the behaviour of the function of the minimum required discount factor to guarantee cooperation as the equilibrium outcome when the resource level goes from zero to infinity. Conditions are derived under which cooperation is most likely to occur for the minimal required discount factor as a function of resources in the prisoners' dilemma case. It is argued that this function can de divided in five different phases. As the level of resource increases (phase 2), the gross incentive to pool will increase from zero (phase 1) to a maximum which possibly can be one (phase 3) and after that decrease (phase 4) again to zero (phase 5). Two examples illustrate a situation where some phases will be attained depending on the specification of the payoff functions.

The possibilities of obtaining cooperation in the repeated prisoners' dilemma is discussed in section 6. This is possible when there is a change in the punishment structure for defection, when the termination time is unknown, and under incomplete information: each others' private information do not necessarily coincide.

The pooling game as formalized in section 1 is analyzed in section 7. The prisoners' dilemma has been imbedded in this finitely repeated game. Under the assumption of a simple trigger strategy, two critical discount factors have been derived for this case. It is concluded that, in a comparison with section 4, there were no qualitative changes in the results.

In the final section it is discussed how the present analysis of the simple two-person three-strategy repeated game can be extended.

2 Formalizing the pooling game

In many places of rural India degraded waste lands can be found. It either belongs to the Government or it is privately owned. The private lands remain fallow, because the owners have no financial resources or are not motivated to undertake necessary investments to restore the biomass such that the land can be used for production again. These private fallow lands can be utilized when a number of owners pool land, water, labour and other resources to create a common pool[2]. This pooling process is possible at the village level once an institution has been established, either spontaneously from inside the village or from outside. Once a common pool has been created, they can start a process of collectively planting trees protected by social fencing. This is possible, since several tree species can even grow when the soil quality is poor. The most important factor is that the trees are properly cared for and protected. That calls for social fencing and collectivisation of labour. Once the quality of resources in the pool starts improving, other products like grass and vegetables can also be grown. The land poolers remain the owner of their voluntarily pooled land. They can decide whether they want to add their private land to the common pool, or they do not want to pool

[2]A number of cases can be found in Lise (1995, forthcoming). There I make a comparison between four kinds of cases in rural India. That approach was supported by discrete time modelling and simulation results were derived.

at all, only once a year. The pooling game is formalized as a symmetric[3] 2-person 3-strategy noncooperative repeated game $\Gamma = (S, \delta, R, P)$ in the following manner.

Let us consider a community and focus on 2 landowners only: 2 persons, who own a parcel of degraded waste land, $i \in I = \{1, 2\}$.

a) The possible actions of all players are identical: they have to decide whether they want to pool their land, to withdraw their land some time after pooling, or not to pool. $S^* = \{\text{pool, withdraw, not}\} = \{s^1, s^2, s^3\} = S_i$ for all $i \in I$. Then the joint strategy space is given by $S = \Sigma_{i=1}{}^2 S_i = S^* . S^*$. Let s_{it} be the action of player i at time t, then $s_t = (s_{1t}, s_{2t})$, the vector of actions at time t chosen by all players.

b) This game is repeated T times, where T is a natural number which can be infinite. The importance of the future of the players is expressed with the following discount factor[4] $\delta = (\delta_1, \delta_2)$, which is different for both players in general.

c) It is assumed that a players' payoff does not only depend on the strategies, but also on the level of natural resources R. At time t, $R = R_t$ is a real non-negative number, $R_t \in [0,\infty)$. Notice here that R_t crucially depends on all past actions and therefore summarizes an infinite history in a single numerary.

d) For all $s_t \in S$ and for all $R_t \in [0,\infty)$, $P_{it}(s_t, R_t)$ denotes the payoff of player i at time t when all players selected their action according to s_t, where the resource level is R_t. Hence, R_t changes endogenously, dependent on the past choices of the players and has as such an indirect effect on the payoffs. Under discounting the net present value of players i's payoff will have the following structure: $P_i = \Sigma_{t=0}{}^T \delta_i{}^t P_{it}(s_t,R_t)$. $P = \Pi_{i=1}{}^2 P_i$ is the aggregate payoff set.

[3]When the payoff structure is symmetric, both players are assumed to be perfectly equal and both players will face exactly the same dilemma.

[4]$\delta = 1/(1+r)$, where r is the discount rate, $r \in [0,\infty)$, which is logically equivalent to $\delta \in (0,1]$.

It is not always an easy task to find a suitable equilibrium concept for this 2-person game where the payoffs functions can change endogenously. I will use the concept of subgame perfect equilibrium throughout this paper, 'an equilibrium b of Γ is said to be a *subgame perfect equilibrium* if, for every subgame Γ_x of Γ, the restriction b_x of b to Γ constitutes a Nash equilibrium of Γ_x' (Van Damme, 1991, 108). The main emphasis will be on the case of the prisoners' dilemma, but various other cases will pass in review.

3 An assessment of two person dilemmas

Let us consider the preferences of a peasant at the village level. As mentioned in the setting of the last section he can decide every year whether he will pool either labour and/or land, or not pool anything. Once he has made a decision, he will not be able to change his strategy for one year, but the option of pooling (continue pooling) or not pooling (stop pooling) remains at the beginning of each year. Therefore, the problem can be modelled in the framework of a repeated game. The following assumption[5] restricts the pooling to the most simple to strategy case.

Assumption 3.1: Pooled land or labour is not withdrawn during the yearly cycle of the cultivation process.

The requirement of such an assumption can be justified, since only after the creation of a pool, a collective cultivation process can be started.

In this paper only two peasants are considered. When both of them extend the common by pooling private (waste) land and/or labour, a situation of cooperation and peoples' participation is created. When one peasant pools, while the other does not, the single pooler has to do all the work for the common, while the other can focus his attention on other work, while he is still able to use the common. Hence, he can free-ride. Finally, when both do not pool at all, they will leave their waste land and the common barren and it has a marginal utility for whatever is left as grazing land. Then the free-rider problem is most severe.

[5]This assumption will be dropped in section 6, in order to analyze the general pooling game as formulated in the last section.

Consider the following representation of the pooling game, which is a symmetric bimatrix game.

Game P:

2 / 1	Pool	Not
Pool	$x(R_t)$, $x(R_t)$	$b(R_t)$, $a(R_t)$
Not	$a(R_t)$, $b(R_t)$	$y(R_t)$, $y(R_t)$

Here $x(R_t)$ denotes the payoff to player 1 (or 2), when both decide to pool, at resource level R_t, at time t. $a(R_t)$, $b(R_t)$ and $y(R_t)$ can be described analogously.

Assumption 3.2: The payoffs in **game** P are taken to depend on the resource level R_t[6].

Hence, as time goes on the value of R_t can change, which has its indirect effect on the payoff functions. This can even cause a change in payoff structure in the long run. I argue that this very interesting transition process can be described by this simple game. The phase-approach in the next section is illustrative to this in the sense that different phases in the rise and fall of a pooling process reflect different dilemmas. In this context it will be very useful to have an exhaustive overview of all cases which can appear when different payoff orderings are considered. For this purpose let us concentrate, for the time being, on fixed payoff structures:

Notation: $x = x(R_t)$; $y = y(R_t)$; $a = a(R_t)$; $b = b(R_t)$.

Attach the following economic interpretation to three differences in **game** P. These will be used to describe the trade-off between pooling and not pooling.

Definition 3.1: $a - x$ = the net threat to deviate frommutual pooling.

[6]This assumption will not become effective before the section on the comparative statics.

Definition 3.2: $x - y$ = the net incentive to pool.

Definition 3.3: $a - y$ = the net threat of deviation by the other, when one decides not to pool.

Prime (') is used for the derivative with respect to R.

Notation: $x' = dx/dR$; $y' = dy/dR$; $a' = da/dR$; $b' = db/dR$.

x' is the rate of change in the (pool,pool)-payoff with respect to the level of resources.

Assumption 3.3: The payoffs are *incredsing functions* in the level of the resources in the common pool.

Hence, it follows from assumption 3.3 that $a', b', x', y' > 0$. But this does not mean that the productivity of the pool will increase. The response of the pool to a change in the level of resources is the focus of comparative statics in the next section.

Assumption 3.4: $x > y$: the net incentive to pool is positive.

A direct implication of assumption 3.4 is that the net threat of deviation by the other, when one decides not to pool, is strictly larger than the net threat to deviate from mutual pooling, or the net incentive to pool. It also restricts us to consider only those cases where mutual pooling is always more beneficial than mutual not pooling. This is in accordance with the argument in the beginning of this section. Namely, in the situation of 'not pooling', the land is neglected and hence it gets degraded. This assumption guarantees the presence of a strictly positive net incentive to pool. It rules out a second class of dilemmas, where the reverse of assumption 3.4 is true. Therefore, only 12 (4!/2) possibilities are left for ordering a, b, x, y. These can be dividedinto six distinct cases. For the first two cases, it is possible to derive non-trivial minimum required discount factors $\phi = \phi(R_t) \in (0,1)$. ϕ is very important and has the following characteristic.

