INVESTMENT IN A TWO-SECTOR DEPENDENT ECONOMY

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ABSTRACT

This paper analyzes capital accumulation in a dependent economy model. When the non-traded good is used for investment we obtain a saddle-point structure irrespective of sectoral capital intensities. But relative capital intensities determine how the real exchange rate moves over time. Some comparative dynamic exercises are performed with different capital intensity assumptions.
1. INTRODUCTION

The dependent economy model of Salter (1959) and Swan (1960) is a basic workhorse of international macroeconomics. By distinguishing between traded and nontraded goods, it provides a convenient general equilibrium framework for analyzing the behavior of the real exchange rate both in a static and dynamic context. While the earlier studies can be characterized as being purely descriptive, in more recent years, these models have become increasingly grounded in optimizing behavior; see e.g. Dornbusch (1983), Edwards (1989).

Several authors have begun to incorporate capital formation into this framework; see e.g. Razin (1984), Murphy (1986), Brock (1988), and Obstfeld (1989). This development, while being of considerable importance for policy analysis, is also of interest to those who want to relate this literature to the standard two-sector optimal growth model, and to earlier open economy extensions which were based on the Heckscher-Ohlin technology; see e.g. Fischer and Frenkel (1972), Bazdarich (1978), and Matsuyama (1988).

Once the distinction between traded and nontraded goods is introduced, how investment is to be classified becomes important. At an intuitive level, investment can reasonably fall into either category. Capital goods, taking the form of infrastructure and construction, are presumably nontraded; investment goods in the form of machinery or inventories are obviously potentially tradeable. Different treatments of investment, reflecting these different possibilities, can be found in the literature. For example, Obstfeld (1989), while allowing for capital to be instantaneously moveable between sectors, assumes that only the traded good is used for investment. He therefore allows the capital stock to be instantaneously augmented at any point in time by an exchange of traded financial assets for capital. Brock (1988) also treats capital as being traded, though the investment process involves convex costs of adjustment, thereby constraining the rate of investment at any point in time to remain finite. By contrast, early authors such as Frenkel and Fischer (1972), and more recently Marion (1984), Murphy (1986), Turnovsky (1991), van Wincoop (1993), and Brock (1993) also analyze models in which investment is treated as being nontraded.
In this paper, we further analyze the process of capital accumulation in a two sector model of a small open economy producing nontraded as well as traded goods. Most of our attention is devoted to the case where capital is nontraded. However, the case of traded capital is also briefly discussed, with the purpose of trying to draw out the similarities, as well as the differences of the structures. In the absence of any installation costs, it is immediately seen that if traded goods are used for investment, then the instantaneous adjustment of the capital stock obtains.

But on the other hand, if the capital accumulation is in the form of the nontraded good, then even in the absence of adjustment costs associated with investment, nondegenerate dynamics are obtained. The rate of investment remains finite due to the fact that the supply of traded goods is subject to increasing marginal cost. In other words, these increasing marginal costs play the same role as adjustment costs in the traded case. But the most interesting aspect of the dynamics is that it involves a saddlepoint structure, irrespective of the relative capital intensities of the two sectors. This is in contrast to the early model, with fixed terms of trade, in which the dynamics is known to be unstable, if the sector producing the investment good is relatively capital intensive.3

Although a saddle point is always obtained, the nature of the dynamics turns out to depend critically upon the relative capital intensities of the two sectors. First, if the traded good is the more capital intensive, the adjustment of the real exchange rate to any unanticipated permanent shock occurs immediately. The subsequent accumulation or decumulation of capital in response to such a shock takes place with no concurrent change in the real exchange rate. By contrast, if the nontraded sector is the more capital intensive, then any initial adjustment in the real exchange rate is only partial. The transitional adjustment in the capital stock is accompanied by an accompanying change in the real exchange rate. In this respect, we find that some of the characteristics found to hold in the more complex three sector models of Turnovsky (1991) and van Wincoop (1993) apply in this simpler two sector set up.

Before proceeding with the analysis, we wish to relate this paper to the literature, and in particular those studies which treat investment as being nontraded. There is a wide divergence among these papers in terms of: (i) the types of disturbances they consider; (ii) the time horizon of the
analysis; and (iii) the specifics of the production structure. In relating the present contribution to the literature, we wish to make it clear that the primary objective of the present study is to provide a characterization of the dynamics of the economy in the face of demand and supply shocks, within an infinite horizon intertemporal optimizing framework, emphasizing the role of the relative sectoral capital intensities in this process. Marion (1984) and Murphy (1986) are both restricted to two period time horizons; Marion analyzes oil shocks, while Murphy discusses productivity shocks. Transversality conditions, which play a central role in the present analysis, are much less significant in these two period models. Some of the specific differences between the results obtained by Murphy and those obtained in this paper with respect to productivity shocks, are noted in the concluding section. Turnovsky (1991) and van Wincoop (1993) both incorporate nontraded investment into a fully intertemporal three-sector framework. However, both address very different sets of issues from those to be discussed here. Turnovsky is concerned with analyzing the sectoral impacts of tariffs, while van Wincoop discusses the impact of a resource discovery, or the so-called "Dutch disease" problem. Finally, Brock (1993) also considers the Dutch disease issue, modeling it as a transfer of income from abroad. His particular focus is on analyzing its impact on the current account, under alternative production structures, rather than just the pure Heckscher-Ohlin technology of this paper.

The emphasis of the analysis on nontraded investment is much more general than may at first appear. A recent paper by Brock and Turnovsky (1993) has begun to integrate both traded and nontraded investment into a single unified framework, a task which had previously been generally thought to be intractable. One of their initial conclusions is that the fundamental dynamic characteristics of this integrated model are determined exclusively by the relative sectoral intensities in nontraded capital alone, as in this model. This implies that as long as the economy utilizes some nontraded capital in production, the exclusion of traded investment involves no essential loss of generality, at least insofar as the fundamental dynamic structural characteristics are concerned. While Brock and Turnovsky illustrate their model by analyzing the transfer of foreign income, as in Brock (1993) and van Wincoop (1993), the dynamic structure they identify supports the analytical framework of the present model as being relevant to a variety of real world issues.
The remainder of the paper is structured as follows. Section 2 sets out the model in the case where investment goods are assumed to be nontradeable. Section 3 and 4 illustrate the behavior of such an economy by analyzing the dynamic responses to: (i) permanent demand shocks, taking the form of fiscal expenditures on the traded and nontraded good; and (ii) permanent supply shocks, taking the form of productivity disturbances in the two sectors. One characteristic of the equilibrium is that it depends upon the initial stocks of assets. In most instances this gives rise to hysteresis; i.e. temporary shocks have permanent effects. This issue is discussed briefly in Section 5. Section 6 briefly considers the case where the investment good is traded, while conclusions are reviewed in the final section.

2. TWO-SECTOR SMALL OPEN ECONOMY

A. Economic Structure

Consider a small economy inhabited by a single infinitely-lived representative agent who is endowed with a fixed supply of labor, (normalized to be one unit), which he sells at the competitive wage, and who accumulates capital, $K$, for rental at the competitively determined rental rate. The agent produces a traded good $T$ (taken to be the numeraire) using a quantity of capital $K_T$ and labor $L_T$, by means of a neoclassical production function $F(K_T, L_T)$. That is, both capital and labor are assumed to have positive, but diminishing, marginal physical products and to be subject to constant returns to scale. He also produces a nontraded good using a quantity of capital $K_N$ and labor $L_N$, by means of a second production function $G(K_N, L_N)$, having the same neoclassical properties. Until Section 6 below, we assume that the traded good is used for consumption, while the nontraded good may be used either for consumption or investment.

The agent also accumulates net foreign bonds, $B$, that pay an exogenously given world interest rate $r$. Equation (1a) describes the agent’s instantaneous budget constraint:

$$ \dot{B} = F(K_T, L_T) + \sigma G(K_N, L_N) - C_T - \sigma C_N - \sigma I - Z + rB \quad (1a) $$
where \( C_T, C_N \) are the agent's consumption of the traded and nontraded good respectively; \( \sigma \) is the relative price of the nontraded to the traded good, or the real exchange rate; \( I \) denotes investment; \( Z \) denotes lump-sum taxes. We assume that the capital stock does not depreciate, implying the standard capital accumulation constraint:

\[
\dot{K} = I
\]  

(1b)

As formulated, (1b) permits negative investment. The usual interpretation of this is that the agent is permitted to consume his capital stock or to sell it in the market for new output. Alternatively, one can incorporate negative net investment, while constraining gross investment to be nonnegative, by allowing capital to depreciate. However, no significant losses are incurred by adopting the simpler formulation. The allocation of labor and capital between the two sectors is constrained by

\[
L_T + L_N = 1
\]  

(1c)

\[
K_T + K_N = 1
\]  

(1d)

The agent’s decisions are to choose his consumption levels \( C_T, C_N \), labor allocation decisions, \( L_T, L_N \), the rate of investment \( I \), and the capital allocation decisions, \( K_T, K_N \), to maximize the intertemporal utility function:

\[
\int_0^\infty U(C_T, C_N) e^{-\beta t} dt
\]

(2)

subject to the constraints (1a) - (1d) and given initial stocks \( K(0) = K, B(0) = B \). The instantaneous utility function is assumed to be concave and the two consumption goods are assumed to be normal. The agent’s rate of time preference is \( \beta \) and is taken to be constant.

This is a standard intertemporal optimization problem. It is straightforward to show that the optimality conditions are:

\[
U_T(C_T, C_N) = \lambda
\]  

(3a)
\[ U_N(C_T, C_N) = \lambda \sigma \]  
\[ \frac{1}{\sigma} F_K(K_T, L_T) = G_K(K_N, L_N) \]  
\[ \frac{1}{\sigma} F_L(K_T, L_T) = G_L(K_N, L_N) = w \]  
\[ \frac{\dot{\lambda}}{\lambda} = \beta - r \]  
\[ \frac{\dot{\sigma}}{\sigma} + G_K(K_N, L_N) = r \]

together with the transversality conditions

\[ \lim_{t \to \infty} \lambda Be^{-\beta t} = \lim_{t \to \infty} \lambda \sigma Ke^{-\beta t} = 0 \]

where \( \lambda \), the Lagrange multiplier associated with the wealth constraint (1a), is the shadow value of wealth.

One important issue in models of small open economies such as the present concerns the relationship between the rate of time discount \( \beta \) and the world interest rate \( r \). With both of these being exogenously given constants, in order for (3e) to imply a non-zero finite steady-state value for the marginal utility \( \lambda \), and therefore consumption, we require \( \beta = r \). But this further implies \( \dot{\lambda} = 0 \), for all \( t \), so that the marginal utility \( \lambda \) remains constant over all time, i.e. \( \lambda = \overline{\lambda} \), say. As discussed by Sen and Turnovsky (1990) this has important consequences for the dynamics, some of which will be explored below, in the context of this model.

The assumption that the rate of time preference in the small economy equals the given world rate of interest is the standard assumption in virtually all of this literature of a small open economy, based on intertemporal optimization. But this is what is required if an interior equilibrium is to be attained, when \( \beta \) and \( r \) are both constant. One justification is that a small open economy, facing a perfect world capital market, must constrain its rate of time preference by the investment opportunities available to it, which are ultimately determined by the exogenously given rate of return in the world
capital market. For if that were not the case, the domestic agent would end up either in infinite debt or in infinite credit to the rest of the world and that would not represent a viable interior equilibrium. The economy would cease to be a small open economy.

While the assumption $\beta = r$ is not unreasonable, it is nevertheless restrictive and has been a point of criticism of this model. How acceptable it is depends in part upon the specific shock one is analyzing. For the demand and supply shocks we shall consider, both of which leave $\beta$ and $r$ unchanged, it is adequate. However, it would be inappropriate if one wished to analyze changes in either $\beta$ or $r$, which would break the assumed equality between them. In view of this, one alternative has been to allow the rate of time preference to be variable. This approach was first adopted by Obstfeld (1981), though in the absence of capital, where he does so by endogenizing the consumer rate of time preference through the introduction of Uzawa (1968) preferences. Assuming $\beta = \beta[U(C_T, C_N)]$, one modification to the optimality conditions involves replacing (3e) by the relationship

$$\frac{\dot{\lambda}}{\lambda} = \beta[U(C_T, C_N)] - r \quad (3e')$$

There are two persuasive reasons for not pursuing this approach. First, the rationale for the restrictions on the function $\beta$ necessary to ensure stability are themselves not particularly convincing and subject to their own criticisms; see e.g. Obstfeld (1981), Razin and Svensson (1983). Secondly, it turns out that our main result -- namely the dependence of transitional dynamics of the real exchange rate upon the relative capital intensities of the two sectors -- remains fully intact even if the rate of preference is modified in this way. This is because the equilibrium real exchange rate is determined entirely by the production structure and is thus independent of demand conditions.