Definition 3.4: Pooling is individually most beneficial, if and only if $\min\{\delta_1, \delta_2\} > \phi$.

Case 1. $a > x > y > b \Leftrightarrow$ prisoners' dilemma.

This is the most intriguing case. The prisoners' dilemma game has the characteristic that the most attractive outcome (pool,pool) is not self-enforceable but self-destabilizing, due to an individual dominating strategy of not pooling. Hence (not,not) is the Nash equilibrium, which is Parieto dominated by (pool,pool). Even when the game is repeated, (not,not) is still a subgame perfect equilibrium. The backward induction principle prescribes to play (not,not) in every period as an equilibrium strategy when the termination time is common knowledge (see Luce and Raiffa, 1957). One way out of this dilemma is by repeating the game an infinite number of times. (Pool,pool) can result as an equilibrium outcome when the following trigger strategy σ_1 is followed by all players.

$$\sigma_1 = \begin{cases} \text{Pool} & \text{as long as no defection took place;} \\ \text{Not} & \text{otherwise.} \end{cases}$$

Continuing to work, for the time being, with fixed R_t, it is possible to obtain mutual pooling as an equilibrium outcome, when the importance of future payoffs is large enough. A lower bound or minimal required discount factor function can be derived by interpreting the consequences of σ_1. Moreover, always pooling should yield a strictly higher payoff than pooling in the first $(\tau-1)$ periods until defection takes place at time τ, directly followed by not pooling after time τ as prescribed by the trigger strategy. Whenever inequality 1 is satisfied, mutual pooling will be an equilibrium outcome.

$$x\sum_{t=0}^{\infty} \delta^t > x\sum_{t=0}^{\tau-1} \delta^t + a\delta^\tau + y\sum_{t=\tau+1}^{\infty} \delta^t \qquad (1)$$

Rewriting inequality 1 yields the following function on the minimal required discount factor for the case of the prisoners' dilemma.

$$\delta > \frac{a - x}{a - y} = \phi_p \qquad (2)$$

Hence, condition 2 is a sufficient and necessary condition for mutual pooling to be a subgame perfect Nash equilibrium[7]. The behavior of this function will get major attention in the remaining sections, since it is very important to derive exact conditions under which pooling is beneficial. This is also important in connection with the chance of survival of the local management institution.

The next few sections are mainly based on the prisoners' dilemma, where it will be analyzed in depth. The remaining five cases are only mentioned to get an exhaustive overview of possible cases. These can be analyzed with similar techniques as applied to the prisoners' dilemma case. Dualities with other cases will be mentioned when applicable.

<u>Case 2.</u> <u>a > x > b > y ⇔ hawk/dove or chicken game.</u>

This game has two pure Nash equilibria, (pool,not), (not,pool). Let us define another trigger strategy, σ_2, under which mutual pooling can emerge as an equilibrium strategy in this case.

$$\sigma_2 = \begin{cases} \text{Pool} & \text{as long as the other did not defect first;} \\ \text{Not} & \text{otherwise.} \end{cases}$$

This trigger strategy differs from σ_1, because it is more beneficial for the defector to return to pooling, when the other sticks to the not pooling strategy, because the payoff increases from y to b. When both players follow this behavior, then a function of the minimal required discount factor can be derived in similar manners as the last case on the prisoners' dilemma[8].

$$\delta > \frac{a - x}{a - b} = \phi_h \qquad (3)$$

At this point it must be remarked that ϕ_h of case 2. is based on the assumption that this trigger strategy is strictly carried out. Only then it will be beneficial for the defector to pool

[7]This is a well-known result. See for example proposition 1 in Stahl (1991).

[8]It has been derived from inequality 1 with a small alteration, where b should be substituted for y.

again. However, the former argument can be criticized, because the *defector* can observe in the consecutive period that the *defectee* improved her payoff in such a manner that the defectee even overtook the payoff she got under mutual pooling. This can tempt the defector to select not pooling to minimize the payoff to the defectee to punish the defectee for implementing the trigger strategy. This argument can be supported by the following two-step trigger strategy, $\sigma_{2\text{-step}}$.

$$\sigma_{2\text{-step}} = \begin{cases} \sigma_1 & \text{for the defectee;} \\ \sigma_3 & \text{for the defector.} \end{cases}$$

Where $$\sigma_3 = \begin{cases} \text{Pool} & \text{as long as the other never defected;} \\ \text{Not} & \text{otherwise.} \end{cases}$$

Here σ_3 is the strategy of the defector to punish the defectee for implementing the trigger strategy σ_1.

Under the two-step trigger strategy the defector and the defectee will have a different discount factor, due to the role assignment of defector and defectee, which are essentially different. Assume that player i defects first[9], then he will be assigned the role of defector and player j ($\neq$i) will get the role of defectee. This will yield the payoff scheme as depicted in table 3.1.

Period	s_{it}	$P_i(s_t)$	s_{jt}	$P_j(s_t)$
$\{...,\tau-1\}$	Pool	x	Pool	x
τ	Not	a	Pool	b
$\tau+1$	Pool	b	Not	a
$\{\tau+2,...\}$	Not	y	Not	y

Table 3.1. The strategy choice of defector i and defectee j.

The net present value of the payoffs are expressed in equations 4 and 5.

$$P_i = x\sum_{t=0}^{\tau-1} + a\delta^{\tau} + b\delta^{\tau+1} + y\sum_{\tau+2}^{\infty}\delta^{t} \qquad (4)$$

[9]Neglect the first defection when it is mutual. In that case no role assignment is possible, but more importantly, since both are to blame, there is no need for punishment.

$$P_j = x\sum_{t=0}^{T-1} + b\delta^T + a\delta^{T+1} + y\sum_{T+2}^{\infty}\delta^t \qquad (5)$$

As soon as $x\sum_{t=0}^{\infty}\delta^t$ is bigger than equation 4 or 5, cooperation will again be an equilibrium outcome. this yields the following discount factor functions.

$$\delta_i > \frac{-(a-b) + \sqrt{(a-b)^2 + 4(b-y)(a-x)}}{2(b-y)} = \phi_{defector} \qquad (6)$$

$$\delta_j > \frac{(a-b) + \sqrt{(a-b)^2 - 4(a-y)(x-b)}}{2(a-y)} = \phi_{defectee} \qquad (7)$$

With some algebra, it can be derived that $\{\phi_{defector}, \phi_{defectee}\} < \phi_h$ always holds. Hence, the two-step trigger strategy does always lead to a more severe punishment for both the defector and the defectee.

There is still a basic problem which bothers the two-step trigger strategy. It requires both players to be individually irrational, because one of them can always improve on his/her payoff by deviating from the two-step trigger strategy unilaterally. Deviation yields a payoff of b instead of y, which is strictly bigger in the case of the hawk/dove game. Consider discount factor ϕ_h, derived from trigger strategy σ_2, as the crucial one.

Case 3. $x > a > y > b \Leftrightarrow$ an assurance game.

Here it is more beneficial to either mutually pool or mutually defect, which are both Nash equilibria. This game has no individual dominating strategies (as in the case of the prisoners' dilemma), because x > a, while b < y. Strategy (pool,pool) Parieto dominates (not,not) and is self-enforcing, because once the players have agreed upon (pool,pool), they can never do better. Trigger strategy σ_1 is sufficient to arrive upon mutual pooling as an equilibrium outcome. In other words, it never pays to defect. Hence, $\phi_a = 0$.

Case 4. x > max {a, y} and b > y ⇔ Pareto game.

The mutual pooling outcome is the only Nash equilibrium, because it is an individual dominating strategy which also Pareto dominates (not,not). Therefore (pool,pool) is always more beneficial in the case of the Pareto game. Hence, $\phi_{par} = 0$.

Case 5. x > y > max {a, b}[10] ⇔ coordination or battle of the sexes game.

The same reasoning as followed by the assurance game applies here. In fact, it is exactly the same situation in a game theoretic sense. It is always more profitable for both to pool and the cooperative outcome is self-enforcing. Hence, $\phi_c = 0$.

Case 6. min {a, b} > x > y ⇔ reverse battle of the sexes game.