The optimality conditions (3) are familiar and require little comment. Equations (3a) and (3b) are the usual conditions equating the marginal utility of consumption to the shadow value of wealth. Equations (3c) and (3d) equate the marginal physical products of the two factors in the two sectors across which they are mobile. Equations (3e) and (3f) are arbitrage conditions. The latter equates the
instantaneous rate of return on nontraded capital, which consists of its marginal physical product plus capital gain, to the rate of return on the traded bond.

The other agent in the economy is the government, which plays a simple role. It simply raises lump sum taxes to finance its expenditures on the traded and nontraded good, \( G_T \) and \( G_N \), respectively, in accordance with

\[
G_T + \sigma G_N = Z \tag{4}
\]

### B. Macroeconomic Equilibrium

Defining:

\[
k_i = K_i / L_i \text{ to be the capital-labor ratio in sector } i, \quad i = T,N,
\]

\[
\rho(\equiv L_T) \quad \text{to be the fraction of labor employed in the traded good sector,}
\]

\[
f(k_T) = F(K_T, L_T) / L_T, \quad g(k_N) = G(K_N, L_N) / L_N,
\]

be the corresponding production functions expressed in intensive form, enables the macroeconomic equilibrium to be summarized by the following set of relationships:

\[
U_T(C_T, C_N) = \bar{\lambda} \tag{5a}
\]

\[
U_N(C_T, C_N) = \bar{\lambda} \sigma \tag{5b}
\]

\[
f'(k_T) = \sigma g'(k_N) \tag{5c}
\]

\[
f(k_T) - k_T f'(k_T) = \sigma [g(k_N) - k_N g'(k_N)] \tag{5d}
\]

\[
\rho k_T + (1-\rho) k_N = K \tag{5e}
\]

\[
\dot{\sigma} = \sigma [r - g'(k_N)] \tag{6a}
\]

\[
\dot{K} = (1-\rho)g(k_N) - C_N - G_N \tag{6b}
\]

\[
\dot{B} = \rho f(k_T) - C_T - G_T + rB \tag{6c}
\]
Equations (5a)-(5d), (6a) correspond to (3a)-(3d), (3f) respectively, while (5e) describes the capital allocation relationship in sectoral per capita terms. Equations (6b), (6c) specify market clearing conditions. The former describes equilibrium in the nontraded goods market. Any output in excess of domestic private or government consumption is accumulated as capital. The latter describes the economy's current account. The rate of accumulation of traded bonds equals the excess of the domestic supply of the traded good over domestic consumption of that good, plus the interest earned on the outstanding stock of foreign bonds.

The set of equations (5a) - (5e) define a short-run equilibrium, which may be solved very simply, as follows. First, the marginal utility conditions (5a), (5b) may be solved for consumptions $C_T, C_N$ in the form

$$C_T = C_T(\bar{\lambda}, \sigma)$$

$$C_N = C_N(\bar{\lambda}, \sigma)$$

where

$$\frac{\partial C_T}{\partial \lambda} < 0; \quad \frac{\partial C_T}{\partial \sigma} > 0; \quad \frac{\partial C_N}{\partial \lambda} < 0; \quad \frac{\partial C_N}{\partial \sigma} < 0.$$ 

Secondly, from the production block (5c) - (5e), we may derive

$$k_T = k_T(\sigma)$$

$$k_N = k_N(\sigma)$$

$$\rho = \rho(K, \sigma)$$

where

$$k'_T = \frac{g}{f''(k_N - k_T)}; \quad k'_N = \frac{g}{\sigma^2 g''(k_N - k_T)}$$

$$\frac{\partial \rho}{\partial K} = \frac{1}{k_T - k_N}; \quad \frac{\partial \rho}{\partial \sigma} = \left[ \frac{(1-\rho)g}{\sigma^2 g''} + \frac{g}{f''} \right] \frac{1}{(k_T - k_N)^2} < 0.$$
As is well known from two-sector trade models, the signs in (8a) - (8c) depend upon sectoral capital intensities. For example, a rise in the relative price of the nontraded good \( \sigma \) will cause resources to move from the traded to the nontraded sector. If the latter sector is more capital intensive, capital increases in relative scarcity, causing the wage-rental ratio to fall and inducing the substitution of labor for capital in both sectors.

Equations (6a) - (6c) describe the dynamics and can be solved recursively as followings. First, substituting the solutions for \( C_T, C_N, k_T, k_N, \rho \) into (6a) and (6b) leads to two equations describing the dynamics of the evolution of capital \( k \) and the real exchange rate \( \sigma \). Next, substituting the solutions obtained for \( k \) and \( \sigma \) into (6c) one can obtain the evolution of the economy's claims against the rest of the world.

C. Equilibrium Dynamics

Performing the substitution into (6a) and (6b), and linearizing about steady state (denoted by tildes), the dynamics of \( K \) and \( \sigma \) can be approximated by

\[
\begin{pmatrix}
\dot{\sigma} \\
\dot{K}
\end{pmatrix} =
\begin{pmatrix}
a_{11} & 0 \\
a_{21} & a_{22}
\end{pmatrix}
\begin{pmatrix}
\sigma - \bar{\sigma} \\
K - \bar{K}
\end{pmatrix}
\]

(9)

where

\[
a_{11} \equiv -\frac{f}{\sigma(k_N - k_T)}; \quad a_{22} \equiv \frac{g}{k_N - k_T}
\]

\[
a_{21} \equiv -\frac{1}{(k_T - k_N)}\left[ \frac{(1-\rho)f^2 + \rho g^2}{\sigma^3 g'' + f''} \right] - \frac{\partial C_N}{\partial \sigma} > 0.
\]

Since \( a_{11}a_{22} < 0 \), the dynamics is always a saddlepoint, irrespective of the relative capital intensities \( k_T, k_N \). We shall denote the eigenvalues by \( \mu_1 < 0, \mu_2 > 0 \). While the capital stock always evolves gradually, the relative price \( \sigma \) may jump in response to new information. The stable solution is of the form

\[
K(t) = \bar{K} + (K_0 - \bar{K})e^{\mu_1 t}
\]

(10a)
The dynamic behavior of the economy depends crucially upon the relative sectoral capital intensities and the two cases $k_t > k_N, k_N > k_t$, need to be considered separately.

**Case (i):** $k_t > k_N$  
This assumption asserts that the capital intensity of the traded good sector exceeds that of the nontraded good sector. It implies that $\mu_1 = a_{22} < 0, \mu_2 = a_{11} > 0$ so that the stable path (10a), (10b) is

$$K(t) = \tilde{K} + (K_0 - \tilde{K})e^{\mu t}$$  
$$\sigma(t) = \tilde{\sigma}$$

In this case, the relative price of the nontraded good remains constant at its steady-state level during the dynamic evolution of the economy. It just moves along a Rybczynski line.

**Case (ii):** $k_N > k_t$  
The contrary case, where the nontraded sector is more capital intensive yields $\mu_1 = a_{11} < 0, \mu_2 = a_{22} > 0$ and the stable adjustment path now becomes

$$K(t) = \tilde{K} + (K_0 - \tilde{K})e^{\mu t}$$  
$$\sigma(t) - \tilde{\sigma} = \left(\frac{a_{22} - a_{11}}{a_{21}}\right)(K(t) - \tilde{K})$$

The stable arm is now negatively sloped. In this case, a shock (such as an increase in demand) which leaves the steady state real exchange rate $\tilde{\sigma}$ unchanged, causes a rise in $\sigma$ in the short run, (i.e. a real appreciation), so that resources can move to the nontraded sector and enable capital accumulation to take place. Here, while the long-run equilibria are connected by a Rybczynski line, during the transition, the economy is necessarily off this locus.

The striking feature of the stable transitional adjustment paths described by (10a'), (10b') and (10a''), (10b'') is the qualitative dependence of the behavior of the relative price $\sigma$ on the relative
capital intensities of the two sectors. In part, this is because \( \sigma \) is also playing the role of an asset price. The fact that in the case where \( k_T > k_N \), \( \sigma \) remains unchanged during the transition can be seen by considering the arbitrage relationship (6a) in the form

\[
\frac{\dot{\sigma}}{\sigma} + g'(k_N) = r
\]

Suppose that instead of remaining fixed over time, \( \sigma \) were increasing. Then as \( \sigma \) increases, \( k_N \) increases, so that the marginal physical product \( g'(k_N) \) declines. In order to ensure that the rate of return on capital equals the exogenously given return on bonds, this requires \( \dot{\sigma} > 0 \), that is, a further increase in \( \sigma \), and this is clearly an unstable path. The same applies if \( \sigma \) is decreasing over time. An unchanging relative price is the only stable alternative. On the other hand, if \( k_N > k_T \), then an increasing \( \sigma \) is associated with \( \dot{\sigma} < 0 \) and this is clearly a stable process.

D. Foreign Asset Accumulation, Investment, and Savings

To determine the accumulation of foreign bonds, we consider (6c) expressed in terms of \( \sigma, K \) as follows:

\[
\dot{B} = \rho(K, \sigma) f[k_T(\sigma)] - C_T(\bar{K}, \sigma) - G_T + rB \tag{6c'}
\]

and apply the procedure discussed by Sen and Turnovsky (1990), Turnovsky (1991). This involves linearizing this equation, substituting for (10a), (10b), and invoking the transversality condition (3g). The upshot is that starting from an initial stock of traded bonds \( B_o \), the stable adjustment, consistent with intertemporal solvency, is

\[
B(t) - \bar{B} = \frac{\Omega}{\mu_1 - r} [K(t) - \bar{K}] = \frac{\Omega}{\mu_1 - r} [K_o - \bar{K}] e^{\mu_1 t} \tag{11a}
\]

with

\[
B_o - \bar{B} = \frac{\Omega}{\mu_1 - r} [K_o - \bar{K}] \tag{11b}
\]
\[
\Omega = f \frac{\partial \rho}{\partial K} + \left[ f \frac{\partial \rho}{\partial \sigma} + \sigma f' \frac{\partial k_f}{\partial \sigma} - \frac{\partial C_f}{\partial \sigma} \right] \left( \mu_1 - a_{12} \right) \rho
\]

The expression \( \Omega \) describes the instantaneous effect of an increase in the capital stock on the current account. This may operate through two channels, directly and indirectly through the real exchange rate \( \sigma \).

If \( k_T > k_N \), so that \( \sigma \) remains fixed over time, only the first effect is operative. In this case \( \Omega \equiv \frac{f}{(k_T - k_N)} > 0 \), and using the steady-state condition (12a) below, one can further show \( 10 \)

\[
\frac{\Omega}{\mu_1 - \rho} = -\sigma; \quad \text{i.e.} \quad \dot{B}(t) = -\sigma \dot{K}(t)
\]

An increase in \( K \), lowers \( \dot{K} \), while increasing the rate of output in the traded sector and increasing the current account balance. A decumulating capital stock is therefore accompanied by an accumulating stock of foreign bonds. Moreover, since \( \dot{B}(t) = -\sigma \dot{K}(t) \), these flows are exactly offsetting, so that with \( \sigma \) fixed over time, this implies a zero net rate of savings. There is no correlation between the rate of investment and savings.

But if \( k_N > k_T \), this direct effect is reversed; it will now generate a positive relationship between \( \dot{K} \) and \( \dot{B} \). At the same time, an increasing capital stock is now associated with a declining relative price of nontraded goods (appreciating real exchange rate). This leads to a declining trade balance which offsets the direct effect. The net relationship between the rate of accumulation of capital and the current account balance, as summarized by \( \Omega \), is thus quite ambiguous, and the same applies to the overall savings rate.\(^{11}\)

Finally, (11b) describes the economy's intertemporal solvency condition. It is in effect linear approximation to the economy's intertemporal budget constraint, which corresponds to the linear approximation to the adjustment paths described by (10a), (10b) and (11a).
E. Steady State

The steady state equilibrium of the economy, reached when $\dot{K} = \dot{\sigma} = \dot{B} = 0$, implies

$$\frac{f'(\tilde{k}_T)}{\tilde{\sigma}} = g'(\tilde{k}_N) = r \quad \text{(12a)}$$

$$(1 - \tilde{\rho})g(\tilde{k}_N) - \tilde{C}_N - G_N = 0 \quad \text{(12b)}$$

$$\tilde{pf}(\tilde{k}_T) - \tilde{C}_T - G_T + r\tilde{B} = 0 \quad \text{(12c)}$$

where tildes denote steady-state values. Equation (12a) asserts that the long-run marginal physical product of capital in the traded sector must equal the exogenously given world interest rate. The second equation requires that the output of the nontraded sector equal total consumption demand, while the third equation requires that the long-run current-account balance must be zero.