Strategy combinations (not,pool) and (pool,not) are two pure strategy Nash equilibria. The implementation of trigger strategy σ_1 is useless in this case, since the defector will start pooling again as soon as "he is punished". But why should the defectee want to punish the defector for defection? Notice that it is more beneficial for the defectee to be defected. Moreover, there exists no discount factor by which the cooperative outcome can be enforced. Hence, $\phi_r = 1$.

[10]This payoff ordering was taken by Runge (1984) as the one belonging to an assurance game.

All cases are summarized in table 2.2.

Case	Name of game	Payoff ordering	Critical discount factor
1 (1[11])	Prisoners' dilemma	a > x > y > b	$\frac{a-x}{a-y} = \phi_p$
2 (1)	Hawk/dove or chicken	a > x > b > y	$\frac{a-x}{a-b} = \phi_h$
3 (1)	Assurance	x > a > y > b	$0 = \phi_a$
4 (5)	Pareto	x > max{a,y}; b>y	$0 = \phi_{par}$
5 (2)	Coordination	x>y>max{a,b}	$0 = \phi_c$
6 (2)	Reverse Battle of the sexes	min{a,b}>x>y	$1 = \phi_r$

Table 3.2: The value of the critical discount factor function in all possible cases[12].

4 Number of punishments versus permissible discount factor

It has been a hot issue in economic theory for a long time why there is always so much discrepancy between finite and infinite horizon problems. Intuitively, it should be true that the finite case will converge to the infinite case. The main difficulty with this statement is that it is sometimes very hard to get an explicit expression for the finite case, while it is possible to get one for the infinite case. This is also true for the repeated prisoners' dilemma, where, again, it is impossible to find an explicit expression for the critical discount factor of a function of the number of punishments.

[11]The number in the brackets represents the number of the possibilities of ordering the payoffs a, b, x, y in each case.

[12]For case 2 to 6 it holds that whenever x > y (the reverse of assumption 4) the critical discount factor will always be one. This is not true for the *reverse* prisoners' dilemma (b > y > x > a), because whenever δ < (x-a)/(y-a) cooperation is feasible. This reasoning is due to Stahl (1991, 368).

In this section we will focus on the infinitely repeated prisoners' dilemma with a trigger strategy of finite period punishments[13]. It is shown that, under finite punishments, a sufficiently large discount factor can be found, under which cooperation will become an equilibrium outcome. A sufficient condition for the existence of such an equilibrium is easy to derive, but the analytical construction of an exact critical discount factor with more than two punishments is tough. The focus in this section will be on those cases where it can be derived.

The objective is to find an expression for the minimal required discount factor for which cooperation is the equilibrium outcome, when the number of punishments varies. The same approach as in section 3 can be followed to arrive at such an expression. Define n as the number of punishments Further, assuming that one player defects in period τ, we have to solve inequality 8 which is the finite version of inequality 2.

$$x\sum_{t=0}^{\tau+n} \delta^t > x\sum_{t=0}^{\tau-1} \delta^t + a\delta^\tau + y\sum_{t=\tau+1}^{\tau+n} \delta^t \qquad (8)$$

This will yield an implicit relation between δ and n+1. Substitute the inequality sign for the equality sign and call the solution of the resulting equation ϕ_n.

$$\delta > \frac{a - x}{a - y} + \frac{x - y}{a - y}\delta^{n+1} \qquad (9)$$

Condition 9 can be rewritten as an (n+1)-degree polynomial.

$$f_n(\delta) = \delta^{n+1} - (1+\phi_1)\delta + \phi_1 < 0; \quad \text{Where } \phi_1 = \frac{a-x}{x-y} \qquad (10)$$

Definition 4.1: ϕ_n is the solution of $f_n(\delta) = 0$.

Lemma 4.1: $\lim_{n\to\infty} \phi_n = \phi_\infty$.

---

[13]The possibility of finite period punishments was already recognized and worked out theoretically by Green and Porter (1984).

Proof: It follows directly from condition 9 when n goes to infinity, since $\delta \in (0,1)$. Q.E.D.

In general an $(n+1)^{th}$-degree polynomial will have n roots which can be either real, complex and/or multiple by the fundamental theorem of algebra. To solve equation "$f_n(\delta) = 0$" is a pure mathematical problem. The general solution of a $(n+1)^{th}$-degree polynomial has not been found in mathematical theory so far. But here we will be dealing with a special case of this general problem. For $n \leq 2$, there exists a solution, from which we can derive the main idea for the general solution.

Case n=1

In this case cooperation is a possibility. $f_2(\delta)$ has two roots: $\delta_1=1$ or $\delta_2=\phi_1$. All values between ϕ_1 and 1 satisfy $f_1(\delta)<0$.

$$\phi_1 = \frac{a - x}{x - y} < \delta < 1 \qquad \textbf{(11)}$$

Hence, if a particular discount factor, δ, happens to be in that region, then cooperation can indeed be an equilibrium outcome. This is a very remarkable solution, since we have arrived at a case where cooperation can be enforced with the threat of only one punishment, provided that the discount factor satisfies condition 11 and the termination time is unknown. This is a very interesting result and the requirement of a quite irrational trigger strategy of infinite punishments (Fudenberg and Maskin (1986, pg.545) already called such a player crazy), can be substituted by one single punishment, but the critical discount factor has risen to ϕ_n which is strictly larger than ϕ_∞[14]. This means that the gross incentive to pool has reduced and the likelihood of cooperation has decreased.

[14]This is easily checked by comparing conditions 11 and 2. It suffices to show that $\phi_\infty < \phi_1$.

$$\frac{a-x}{a-y} < \frac{a-x}{x-y} \Leftrightarrow x-y < a-y \Leftrightarrow x < a$$

This is true for every payoff-ordering of the prisoners' dilemma.

In the last case we have found $\delta=1$ to be the root of the equation $f_n(\delta)=0$. It is easy to verify that this is true for all n. Therefore, condition 10 can be rewritten as follows.

$$f_n(\delta) = (\delta - 1)(\delta^n + \delta^{n-1} + \ldots + \delta^2 + \delta - \phi_1) < 0 \quad (12)$$

Case n=2

In this case $f_2(\delta)=0$ has three different real roots. Cooperation is, again, a possibility. The negative root is not considered here, since only values of $\delta \in (0,1)$ are of interest.

$$\phi_2 = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{a-x}{x-y}} = -\frac{1}{2} + \sqrt{\frac{1}{4} + \phi_1} < \delta < 1 \quad (13)$$

Case n≥3

It is not possible to get an explicit expression for the roots of $f_n(\delta)=0$. But the following can be said about these roots.

- When n is odd, $f_n(\delta)=0$ has two real roots: $\delta=1$, $\phi_n \in (0,1)$ and (n-2)/2 pairs of complex conjugate roots.
- When n is even, $f_n(\delta)=0$ has three real roots: $\delta=1$, $\phi_n \in (0,1)$, the third root is negative and (n-3)/2 pairs of complex conjugate roots.

There is a strict ordering on the positive roots of $f_n(\delta)$:

Theorem 4.1: Let ϕ_n be the real root of

$$\frac{f_n(\delta)}{(\delta-1)} = \delta^n + \delta^{n-1} + \ldots + \delta^2 + \delta - \phi_1 = 0 \quad (12')$$

where ϕ_∞ and ϕ_1 are defined as before. Then the following equivalence relation holds

$$\forall n: \phi_n \in (\phi_\infty, \phi_1] \Leftrightarrow \phi_{n+1} < \phi_n \quad (14)$$

Proof: See appendix A1.

The intuitive idea behind theorem 4.1 is that whenever the number of punishments increases, the likelihood of getting mutual pooling as the equilibrium outcome should increase as well, because the punishment is becoming more severe. This idea has been formalized in theorem 4.1 along with its proof.

<u>Corollary:</u> If $\phi_1<1$ and $\delta>\phi_1$, then (pool, pool) is an equilibrium outcome for any number of punishments.

<u>Proof:</u> If $\phi_1<1$, then for all n: $\phi_n < \phi_1$, by theorem 4.1. This is a sufficient condition to have (pool, pool) as an equilibrium outcome. Q.E.D.

The condition of the corollary $\phi_1=(a-x)/(x-y)<1$ is logically equivalent to $a-x < x-y$ which means in words that the net threat to deviate from mutual pooling should be smaller than the net incentive to pool.

5 Comparative statics on the critical discount factor

With the conditions on the discount factor, as summarized in table 3.2, we reach at the most striking issue which is to analyze the behaviour of $\phi(R)$. This section will rely solely on the case of the prisoners' dilemma[15]. Assume the following shape for $\phi(R)$:

$$\phi(R) = \phi_\infty = \phi_p(R_t) = \frac{a(R_t) - x(R_t)}{a(R_t) - y(R_t)} \qquad (15)$$

The crucial question is how $\phi(R)$ will vary over time. It is an empirical question and it can differ from case to case. For the present analysis let us concentrate on the values of the level of resources only, to derive the possible effects on $\phi(R)$ when R varies from zero to infinity.