The steady-state equilibrium can be determined in the following simple way. First, equations (5c), and (5d), which hold at each instant of time, together with (12a), jointly determine the steady-state sectoral capital intensities, $\tilde{k}_T, \tilde{k}_N$, and the relative price, $\tilde{\sigma}$. These quantities, being determined by production conditions, depend only upon supply shocks; they are therefore independent of any form of demand disturbance. Secondly, equations (7a), (7b) determine long-run consumptions $\tilde{C}_T, \tilde{C}_N$, as functions of $\tilde{\lambda}, \tilde{\sigma}$. Thirdly, substituting these expressions, together with the intertemporal solvency condition (11b) into the sectoral capital allocation condition (5e), and (12b), (12c) yields:

$$\tilde{\rho}\tilde{k}_T + (1 - \tilde{\rho})\tilde{k}_N = \tilde{K} \quad \text{(13a)}$$

$$(1 - \tilde{\rho})g(\tilde{k}_N) - \tilde{C}_N(\tilde{\lambda}, \tilde{\sigma}) - G_N = 0 \quad \text{(13b)}$$

$$\tilde{pf}(\tilde{k}_T) - \tilde{C}_T(\tilde{\lambda}, \tilde{\sigma}) - G_T + r\left[B_s + \frac{\Omega}{r - \mu_1}(K_s - \tilde{K})\right] = 0 \quad \text{(13c)}$$

which jointly determine $\tilde{\rho}, \tilde{K}$, and $\tilde{\lambda}$. The equilibrium stock of bonds, and consumption can then be immediately derived.
At this point, two additional observations merit comment. First, equation (13c) highlights the fact that the steady-state equilibrium depends upon the initial stocks of assets $K_a, B_a$. This dependence upon initial conditions is a consequence of the constant marginal utility and raises the potential for temporary shocks to have permanent effects. Secondly, it is worth recalling that $\Omega$ depends upon the relative capital intensities in the two sectors.

3. DEMAND SHOCKS

In this section we outline the effects of demand shocks, taking the form of permanent fiscal expenditures, directed towards the traded and nontraded good, respectively. The qualitative long-run effects of these policies are summarized in Table 1 and are straightforward.

Neither form of fiscal expansion has any long-run effect on the relative price, $\bar{\sigma}$, or sectoral capital intensities, $\bar{k}_a, \bar{k}_N$, which are determined by production conditions alone. An increase in $G_T$ say, raises the demand for traded goods. With the sectoral capital intensities remaining fixed, the additional output necessary to maintain equilibrium, is produced by attracting labor from the nontraded sector, the output of which therefore declines. A further consequence of the sectoral capital intensities remaining fixed is that the effect of the fiscal expansion on the long-run aggregate capital stock depends upon whether labor is moving from a relatively less, to a relatively more, capital intensive sector. If it is, then $\bar{K}$ will rise, if not $\bar{K}$ will fall. The implications for the long-run stock of foreign bonds in turn depends upon the relationship between the rates of asset accumulation, described by $\Omega$. With the balanced government budget, the increase in $G_T$ implies a reduction in private wealth and an increase in its constant shadow value. This leads to a reduction in the private consumption of both goods, with the reduction in $C_N$ matching the reduction in the output of the nontraded good.

Essentially a parallel argument applies with respect to an increase in government expenditure on the nontraded good, $G_N$. The major point worth noting is that the reversal of the employment effect is obviously reflected in the adjustment of the long-run capital stock and holdings of foreign bonds.12

The dynamic adjustment paths are illustrated in Figs 1 and 2 and depend critically upon the relative sectoral capital intensities. If $k_T > k_N$, an increase in $G_T$ say, will lead to a grad
accumulation of capital, accompanied by a gradual decumulation of foreign bonds. With the long-run relative price (real exchange rate) remaining unchanged, and no transitional dynamic adjustment, \( \sigma \) remains fixed throughout. So do the sectoral capital-labor ratios. The adjustment for \( K \) and \( \sigma \) is the locus \( AP \) in the upper panel of Figure 1.A, with the corresponding decumulation of bonds being represented by the path \( LM \) in the lower panel. In the absence of any instantaneous response in \( \sigma \), the adjustment of labor occurs gradually, as resources are attracted to the traded sector. The adjustment in response to an increase in \( G_w \) is just the reverse, as illustrated in the figure.

With the reversal of capital intensities, \( k_N > k_T \), an increase in government expenditure on the traded good will lead to an initial real depreciation in the exchange rate; i.e. \( \sigma(0) \) will drop. This causes an immediate shifting of resources away from the nontraded to the traded sector. With \( k_N > k_T \), capital increases in relative abundance, the wage-rental ratio rises, firms substitute capital for labor, and the capital-labor ratio in both sectors increases. The drop in the relative price \( \sigma \) causes an immediate shift of labor to the traded sector. Output of the nontraded sector immediately falls and investment begins to decline. Along the adjustment path the capital stock declines steadily, while the relative price is gradually restored to its original level. This is because the initial increase in \( k_N \) reduces the marginal physical product \( g'(k_N) \), requiring a continuous rise in \( \sigma \), in order for the rates of return on the assets to be equalized. The adjustment in \( \sigma \) and \( K \) is illustrated by the initial jump \( AE \), followed by the continuous adjustment \( EQ \) in Figure 1.B. The corresponding path for bonds is illustrated by \( LM \), and is drawn as downward sloping, although now a positive slope is quite possible. Again, the dynamic response to an increase in \( G_w \) is just the mirror image.

4. SUPPLY SHOCKS

We turn now to supply shocks, which are assumed to take the form of multiplicative shifts in the production functions of the two sectors. Consider first the production function in the traded goods sector, expressed in intensive form as \( uf(k_T) \), with a proportional shift being parameterized by \( du > 0 \). Such a shift, as well as increasing the level of output, increases the marginal product of both factors proportionately. It is therefore a representation of a Hicks-neutral technological improvement.
Since the steady-state capital intensity in the nontraded sector, $\tilde{k}_n$, is determined by conditions in that sector alone, it is independent of the shift $du$. There is therefore no change in $\tilde{k}_n$. It follows from the equilibrium conditions (5c), (5d), that a proportional shift such as this leads to proportional adjustments in the capital-labor ratio in all sectors. In this case, $\tilde{k}_T$ remains unchanged as well. On the production side, all that happens is that the relative price of the nontraded good rises, in order to maintain equality among rates of return; i.e.

$$\frac{d\tilde{k}_T}{du} = \frac{d\tilde{k}_N}{du} = 0; \quad \frac{d\tilde{\sigma}}{du} > 0 \quad (14)$$

From the steady-state relationships summarized in (13a) - (13c), one can determine the rest of the long-run responses. In contrast to the demand shocks, the rise in the relative price $\tilde{\sigma}$ introduces further effects, which counter the direct effects of the productivity shift $du$. The qualitative responses to the direct and relative price effects are summarized in the first two columns of Table 2.

One immediate effect of an increase in productivity in the traded sector is to increase the flow of output from the resources available to the economy. The economy's wealth increases, leading to a decrease in the shadow value of wealth, $\tilde{\lambda}$. In the absence of any change in the relative price, this wealth effect will increase the consumption of both traded and nontraded goods. With the productivity of labor, and the capital-labor ratio in the nontraded sector remaining fixed, this additional output is obtained by causing labor to shift from the traded to the nontraded sector. But the concurrent rise in the relative price $\tilde{\sigma}$ has an offsetting effect. It tends to reduce the demand for the nontraded good, and therefore the equilibrium output of the nontraded sector. The net effect upon the output of that sector, and upon the allocation of labor which determines it, depends upon whether or not the direct effect dominates the relative price effect. In the special case of a homogeneous utility function, with the initial stock of foreign bonds being zero, and no government expenditure, one can show that the relative size of these two effects can be parameterized simply in terms of the elasticity of substitution in consumption, $\eta$ say. If $\eta > 1$, the relative price effect dominates, and the net demand for, and supply of, nontraded goods declines, and labor shifts from the nontraded to the traded goods sector, i.e. $\tilde{\sigma}$ rises. If $\eta < 1$, the reverse is true. With the steady-state sectoral capital-labor ratios remaining...
fixed, the response of the aggregate capital stock depends upon (i) the net effect on the allocation of labor (i.e. \( \bar{p} \)), and (ii) whether the movement of labor entails a move from a relatively more, to a relatively less, capital intensive sector. Once the adjustment in \( K \) is determined, the net effect of the equilibrium stock of bonds follows, and depends upon whether \( \Omega \geq 0 \), in accordance with the considerations discussed in Section 2.0.

Phase diagrams summarizing the adjustments in \( K \) and \( \sigma \) are provided in Figure 2.A. There are four possible scenarios, depending upon whether: (i) \( k_T \geq k_N \), and (ii) the relative price effect dominates the demand effect. Corresponding to these adjustment paths, are adjustment paths relating \( B \) to \( K \), in accordance with (11a). However, these are not drawn.

In the case where \( k_T > k_N \), the relative price immediately increases by its full amount. The capital stock steadily decreases or increases, with no further adjustment in \( \sigma \), depending upon whether the direct effect, or the relative price effect, of the productivity shock is the dominant one. The dynamics are represented by the upper two panels in Fig. 2.A. But if the relative sectoral capital intensities are reversed, the relative price \( \sigma \) does undergo transitional dynamics. If the direct effect of the productivity shock dominates, it actually overshoots its long-run response on impact; \( \sigma \) declines over time, i.e. the real exchange rate appreciates as the capital stock is being accumulated. But in the other case, where the relative price effect prevails, the initial response in \( \sigma \) is partial; it continues to rise while the capital stock decumulates.

The long-run responses to a productivity shock in the nontraded sector are reported in the latter part of Table 2. In contrast to a shock in the traded sector, a shift in the production function \( v g(k_N), dv > 0 \), raises the marginal product of capital in the nontraded sector above the world interest rate. This leads to an increase in the capital intensity in that sector, \( \bar{k}_N \), and given the proportionality of the shock, in the traded sector \( \bar{k}_T \), as well. This in turn causes a decline in the marginal product \( f'(\bar{k}_T) \), requires a decrease in the relative price \( \bar{\sigma} \), in order for the arbitrage condition (12a) to be maintained:

\[
\frac{dk_T}{dv} = \frac{dk_N}{dv} > 0; \quad \frac{d\bar{\sigma}}{dv} < 0 \tag{15}
\]
The productivity shock in the traded sector impacts on the remainder of the steady state in three ways, through: (i) the direct effect, (ii) the relative price effect, and in addition (iii) adjustments stemming from changes in the sectoral capital intensities. The direct effects are essentially analogous to those associated with the productivity shock in the traded sector. The only substantive difference is that it attracts labor to the traded sector. The response of the equilibrium stocks of capital and traded bonds to this effect follow as before. The relative price effects are directly opposite to those arising from an analogous shock in the traded sector. However, the impacts resulting from the induced changes in the sectoral capital intensities are not straightforward. Many different patterns of response may result and these cannot be determined without imposing further specific restrictions. Finally, the dynamic adjustment paths for $K$ and $\sigma$ are illustrated in Fig. 2.B. Again there are four scenarios, corresponding to whether: $k_T > k_N$ or whether $K$ rises or falls in the long run.

5. TEMPORARY SHOCKS

It is clear from the steady-state equilibrium described in Section 2.E, that if any temporary shock has any effects on the sectoral capital intensities $k_T, k_N$ or the relative price $\sigma$, that these are only temporary. When the shock ceases, these variables will return to their original levels. This is not generally the case with other variables, such as the aggregate capital stock, $K$, or the labor allocation $\rho$, which are determined by (13a) - (13c). This is because, through the expression

$$V_o \equiv B_o + \frac{\Omega}{r - \mu_1} K_o$$

in (13c), these equilibrium values depend upon the initial stocks $K_o, B_o$ in existence at the time a permanent change is put into effect.

We shall focus our discussion on the capital stock $K$. Suppose that the economy starts out with an initial steady-state stock of capital $\bar{K}_o$, say. Assume further that some temporary shock is introduced at time 0, to be kept in effect until time $S$, when it reverts to its original level. Thereafter, the capital stock may, but more likely will not, revert to its original steady state level $\bar{K}_o$. If not, the
temporary shock will have a permanent effect. The reason is that the long-run capital stock \( \bar{K}_2 \), say to which \( K \) will converge following the permanent removal of the temporary shock at time \( S \), depends upon

\[
V_s = B_s + \frac{\Omega}{r - \mu_1} K_s
\]

which serves as the initial value for the permanent phase thereafter. The equilibrium \( \bar{K}_2 \), will coincide with \( \bar{K}_o \), and capital will therefore converge to its original level if and only if

\[
V_s = V_o
\]

Condition (16) is a necessary and sufficient condition for a temporary shock to have only a temporary effect.

From (11a), the equation \( V(t) = V_o \), describes the comovement of \( B \) and \( K \) along the stable adjustment path. It corresponds to a movement along the locus \( BB \) in Fig. 1. In general, however, \( V_s \neq V_o \), following a temporary shock. This is because during the period \((0, S)\) while the temporary shock is in effect, the economy will follow an \textit{unstable} path, taking it off the locus \( BB \) at time \( S \). It will revert to a new stable path only after time \( S \), when the temporary shock has been permanently removed.