The theoretical possibilities of the critical discount factor of cases 1 to 6 can be used when we consider the behaviour of $\phi(R)$ in practice. In theory something more can be said about

---

[15]The same analysis can be done for the hawk/dove or chicken game by assuming $\phi(R) = \phi_h$.

the behaviour of $\phi(R)$. As R changes, the net threat to deviate from mutual pooling and the net incentive to pool will also change. These changes can be specified by considering the first and second order conditions of $\phi(R)$.

$$\phi'(R) \lesseqqgtr 0 \Leftrightarrow \frac{a' - x'}{a - x} \lesseqqgtr \frac{a' - y'}{a - y} \qquad (16)$$

$$\phi''(R)\,|_{\{\phi'(R)=0\}} \lesseqqgtr 0 \Leftrightarrow \frac{a'' - x''}{a - x} \lesseqqgtr \frac{a'' - y''}{a - y} \qquad (17)$$

In definitions 3.1 to 3.3 is given an economic interpretation to the differences between the payoffs. The fraction (a'-x')/(a-x) can be interpreted as *the rate of change* of the net threat to deviate from mutual pooling.

In words condition 16 reads as follows. The value of the critical discount factor $\phi(R)$ is increasing (decreasing) when the rate of change of the net threat to deviate from mutual pooling is larger (smaller) than the rate of change of the net incentive to pool. From this the optimality result in lemma 5.1 can be derived.

Definition 5.1: 1 - $\phi(R)$ = the gross incentive to pool.

Lemma 5.1: $\phi''(R) > 0$; 1 - $\phi(R)$ is maximized if and only if (a'-x')/(a-x) = (a'-y')/(a-y).

In words: Provided that the second order condition is met ($\phi''(R)>0$), the gross incentive to pool will be highest when the rate of change of the net threat to deviate from mutual pooling is equal to the rate of change of the net threat of deviation by the other, when one decides not to pool.

Proof: The threat rate to deviate from mutual pooling is by definition 3.1 equal to a-x. Its rate of change can be written as (a'-x')/(a-x). The incentive to pool is by definition 3.2 equal to a-y. Its rate of change can be written as (a'-y')/(a-y). Whenever these two are equal then $\phi'(R^*)=0$ by condition 16 and $\phi''(R^*)>0$ by assumption. This is a necessary and sufficient condition for a minimum of $\phi(R)$ in R^*. Q.E.D.

The simultaneous effects of changes in x, y, a as R changes can be analyzed as well with condition 16.

<u>Theorem 5.1</u>: $x' > \max\{a', y'\} \Rightarrow \phi'(R) < 0$.

In words: the statement that the payoff of mutual pooling x is growing faster in R than the threat payoffs a and y is a sufficient condition to have a strictly increasing gross incentive to pool.

<u>Proof</u>: It suffices to interpret condition 16. Both numerators are positive, but the left denominator is negative, while the right is still positive. This implies that $\phi'(R) < 0$. Q.E.D.

It is possible to verify the indirect effects of a one by one increase in x, y, a on $\phi(R)$, by taking first and second order derivatives[16] with respect to x, y, a. These effects are of interest, because as R changes the payoff functions x, y, a will change as well.

$\frac{\partial\phi}{\partial x}=\frac{-1}{a-y}<0;\quad \frac{\partial^2\phi}{\partial x^2}=0$	linear decreasing in x
$\frac{\partial\phi}{\partial y}=\frac{a-x}{(a-y)^2}>0;\quad \frac{\partial^2\phi}{\partial y^2}=\frac{2(a-x)}{(a-y)^3}>0$	convex increasing in y
$\frac{\partial\phi}{\partial a}=\frac{x-y}{(a-y)^2}>0;\quad \frac{\partial^2\phi}{\partial a^2}=\frac{-2(x-y)}{(a-y)^3}<0$	concave increasing in a

<u>Table 5.1</u>: The indirect effects on ϕ by increases in x,y,a.

[16]Further elaboration on higher order derivatives is not useful here, because it will never give a definite answer on the simultaneous effects of x, y, a on the behaviour of $\phi(R)$, which is essentially a function of R.

From table 5.1, it can be seen that $\phi(x,y,a)$ is increasing in both y, a, but decreasing in x. The increase in y is convex hyperbolic, the increase in a is concave hyperbolic, while the decrease in x is simply linear. To arrive at an economic interpretation of changes in x,y,a on ϕ, consider the general form of the n^{th} order derivatives.

$$Let \quad \xi_i^n = \frac{(a-y)^n}{n!} \frac{\partial^n \phi}{\partial i^n}; \quad where \quad i \in \{x, \ y, \ a\};$$

$$then: \begin{cases} \xi_x^n = \begin{cases} -1 & n=1; \\ 0 & otherwise \end{cases} \\ \xi_y^n = \phi_\infty = \frac{a-x}{a-y} \in (0, 1) \\ \xi_a^n = (-1)^{n+1} \dfrac{\phi_\infty}{\phi_1} = (-1)^{n+1} \frac{x-y}{a-y} \in (-1, 1) \end{cases} \qquad (19)$$

This gives a complete qualitative expression for first and higher order changes in $\phi(x,y,a)$ with respect to x,y,a. Changes with respect to x sort out the highest effect on ϕ.

Lemma 5.2: $|\partial\phi/\partial x| > \max \{|\partial\phi/\partial y|, |\partial\phi/\partial a|\}$

Proof: It follows directly by comparing the differences in table 5.1 and by using the payoff-ordering of the prisoners' dilemma. Q.E.D.

Lemma 5.2 provides us a strong result. Increases in x do not only lower the critical discount factor, but the critical discount factor is also most sensitive to changes in x. This means that efforts to increase the payoff under pooling have the highest impact on the critical discount factor.

Changes with respect to y can be qualified as the trade-off between the net threat to deviate from mutual pooling and the net threat of deviation by the other, when one decides to pool. Changes with respect to a are qualified by the fraction of the critical discount factor under finite punishments over the critical discount factor under a single punishment.

This is the trade-off between the net incentive to pool and the net threat of deviation by the other, when one decides not to pool.

The behaviour of the critical discount factor $\phi(R)$ as a function of the level of natural resources R can be broadly divided into five different phases, when R varies from a value of total degradation ($R \approx 0$) to a value of maximal productivity ($R \to \infty$).

Phase 1.

Whenever the resources are too degraded it is very unlikely that any pooling will be realized. If the level of natural resources decreases even further, the *net* incentive to pool will be zero. This means that whenever the level of resources falls below a certain value (γ), it will not pay to pool mutually for any discount factor whatsoever. Formally, it can be written as follows.

$$\exists \gamma > 0 \ \forall \epsilon \leq \gamma \ \lim_{R \downarrow \epsilon} (x - y) = 0 \Rightarrow \lim_{R \downarrow \epsilon} \frac{a - x}{a - y} = \phi(R) = 1 \quad (18)$$

Therefore, whenever the level of resources are in the region $[0, \gamma]$, the critical discount factor $\phi(R) = 1$.

Phase 2.

As the resource level increases, the incentive to pool will also increase. This is logically equivalent to $\phi'(R) < 0$. This increase will continue up to a certain level of resources, say R^*. Either the first order derivative of the critical discount factor or the critical discount factor itself will become zero. In both cases the players will be entering phase 3. This minimum of the critical discount factor does not have to be unique. Whenever it is unique, phase 3 will consist of a single point, otherwise $\phi'(\check{R}) = 0$ or $\phi(\check{R}) = 0$ can hold for a particular region $\check{R} \in [R_*, R^*]$.

Phase 3.

In phase 3 the gross incentive to pool has reached its maximum. It will stay at its maximum until it leaves phase 3. When $\phi'(\check{R}) = 0$ for $\check{R} \in [R_*, R^*]$, $\phi(R)$ $(R > R^*)$ can enter either phase 2 ($\phi'(R) < 0$) or phase 4 ($\phi'(R) > 0$). When $\phi(\check{R}) = 0$ for $\check{R} \in [R_*, R^*]$, $\phi(R)$ $(\check{R} > R_*)$ can only enter phase 4[17].

Phase 4.

As the resource level increases further, the gross incentive to pool will decrease and the temptation to withdraw the pooled resources will grow. This is logically equivalent to $\phi'(R) > 0$. It is possible as long as $\phi(R) < 1$. In this phase, there are again two possibilities for the critical discount factor. Either the first order derivative will become zero or the critical discount factor will become one. The former means a maximum after which phase 2 will be entered again. In the latter case phase 5 will be entered.