The formal solution for describing the dynamic adjustment paths in response to temporary disturbances are spelled out in detail in Sen and Turnovsky (1990) and these methods can easily be applied here.\textsuperscript{15} We do not pursue this, however, except to point out that for most disturbances \( V_s \neq V_o \). This is \( V_o \) because represents an approximation to the present value of total resources available to the economy, national wealth say, starting from an initial endowment at time 0. The same applies to \( V_s \), relative to time \( S \). Typically, the wealth effects generated while a shock is temporarily in effect will permanently change the intertemporal budget constraint facing the economy after the time the shock is removed. This will cause the capital stock to return to some point other than where it initially began, thereby giving rise to a permanent effect.
However, it is possible for a temporary shock to have only a temporary impact on the capital stock. Consider the case of a demand shock, with the sectoral capital intensities satisfying \( k_T > k_N \). We showed previously how in that case (i) \( \sigma \) never changes, and (ii) \( \Omega/(r - \mu_t) = \sigma \). One can establish that in this case, \( V_s = V_c \) so that the temporary demand shock has only a temporary effect. However, this does not apply to a demand shock if the capital intensities are reversed. Nor does it apply to supply shocks. In either case, a temporary shock will give rise to a permanent effect.

6. TRADED INVESTMENT

We now briefly outline the consequences of assuming that the investment good is traded. The static equilibrium conditions (5a) - (5e), remain unchanged. Equations (6a) - (6c), however, are modified as follows. First, the arbitrage condition (6a) now becomes:

\[
 f'(k_T) = r \quad (6a')
\]

Secondly, with the nontraded good being a pure consumption good, the nontraded market equilibrium condition is now:

\[
 (1 - \rho)g(k_N) = C_N + G_N \quad (6b')
\]

while thirdly, the accumulation of traded bonds is now described by:

\[
 \dot{B} = \rho f(k_T) - C_T - G_T - \dot{K} + rB \quad (6c')
\]

Equations (5a) - (5e), (6a'), (6b'), which are now all static, determine solutions for \( k_T, k_N, \rho, C_T, C_N, \sigma, \) and \( K \), all of which remain constant over time. In particular \( \dot{K} = 0 \). The solution to the accumulation equation, consistent with the transversality condition (3g), is

\[
 rB_0 + \rho f(k_T) = C_T + G_T \quad (17)
\]

The system is therefore always in steady-state equilibrium. If a shock requires a change in \( K \), then that is achieved by a one-time swap of traded bonds \( B \) for capital; see e.g. Obstfeld (1989).
Nondegenerate dynamics are introduced by imposing convex costs of adjustment on investment; see e.g. Matsuyama (1987), Brock (1988), Sen and Turnovsky (1989, 1990), among others. This leads to a saddle path in terms of the capital stock and its shadow value, having a negatively sloped stable arm. The dynamics of the system is almost identical to that obtained in Section 2 above, in the case where the nontraded good is the more capital intensive. Only in this case it applies whether $k_T > k_N$. The steady state consists of (5a) - (5e), (6a'), (12b), (12c), and the intertemporal budget constraint (11b), which reflects the accumulation of assets along the transitional path. The structure is therefore virtually identical to that discussed in Section 2.E. Demand shocks have the precisely the same long-run effects as before. The only difference is that with the equilibrium sectoral capital-intensities being determined by (6a'), rather than (12a), $k_T, k_N$, now depend upon the productivity shocks in the traded, rather than in the nontraded sector, as was the case previously.

7. CONCLUSIONS

This paper has considered a model of a two-sector small open economy with traded and nontraded goods and accumulating capital. The classification of capital in such an economy is important and most of our attention has focused on the case where capital is nontraded, in which case the dynamics is always described by a saddlepath. The interesting feature of this path is that the behavior of the real exchange rate during a transition depends fundamentally on the relative capital intensities of the two sectors. If the traded sector is the more capital intensive, then any permanent disturbance leads to at most an initial one-time jump in the real exchange rate. Thereafter, while the capital stock is undergoing the appropriate continuous adjustment, no further change in the real exchange rate occurs. The transition takes place along a Rybczynski line. This contrasts with the case where these sectoral capital intensities are reversed. In this case, the changing capital stock is accompanied by an appropriately changing relative price.

The specific nature of these adjustments depends upon the particular shocks. To illustrate the model, both demand shocks and productivity shocks have been considered. Fiscal expansions will
have neither transitional, nor long-run, effects on the real exchange rate or on sectoral capital intensities, if the traded sector is more capital intensive. By contrast, they will have transitional, but not permanent, effects on these variables, if the nontraded sector is more capital intensive. The long-run effects on the aggregate capital stock depends upon whether the migration of labor that is entailed involves a move from a relatively more, to a relatively less, capital intensive sector.

Productivity shocks are more complicated. A proportional shift in productivity in the traded sector, while leaving long-run sectoral capital intensities unchanged, leads to a long-run real depreciation of the exchange rate. This latter effect to some degree offsets the impact of the direct effect on the resulting adjustments in the economy. A productivity shift in the nontraded sector has three components. In addition to its direct effect, and to causing an appreciation of the real exchange rate, it also leads to an increase in the sectoral capital intensities of both sectors. Our results describing the productivity shocks contrast to some degree with those obtained by Murphy (1986). He emphasized how the relationship between savings and investment following a productivity shock, depends upon the origin of the shock. That is much less important here. A much more critical factor is the parameter $\Omega$, which incorporates the intertemporal solvency of the economy and depends upon the sectoral capital intensities.