Phase 5.

After a certain level of resources, say R', $\phi(R) \in [R',\rightarrow)$ will become equal to one and remain so after that. Hence, at this level of resources it is not beneficial to pool any more and both peasants will prefer to stop pooling.

Two examples are considered, within the prisoners' dilemma situation to derive some graphical examples of $\phi(R)$.

Example 5.1

Consider fixed threat rates. Then $a' = y' = 0$ which simplifies conditions 16 and 17 considerably.

[17]When the virtual function $\phi(R)$ will "hit the R-bar from below".

$$\phi'(R) \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0 \Leftrightarrow x' \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0$$

$$\phi''(R) \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0 \Leftrightarrow x'' \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0$$

Hence, the choice of x alone dictates the behaviour of $\phi(R)$. The effects of changes in x are depicted in the first row of table 5.1, where y and a are constant. Consider for example a parabola $x = x(R) = -a_1R^2 + a_2R + a_3$ $(a_1>0)$. The individual benefits, due to cooperation, increase until a maximum is reached. At this point, the critical discount factor function attains its lowest value. In other words, it is individually most beneficial to pool resources. After the maximum, the net gain from pooling decreases until the whole gain from pooling vanishes in R′. The graph of $\phi(R)$ has the following shape.

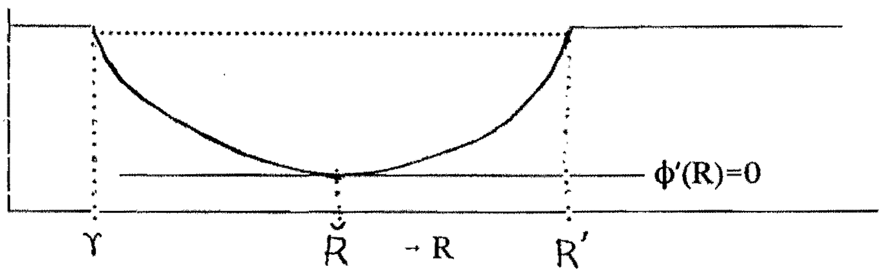

Figure 5.1: $\phi(R)$; a, y fixed; x is a parabola.

Hence this is an example where $\phi(R)$ goes through all phases.

The level of resources can go up and down over time, due to various reasons. It depends on the strategies of the users, but also on natural conditions which define the status quo of the resources, or to what extent the property rights of the community are recognized: Do the owners refrain from overuse? As long as the level of resources is fluctuating in the region between γ and R′, there will be a strictly positive gross incentive to pool and the pooling game is apparent.

Example 5.2

Consider the case where the payoff of mutual defection is zero: $y = 0$. This case still belongs to the class of prisoners' dilemma problems (see Stahl, 1991, 370). This can be realized by subtracting all payoffs in **game** P by y. The critical discount factor function can be written as follows.

$$\phi_{\{y=0\}}(R) = 1 - \frac{x}{a} \Leftrightarrow 1 - \phi(R) = \frac{x}{a}$$

The first and second order derivatives simplify to the following conditions.

$$\phi'_{\{y=0\}}(R) \gtreqless 0 \Leftrightarrow \frac{a'}{a} \gtreqless \frac{x'}{x}$$

$$\phi''|_{\{\phi'_{\{y=0\}}(R)=0\}}(R) \gtreqless 0 \Leftrightarrow \frac{a''}{a} \gtreqless \frac{x''}{x}$$

This means that the rate of change in threat payoff for deviating from pooling and the rate of change in the payoff for pooling determine the changes in $\phi(R)$. In this case the condition has been simplified, but the interpretation is easier than for the general case under conditions 16 and 17.

Consider in this second example linear approximations for a and x. Take $a = a(R) = a_1R + b_1$ and $x = x(R) = a_2R + b_2$. It is easy to verify that $\phi(R)$ has no optimum in this case, because $\phi'(R)$ $(=a_1b_2-a_2b_1)$ is always a constant. The linear approximation of the critical discount factor function has the following shape[18].

[18]This function has in general a vertical asymptote at $R^* = -b_1/a_1$ and a horizontal asymptote at $\delta^* = -(a_1-a_2)/a_1$, but for the present analysis only the part which falls in the regions $R \in [0,\infty)$ and $\delta \in [0,1]$ will be considered.

$$\phi(R) = \frac{-(a_2 - a_1)R + b_1 - b_2}{a_1 R + b_1}$$

$\phi(R)$ will have the following graph, which is part of a hyperbolic.

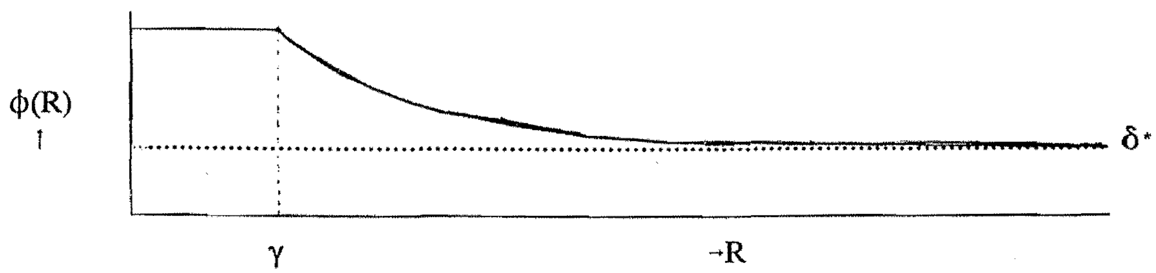

Figure 5.2: $\phi(R)$; y = 0; a and x are linear functions in R.

Here two cases can occur, since the value of $\phi'(R)$ (=constant) can be either positive or negative. If positive, the likelihood of pooling to occur will be ever decreasing in R and approach δ^* in the limit. If $\phi'(R)$ is negative, the likelihood will be ever increasing in R and will also approach δ^* in the limit. Hence, in this example $\phi(R)$ can attain only two or three phases at a time, once the functions x and a are derived by, for example, an estimation.

6 Cooperation in a repeated prisoners' dilemma

If the prisoners' dilemma is repeated a finite number of times with R kept fixed and the terminal time is common knowledge, then the following result is well known from the literature[19]. The equilibrium of subgame perfection is equivalent to the backward induction principle which prescribes to play (not,not) throughout the game. This is not the intuitively expected outcome.

Hardin (1971) shows that cooperation can occur by using the concept of a majority rule, which he calls a Condorcet choice. He shows that there can always be formed a weak or

[19]See, for example, Luce and Raiffa (1957).

strong majority which prefers cooperation as an outcome. Hence, some cooperation can be realized.

The discrepancy between the subgame perfect equilibrium of the finitely repeated prisoners' dilemma and the collective optimal outcome has been given a lot of attention by several authors[20]. This discrepancy can be overcome by relaxing some of the basic assumptions of the repeated prisoners' dilemma game. These ways out can be broadly divided into three classes.

First, a change in the punishment structure under <u>infinite repetitions</u> can yield cooperation as an equilibrium outcome of the finitely repeated prisoners dilemma. Hirschleifer and Rasmusen (1989) analyze an n-person prisoners' dilemma with ostracism. The punishment of a one period exclusion from the community can be severe enough to have cooperation as a long term equilibrium outcome. Matsushima (1990) finds an equilibrium in a random matching game when players continue with cooperating partners and opt for a new partner through a random matching procedure as soon as the present partner defects. Frohlich (1992) uses a notion of fairness "you cut and I choose" to generate cooperation as an equilibrium outcome. The players make a choice first. Their roles will be randomized, such that they cannot be sure what they will get.

Second, cooperation can occur when <u>the number of repetitions are unknown</u>. It has been formalized in Basu (1987) where he uses the concept of common knowledge. When the exact termination time does not have the property of common knowledge, cooperation is a possible equilibrium outcome. It has also been demonstrated experimentally by Axelrod (1990). He invited scientists from different disciplines to design a strategy for playing a finitely repeated prisoners' dilemma where the termination time is unknown. A tit-for-tat-strategy did very well: start with cooperation and do what the other did in the previous period. It also won the tournament over all strategies submitted. The requirement of unknown termination time has been criticized by L. Samuelson (1987). He derived a cooperative solution for the prisoners'

---

[20]For an overview see Seabright (1993, pg. 119) and Van Damme (1991, pg. 170).

dilemma where the termination time is not considered to be common knowledge, but private information. This argument has an overlap with the next class of incomplete information.

Third, Kreps *et al* 'admit a "small amount" of the right kind of incomplete information' to derive that the number of defections is bounded from above (1982, pg. 246). Cooperation can also occur when a players' reputation is taken into account. Kreps and Wilson (1982) analyze reputation, but they do not use the prisoners' dilemma; they base their analysis on the Chain Store Paradox. That game consists of one monopolist and one potential entrant in each period. The likelihood of an entrant to enter depends on the reputation of the monopolist to fight (at high cost) or to acquiesce. The concept of bounded rationality, where a player does not always opt for the expected most optimal outcome, can also lead to cooperation: by 'mistake' both can start cooperating, but pure rational players will never leave the equilibrium of (not,not). Fudenberg and Maskin (1986) analyze a situation where player i will be sane with probability ($1-\epsilon$) and play a 2-person game where mixed strategies are allowed. He will be crazy with probability ϵ and play (pool,pool) until the opponent defects, after which he will play (not,not). They proved this result which is known as a special case of the Folk theorem. The Folk theorem states that any point above the minimax point can be attained when the discount factor is sufficiently large.

Finally, these complications can also be avoided by considering other games. This has for example been done in section 7, where the prisoners' dilemma is imbedded in a game of three strategies.

7 Implications of adding one more strategy

The analysis so far relied mainly on the infinitely repeated prisoners' dilemma. To generalize the results, the prisoners' dilemma will be imbedded in a larger game by extending the strategy set. Let us allow for withdrawal of the pooled land during the process. Hence, the formerly assumed impossibility of withdrawal in *assumption 3.1 is no longer valid.* Two new assumptions are in order now.

Assumption 7.1: Each player decides to pool or not to pool at the beginning of each period.

Assumption 7.2: Withdrawal from the pool can take place within the period in such a way that it can not be observed until the beginning of the next period.

The extension of the prisoners' dilemma game with the withdrawal strategy can be represented by **game** Γ.

Game Γ:

2 / 1	Pool	Not	Withdraw (after Pooling)
Pool	$x(R_t),x(R_t)$	$b(R_t),a(R_t)$	$c(R_t),d(R_t)$
Not	$a(R_t),b(R_t)$	$y(R_t),y(R_t)$	$c(R_t),e(R_t)$
Withdraw (after pooling)	$d(R_t),c(R_t)$	$e(R_t),c(R_t)$	$c(R_t),c(R_t)$

If we keep player 2's strategy choice fixed, then the payoff under withdrawal for player 1 will be a value between pooling and not pooling, since he pools until withdrawal and after withdrawal he will not pool until the next period. Since the payoff matrix is symmetric, the same will hold for player 2. This yields the next payoff ordering.

$$a > d > x > y > e > b$$

Furthermore, if one player decides to withdraw, the other cannot guarantee a higher payoff than c, since the collective effort has yielded nothing. Let us assume the following.

Definition 7.1: y - c = the net increase in punishment payoff.

Assumption 7.3: $y > c$.

Hence, the net increase in punishment payoff is positive. This means that it is better for both not to pool at all, than to have pooling initially. A withdrawal of the pooled land or labour takes place during the process.

Assume that both players played (pool,pool) initially. As soon as one player decides to play either not pooling or withdraw at $t=t'$, the other will select withdraw until t^* and not pooling until the game terminates at time T in the endgame. T is assumed to be common knowledge of both players. This can be summarized in the following behavioral strategy σ_W:

i. Pool as long as both pooled in the previous periods; $t \in \{0, 1, ..., t'\}$.

ii. Withdraw as soon as one defection took place at least once in the previous periods; $t \in \{t'+1, ..., t^*\}$.

iii. Do not pool in the endgame, $t \in \{t^*+1, ..., T\}$.

This constitutes a trigger strategy, since it guarantees the defector significantly less, provided that both players give enough importance to future payoffs. Hence, their discount factor should not be lower than the following critical discount factor[21].

$$\delta > \frac{a - x}{a - c} = \phi_{fs}(R) \Leftrightarrow x \leq y \tag{20}$$

$$\delta > \frac{a - x}{a - x + y - c} = \phi_{f1}(R) \Leftrightarrow x > y \tag{21}$$

---

[21]This can be derived by applying theorem 1 by Friedman (1985, pg. 395-6) which states that the trigger strategy σ_W is subgame perfect. Hence, it is not beneficial to deviate. This is based on the following inequality.

$$x\sum_{t=0}^{t'-1}\delta^t + a\delta^{t'} + c\sum_{t=t'+1}^{T}\delta^t < x\sum_{t=0}^{t^*}\delta^t + y\sum_{t=t^*+1}^{T}\delta^t$$

Here defection takes place at t' and the endgame will start after t^* and the game terminates at time T.

Condition 20 is somewhat confusing, since assumption 3.4 is violated now, since $y \geq x$. One would expect (pool,pool) not to occur, just like case 6 in section 3 of the reverse battle of the sexes game. But even as the payoff of both pooling is lower than both not pooling, the threat of shifting to (withdraw, withdraw) can still make the cooperative outcome more beneficial.

The effects of $\phi_{fs}(x,a,c)$ are exactly same as denoted in table 5.1, when y is substituted by c. The effects of ϕ_{fl} are similar, but the derivatives differ, as denoted in table 7.1. $\phi_{fl}(x,y,a,c)$ is increasing in a,c, but decreasing in x,y. The role of y in ϕ_{∞} is again replaced by c in ϕ_{fl} which is the most severe threat payoff. Further, y is an intermediary value and represents a new effect on ϕ_{fl}.

$\frac{\partial\phi_{fl}}{\partial x}=\frac{-(y-c)}{(a-x+y-c)^2}<0;\quad \frac{\partial^2\phi_{fl}}{\partial x^2}=\frac{-2(y-c)}{(a-x+y-c)^3}<0$	convex decreasing in x
$\frac{\partial\phi_{fl}}{\partial y}=\frac{-(a-x)}{(a-x+y-c)^2}<0;\quad \frac{\partial^2\phi_{fl}}{\partial y^2}=\frac{2(a-x)}{(a-x+y-c)^3}>0$	concave decreasing in y
$\frac{\partial\phi_{fl}}{\partial a}=\frac{y-c}{(a-x+y-c)}>0;\quad \frac{\partial^2\phi_{fl}}{\partial a^2}=\frac{-2(y-c)}{(a-x+y-c)^3}<0$	concave increasing in a
$\frac{\partial\phi_{fl}}{\partial c}=\frac{a-x}{(a-x+y-c)^2}>0;\quad \frac{\partial^2\phi_{fl}}{\partial c^2}=\frac{2(a-x)}{(a-x+y-c)^3}>0$	convex increasing in c

Table 7.1: The direct effect of one by one increases in x, y, a, c on ϕ_{fl}.

Comparative statics can also be applied on ϕ_{fs} and ϕ_{fl} to derive the simultaneous effects of direct changes in R and indirect changes in x,y,a,c. A comparison with the results of section 5 are condensed in table 7.2.

$\gtreqless 0$	i = {p, fs, fl} $\phi_i'(R)$	$\phi_i''(R)\,\vert_{\{\phi_i'(R)=0\}}$
ϕ_p	$\frac{a'-x'}{a-x} \gtreqless \frac{a'-y'}{a-y}$	$\frac{a''-x''}{a-x} \gtreqless \frac{a''-y''}{a-y}$
ϕ_{fs}	$\frac{a'-x'}{a-x} \gtreqless \frac{a'-c'}{a-c}$	$\frac{a''-x''}{a-x} \gtreqless \frac{a''-c''}{a-c}$
ϕ_{fl}	$\frac{a'-x'}{a-x} \gtreqless \frac{y'-c'}{y-c}$	$\frac{a''-x''}{a-x} \gtreqless \frac{y''-c''}{y-c}$

Table 7.2: The first and second order conditions on $\phi_i(R)$.

8 Summary, conclusions and extensions

The main objective of this chapter was to obtain more insight in the possible situations a peasant at the village level can face when he has to decide whether to pool, to withdraw or not to pool.

An assessment of six different two-person dilemmas has been made to find the conditions under which pooling can be an equilibrium outcome. The hawk/dove game leads to a more intricate analysis than is required for the prisoners' dilemma. Two different trigger strategies have been formulated and compared with each other for the hawk/dove game. A two-step trigger strategy, where the defector punishes the defectee for implementing the trigger strategy to minimize the payoff of the defectee, was seen to lead to a smaller critical discount factor than under the ordinary trigger strategy, where one pools as long as the other did not defect first.

The critical discount factor is a decreasing function in the number of punishments and it converges to the critical discount factor of the infinitely repeated prisoners' dilemma when the number of punishments goes to infinity. Mutual pooling can be an equilibrium outcome under the threat of a single punishment. This holds when the net incentive to pool is strictly larger than the net threat to deviate from mutual pooling. When this condition is not satisfied, more punishments are required to make mutual pooling a possible equilibrium outcome.