The analysis has emphasized the importance of the pre-existing sectoral capital intensities in determining the nature of the dynamic adjustment path followed by the economy following some previously unanticipated shock. The responses we have been describing are all based on the assumption that the relative magnitudes of the sectoral capital intensities remain unchanged throughout the adjustment, so that strictly speaking, the changes must be infinitesimally small. But it is entirely possible that for larger changes, the economy will move from one configuration of relative sectoral capital intensities to another. Under perfect foresight, this needs to be taken into account at the outset. The potential for such structural shifts has important implications for the dynamic evolution of the economy and promises to be an interesting avenue for future research.
TABLE 1
LONG RUN EFFECTS OF PERMANENT DEMAND SHOCKS

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<tr>
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<th>Traded Good</th>
<th>Nontraded Good</th>
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<td>$\tilde{k}_T$</td>
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<td>0</td>
</tr>
<tr>
<td>$\tilde{k}_N$</td>
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<td>0</td>
</tr>
<tr>
<td>$\tilde{\sigma}$</td>
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<td>0</td>
</tr>
<tr>
<td>$\tilde{\rho}$</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\tilde{k}$</td>
<td>$\text{sgn}(k_T - k_N)$</td>
<td>$\text{sgn}(k_N - k_T)$</td>
</tr>
<tr>
<td>$\tilde{\beta}$</td>
<td>$\text{sgn}((k_N - k_T)\Omega)$</td>
<td>$\text{sgn}((k_T - k_N)\Omega)$</td>
</tr>
<tr>
<td>$\tilde{\lambda}$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\tilde{C}_T$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\tilde{C}_N$</td>
<td>-</td>
<td>-</td>
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### TABLE 2
**LONG RUN EFFECTS OF PERMANENT SUPPLY SHOCKS**

<table>
<thead>
<tr>
<th>Sector</th>
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<th>Nontrade Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_T$</td>
<td>0</td>
<td>na</td>
</tr>
<tr>
<td>$k_N$</td>
<td>0</td>
<td>na</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$-$</td>
<td>na</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
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<td>$\operatorname{sgn}(k_N-k_T)$</td>
<td>$\operatorname{sgn}(k_T-k_N)$</td>
</tr>
<tr>
<td>$b$</td>
<td>$\operatorname{sgn}((k_T-k_N)\Omega)$</td>
<td>$\operatorname{sgn}((k_T-k_N)\Omega)$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$-$</td>
<td>$?$</td>
</tr>
<tr>
<td>$\theta_T$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\theta_N$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

$\text{na} = \text{not applicable.}$
FIGURE 1. DEMAND SHOCKS
Relative Price Effect Dominates Wealth Effect Dominates

Wealth Effect Dominates Relative Price Effect Dominates

I. \( k_T > k_N \)

II. \( k_N > k_T \)

**FIGURE 2A** TRADED SECTOR PRODUCTIVITY SHOCK
FIGURE 2.B NONTRADED SECTOR PRODUCTIVITY SHOCK

I. $k_T > k_N$

II. $k_N > k_T$
FOOTNOTES

1. We wish to thank Philip Brock, two anonymous referees and the Co-Editor Kotaro Suzumura for helpful comments on this paper.

2. See e.g. McDougall (1965), Neary and Purvis (1982).

3. This procedure is also adopted in a number of single sector, aggregate models with traded capital; see e.g. Matsuyama (1987), Sen and Turnovsky (1989, 1990). It is also adopted in the two sector Heckscher-Ohlin type setup, in which both goods are traded; see e.g. Gavin (1991).

4. See also Devereux and Shi (1991) who introduces a variable rate of time preference into a single-sector two-country model which incorporates physical capital accumulation.

5. The key stability condition is that the rate of time discount $\beta$ must increase with the level of utility and therefore consumption. This assumption implies that as agents become richer and increase their consumption levels, their preference for current consumption over future consumption increases. The rationale for this assumption is by no means clear and has generated some debate.

6. The fact that $\partial C_f / \partial \lambda < 0, \partial C_N / \partial \lambda < 0$, while $\partial C_N / \partial \sigma < 0$ is a consequence of the concavity of the utility function. Also $sgn(\partial C_f / \partial \sigma) = sgn(U_{TN})$.

7. We have the following:

$$a_{11} = -\sigma^2 k_N' = -\frac{f}{\sigma(k_N - k_T)}; \quad a_{22} = -g \frac{\partial \rho}{\partial K} = \frac{g}{k_N - k_T}$$

$$a_{21} = (1 - \rho)gk_N' - g \frac{\partial \rho}{\partial \sigma} - \frac{\partial C_N}{\partial \sigma} = -\frac{1}{(k_T - k_N)^{\sigma}} \left[ \frac{(1 - \rho)f^2}{\sigma^2 g''} + \frac{\rho g^2}{f''} \right] - \frac{\partial C_N}{\partial \sigma} > 0.$$ 

8. Recall that

$$k_N' = \frac{f}{\sigma^2 g''(k_N - k_T)} < 0 \quad \text{for} \quad k_N > k_T$$
In the case where the rate of time preference is endogenized along the lines outlined in Section 2.3, the dynamics corresponding to (9) can now be represented by a fourth order system involving $\sigma, K, \lambda, \mu$, where $\mu$ is the costate variable associated with the differential equation describing the variable rate of time preference. More specifically letting $m(t) = \int_0^t \beta[U(C_T, C_N)] ds$, $\mu$ is the costate variable associated with the derivative of this relationship, namely $\dot{m}(t) = \beta[U(C_T, C_N)]$. The crucial observation is that $a_{11}, a_{22}$ as defined in equation (9) remain eigenvalues of this augmented system, so that the key role played by the relative capital intensities in determining $\sigma$, continues to apply. The complete dynamics involves combining equations (9), (3e'), the equation determining $\mu$, and (6c'). This gives a total of five dynamic equations in the five variables $\sigma, \lambda, \mu, K, B$, having three unstable and two stable eigenvalues. With the first three being "jump variables" a unique stable path can be derived starting from the initial stocks $K_0$ and $B_0$.

In this case, in the neighborhood of steady state

$$\mu_1 - r = a_{22} - g' = \frac{\sigma - g'(k_N - k_T)}{(k_N - k_T)} = \frac{f}{\sigma(k_N - k_T)}$$

The fact that the covariation between the savings rate, investment, and the current account balance depends upon the sectoral capital intensities is also emphasized by Murphy (1986) in his two period analysis of productivity shocks.

$G_N$ can be thought of as generating two demand effects, a direct one and an indirect one through the wealth effect $\lambda$. This contrasts with $G_T$ which generates only the latter.

An increase in $G_T$ is equivalent to a transfer in the Brock (1993) model.

The relative price $\sigma$ performs the two functions of: (i) equilibrating the goods market and (ii) serving as an asset price.

Details of the calculations are available on request. The details are sketched out in Turnovsky (1991) for a three sector model.

This can be established by direct calculation.

See McKenzie (1982).
18 But in contrast with the case where capital is nontraded, demand shocks will now generate transitory effects on the relative price and sectoral capital intensities, irrespective of whether $k_f > k_N$. 

\[
\begin{align*}
\text{...&...&...&...&...&...&...&...&...&...&...&...&...&...&...&...&...&...&...&...&...
\end{align*}
\]
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