The simultaneous changes in the payoff functions when the level of resources change endogenously has been analyzed with comparative statics. A sufficient and necessary condition for a maximum gross incentive to pool is that the rate of change of the net threat to deviate from mutual pooling and the net threat of deviation by the other, when one decides not to pool are equal. The condition that the payoff under mutual pooling is growing faster than other (threat) payoffs is sufficient to have a strictly increasing incentive to pool. Hence, this is the situation where more and more people would be willing to join the pooling process. Concerning the functional behaviour of the critical discount factor function it is useful to distinguish between five distinct phases when the level of resources goes from zero to infinity. This has been illustrated with two examples. This can form a basis for a dynamic behaviour of the effect of changes in the level of resources over time.

When the termination time is common knowledge, cooperation can also appear as an equilibrium outcome. In that case a three-strategy space has been considered. Trigger strategies have been formulated under which the defector is punished sufficiently. Imbedding the prisoners' dilemma, was not seen to lead to a change in the quantitative results under the original prisoners' dilemma.

With the help of the present analysis, it is an easy task to describe a transition process. Consider for example a prisoners' dilemma and an assurance game. In this case only the ordering of b and y differ (as can be seen from table 3.2). Assume that $b' > y'$. An interpretation can be given to this situation. Here player i already had pooled land, while player j did not yet pool, hence, the level of resources is increasing. If $b' > y'$ holds long enough, a transition will take place from the prisoners' dilemma to an assurance game. In similar manners, other transition processes can be analyzed as well.

The incorporation of institutional change into the model is complicated. There are certain difficulties which have to be resolved, before the effects of institutional change and its effect on the society can be understood completely. One difficulty is how to measure the efficiency of a particular institution. Provided that such a measure exists, the effects of endogenous changes in the efficiency of institutions can be modelled by substituting it for the level of resources in the model of this chapter. Hence, there is a need to look for a measure of efficiency based on property rights, transaction costs, norms, etc.

The intention of this chapter was to keep the problem simple. The analysis so far concentrated mainly on the case of two persons and two strategies. An extension to three strategies in the finitely repeated prisoners' dilemma also got attention. The number of repetitions played a major role in this context. The endogenous effects of resource changes on the payoff function was the special feature. Extending the game to higher dimensions is not an easy task and it may lead to essentially different results.

The concept of two persons can be continued even when 2n-person random matching games are considered. There the players change their partners over time (Kandori (1992) and Matsushima (1990)). This can reduce the bias of the partners and, hence, mutual pooling emerging as an equilibrium under a lower discount factor can be a possibility.

In this chapter an assumption of symmetry has been made, which restricted the analysis to equal peasants. This, of course, is not the case in rural India, which is known for its extreme duality between the 'castes' and the 'outcasts'. Heckathorn (1993) based his analysis on an escape from the n-persons prisoners' dilemma by introducing a compliance control level. Zero compliance means that the peasants rely on voluntary action only. Positive compliance also enhances group heterogeneity. Heckathorn demonstrates that cooperation is less likely in heterogenous groups.

Snidal (1985) came up with an important comparison between the prisoners' dilemma and the coordination game (which he applied to international cooperation). These two cases were in almost every aspect opposites. The comparison between these two polar cases also included relaxing the assumptions of the most simple one-shot bimatrix game.

- The effect of a continuous strategy space does not alter the strategic structure of the prisoners' dilemma, but the coordination game becomes a bargaining game with a continuum of equilibria; it is not easy to decide which equilibrium is most efficient.

- The effect of multiple players in the prisoners' dilemma yields the standard representation of the problem of collective action. For the coordination game it can facilitate coalition formation when there is also allowed for heterogeneity in the group as a whole.

In n-person games the notion of penalizing becomes a difficulty. Which punishment scheme is most appropriate? What kind of punishment scheme can be agreed upon within a community? Can such punishment scheme form a basis for another equilibrium analysis? An interesting result is found by Abreu (1988). He introduced simple penal codes in infinitely repeated games to derive that the collusive outcome can be an equilibrium outcome. With optimal penal codes it can be possible to derive lower discount factors than the one derived in this paper, because, penal codes can enhance an even more severe punishment scheme.

In this chapter I assumed that the peasants could not interact with each other concerning their intentions. Such an assumption does not always hold in reality. The very fact that farmers can discuss their intentions can be made a crucial incentive to generate cooperation. Hence, an important extension can be by undertaking a cooperative game approach. Okada (1991) already used such an approach. He demonstrated that the classical n-person prisoners' dilemma can be eliminated in four stages.

- Members decide if they want to participate;
- Members bargain for an enforcement agency;
- Members bargain for a proper punishment mechanism to eliminate the temptation to deviate, to be applied on all group members;
- Members come to an agreement or they do not.

In this game k members ($k \leq n$) participate and n-k members do not. Okada's four stage game has not yet been analyzed in a repeated framework. However, such an approach comes close to the problem studied in our context and can provide important clues for extensions on my present approach.

Appendix A1 Proof of theorem 4.1

In order to derive a proof of theorem 4.1, it suffices to prove both the left and the right implication.

⇒) (by contradiction).

$$\forall n:\ \phi_n \in (\phi_\infty\ \ \phi_1] \Rightarrow \phi_{n+1} < \phi_n \qquad (14)$$

Suppose there exists a ñ such that $\phi_{\tilde{n}+1} \geq \phi_{\tilde{n}}$. Since $\phi_{\tilde{n}}$ is a root of $f_n(\delta)$, we know that: $\phi_{\tilde{n}}^n + \phi_{\tilde{n}}^{n-1} + \dots + \phi_{\tilde{n}} - \phi_1 = 0$. From this follows that $\phi_{\tilde{n}+1}^n + \phi_{\tilde{n}+1}^{n-1} + \dots + \phi_{\tilde{n}+1} - \phi_1 \geq 0$, as supposed, and since $\phi_n > 0$. Multiply this with $\phi_{\tilde{n}+1}$ and subtract it by $f_{\tilde{n}+1}(\phi_{\tilde{n}+1})/(\delta-1)$ (=0). This leads to:

$$\phi_{\tilde{n}+1}^{n+1} + \phi_{\tilde{n}+1}^{n} + \dots + \phi_{\tilde{n}+1}^2 - \phi_1\phi_{\tilde{n}+1} \geq 0$$
$$-\phi_{\tilde{n}+1}^{n+1} - \phi_{\tilde{n}+1}^{n} - \dots - \phi_{\tilde{n}+1}^2 - \phi_{\tilde{n}+1} + \phi_1 = 0.$$

$$-(\phi_1+1)\phi_{\tilde{n}+1} + \phi_1 \geq 0 \text{ or } \phi_{\tilde{n}+1} \leq \phi_1/(1+\phi_1) = \phi_\infty.$$

This contradicts the assumption that for all n: $\phi_n \in (\phi_\infty, \phi_1]$. Q.E.D.

⇐) To prove that for all n: $(\phi_{n+1} < \phi_n) \Rightarrow \phi_n \in (\phi_\infty, \phi_1]$.

Hence, it suffices to prove that a strict ordering on ϕ_n is only possible when it is an element of the interval $(\phi_\infty, \phi_1]$.

There are two cases: i. For all n: $\phi_n \leq \phi_1$;

ii. There is no ñ such that: $\phi_{\tilde{n}} \leq \phi_\infty$.

<u>Case i</u>: We know that for all n: $\phi_{n+1} < \phi_n$, hence for all n: $\phi_1 > \phi_n$. Q.E.D.

<u>Case ii</u>: (by contradiction)

Suppose there is a finite ñ such that: $\phi_{\tilde{n}} \leq \phi_\infty$. Then the strict ordering to be derived can no longer be true, since if it was, then for all n>ñ: $\phi_n < \phi_{\tilde{n}} \leq \phi_\infty$. This means that the sequence $\{\phi_n\}$ will never attain ϕ_∞ as n goes to infinity. This contradicts the result of lemma 4.1.

Q.E.D.

References

Abreu, D., 1988, On the theory of infinitely repeated games with discounting, **Econometrica 56(2)**: 383-396.

Axelrod, Robert, 1984, The evolution of cooperation, Penguin books, London, pp. 229.

Bardhan, Pranab, 1993, Analytics of the institutions of informal cooperation in rural development, **World Development 21(4)**: 633-639.

Basu, Kaushik, 1987, Modelling finitely-repeated games with uncertain termination, **Economic letters 23**: 147-151.

Basu, Kaushik and Ajit Mishra, 1993, Sustainable development and the commons problem, a simple approach, in Pranab Bardhan, Mrinal Datta Chaudhuri, and T.N. Krishnan, (eds.) **Development and Change, essays in honour of K.N. Raj**, Oxford University press, Bombay: 90-101.

Bates, Robert H., 1988, Contra contractarian, some reflection on the new institutionalism, **Politics and Society 16(2-3)**: 387-401.

Bromley, Daniel W. and Michael M. Cernea, 1989, The management of common property natural resources: some conceptual and operational fallacies, The World Bank, Washington, pp. 66.

Friedman, James W., 1985, Cooperative Equilibria in Finite Horizon Noncooperative Supergames, **Journal of Economic theory 35**: 390-398.

Frohlich, Norman, 1992, An impartial reasoning solution to the prisoners' dilemma, **Public Choice 74**: 447-460.

Fudenberg, Drew and Eric Maskin, 1986, The Folk theorem in repeated games with discounting or incomplete information, **Econometrica 54(3)**: 533-554.

Green, Edward J. and Robert H. Porter, 1984, Noncooperative collusion under imperfect price information, **Econometrica 52(1)**: 87-100.

Hardin, Garrett, 1968, The tragedy of the commons, **Science 162**: 1243-1248.

Hardin, Garrett and John Baden (eds.), 1977, Managing the Commons, W.H.Freeman and Company, San Francisco.

Hardin, Russell, 1971, Collective action as an agreeable n-person prisoners' dilemma, **Behavioral Science 16**: 472-81.

Hardin, Russell, 1982, Collective action, John Hopkins University press, Baltimore and London, Resources for the future, Inc.

Heckathorn, D., 1993, Collective action and group heterogeneity, voluntary provision versus selective incentives, **American Sociological Review 58(3)**: 329-350.

Hirschleifer, David and Eric Rasmusen, 1989, Cooperation in a repeated prisoners' dilemma with ostracism, **Journal of Economic Behavior and Organization 12**: 87-106.

Kandori, Michihiro, 1992, Social norms and community enforcement, **Review of Economic Studies 59(1)**: 63-80.

Kreps, David M., Paul Milgrom, John Roberts, and Robert Wilson, 1982, Rational Cooperation in the Finitely Repeated Prisoners' Dilemma, **Journal of Economic theory 27**: 245-252.

Kreps, David M. and Robert Wilson, 1982, Reputation and imperfect information, **Journal of Economic Theory 27**: 253-279.

Lise, Wietze, 1995, Participatory development, a path to ecological preservation, An approach with case studies and modelling, **International Journal of Environment and Pollution 5(1)**, forthcoming.

Luce, Duncan and Howard Raiffa, 1957, Games and Decisions, Introduction and critical survey, Wiley, New York, Chapter 5, pp. 484.

Matsushima, Hitoshi, 1990, Long-term partnership in a repeated prisoners' dilemma with random matching, **Economic Letters 34**: 245-248.

Okada, A., 1993, The possibility of cooperation in an n-person prisoners' dilemma with institutional arrangements, **Public Choice 77(3)**: 629-56.

Olson, M., 1965, The logic of collective action, public goods and the theory of groups, Harvard University press, Cambridge.

Ostrom, E., 1987, Institutional arrangements for resolving the commons dilemma, in McKay, B. and J. Acheson (eds.), The question of the commons: the culture and ecology of communal resources, Tucson, University of Arizona Press: 250-265.

Ostrom, Elinor, 1990, Governing the commons, The Evolution Institutions for Collective Action, Cambridge University Press, Cambridge, pp. 280.

Pearce, David and Jeremy J. Warford, 1993, World without end, economics, environment and sustainable development, Oxford University press, New York, pp.440.

Runge, Carlisle Ford, 1981, Common property externalities: isolation, assurance and resource depletion in a traditional grazing context, **American Journal of Agricultural Economics 63**: 595-606.

Runge, Carlisle Ford, 1984, Institutions and the free rider: the assurance problem in collective action, **The Journal of Politics 46**: 154-81.

Runge, Carlisle Ford, 1986, Common property and collective action in economic development, **World Development 14**: 623-635.

Samuelson, L., 1987, A note on uncertainty and cooperation in a finitely repeated prisoners' dilemma, **International Journal of Game Theory 16(3)**: 187-195.

Seabright, Paul, 1993, Managing Local Commons: Theoretical Issues in Incentive Design, **Journal of Economic Perspectives 7:4**: 113-134.

Snidal, D., 1985, Coordination versus prisoners' dilemma: implications for international cooperation and regimes, **American Political Science Review 79**: 923-47.

Stahl, D., 1991, The graph of the prisoners' dilemma supergame payoff's as a function of the discount factor, **Games and Economic Behavior 3(3)**: 368-84.

Taylor, Michael, 1987, The possibility of cooperation, Cambridge University press, Cambridge.

Taylor, M. and S. Singleton, 1993, The communal resource, transaction costs and the solution of collective action problems, **Policy and Society 21(2)**: 195-214.

Uphoff, N., 1993, Grassroots and NGO's in rural development: opportunities with diminishing states and expanding markets, **World Development 21(4)**: 607-22.

Van Damme, Eric, 1991, Stability and perfection of Nash equilibria, second revised and enlarged edition, Springer Verlag, Berlin, pp. 339.

Wade, Robert, 1988, Village republics, economic conditions for collective action in South India, Cambridge University Press, Cambridge, pp. 220.

CENTRE FOR DEVELOPMENT ECONOMICS
WORKING PAPER SERIES

No.	Author(s)	Title
1	Kaushik Basu Arghya Ghosh Tridip Ray	The Babu and The Boxwallah : Managerial Incentives and Government Intervention (Jan 1994)
2	M.N. Murty Ranjan Ray	Optimal Taxation and Resource Transfers in a Federal Nation (Feb 1994)
3	V. Bhaskar Mushtaq Khan	Privatization and Employment : A Study of The Jute Industry in Bangladesh (Mar 1994)
4	V. Bhaskar	Distributive Justice and The Control of Global Warming (Mar 1994)
5	Bishnupriya Gupta	The Great Depression and Brazil's Capital Goods Sector : A Re-examination (Apr 1994)
6	Kaushik Basu	Where There Is No Economist : Some Institutional and Legal Prerequisites of Economic Reform in India (May 1994)
7	Partha Sen	An Example of Welfare Reducing Tariff Under Monopolistic Competition (May 1994)
8	Partha Sen	Environmental Policies and North-South Trade : A Selected Survey ofThe Issues (May 1994)
9	Partha Sen Arghya Ghosh Abheek Barman	The Possibility of Welfare Gains with Capital Inflows in A Small Tariff-Ridden Economy (June 1994)
10	V. Bhaskar	Sustaining Inter-Generational Altruism when Social Memory is Bounded (June 1994)
11	V. Bhaskar	Repeated Games with Almost Perfect Monitoring by Privately Observed Signals (June 1994)
12	S. Nandeibam	Coalitional Power Structure in Stochastic Social Choice Functions with An Unrestricted Preference Domain (June 1994)
13	Kaushik Basu	The Axiomatic Structure of Knowledge And Perception (July 1994)
14	Kaushik Basu	Bargaining with Set-Valued Disagreement (July 1994)

No.	Author(s)	Title
15	S. Nandeibam	A Note on Randomized Social Choice and Random Dictatorships (July 1994)
16	Mrinal Datta Chaudhuri	Labour Markets As Social Institutions in India (July 1994)
17	S. Nandeibam	Moral Hazard in a Principal-Agent(s) Team (July 1994)
18	D. Jayaraj S. Subramanian	Caste Discrimination in the Distribution of Consumption Expenditure in India: Theory and Evidence (August 1994)
19	K. Ghosh Dastidar	Debt Financing with Limited Liability and Quantity Competition (August 1994)
20	Kaushik Basu	Industrial Organization Theory and Developing Economies (August 1994)
21	Partha Sen	Immiserizing Growth in a Model of Trade with Monopolisitic Competition (August 1994)
22	K. Ghosh Dastidar	Comparing Cournot and Bertrand in a Homogeneous Product Market (Sept. 1994)
23	K. Sundaram S.D. Tendulkar	On Measuring Shelter Deprivation in India (Sept. 1994)
24	Sunil Kanwar	Are Production Risk and Labour Market Risk Covariant? (Oct. 1994)
25	Partha Sen	Welfare-Improving Debt Policy Under Monopolistic Competition (Nov. 1994)
26	Ranjan Ray	The Reform and Design of Commodity Taxes in the presence of Tax Evasion with Illustrative Evidence from India (Dec. 1994)
27	Wietze Lise	Preservation of the Commons by Pooling Resources, Modelled as a Repeated Game (Jan. 1995)