Alternative Forms of Buyer Power in a Vertical Duopoly: Implications for profits and consumer welfare

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ABSTRACT

We derive several variations of a model in which two upstream firms supply a differentiated products to two downstream firms under exclusive contracts of different kinds. We first derive a benchmark model with upstream first-mover pricing. We then compare its outcomes with the benchmark model with upstream first-mover pricing; downstream first-mover pricing; Nash bargaining, alternatively with linear and two-part tariffs; and vertical integration. In each case, we show how the equilibrium values of wholesale and retail prices as well as downstream firms’ profits are affected by changes in the exogenous parameters (degree of product differentiation, bargaining power, and production costs). We evaluate the various vertical regimes from the perspective of downstream firms’ profits as well as consumer welfare, and show how more powerful downstream firms can benefit consumers by exercising “countervailing power” against upstream firms.

KEYWORDS: Buyer power, Bertrand duopoly, Vertical contracts, Nash bargaining, Vertical integration.

JEL Codes: D43, L13, L22

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Implications for profits and consumer welfare

Aditya Bhattacharjea and Srishti Gupta

1. Introduction

In the theory of industrial organization, a lot of emphasis was traditionally given to horizontal market structures, in which the extent of competition between firms producing the same or similar goods, and their resulting market power, affects the prices, quality and variety of the goods they produce. Even models of vertical structure traditionally emphasized the power of large upstream producers over small downstream retailers. More recently, however, the focus has shifted to buyer power in vertical relations. The increasing dominance of downstream retailers such as Wal-Mart and Toys “R” Us is not only because of their market size but also because of their increased buyer power which allows them to get favourable trading terms from their upstream suppliers. With large retailers, there is a change in the structure of power in a supply chain. Such firms may exercise monopsony power in input markets, or monopoly power in output markets, or both.¹

Theoretically, in an upstream manufacturer-downstream retailer model, buyer power involves ability of retailers to obligate manufacturers to provide more favourable contractual terms. These include requiring manufacturers to make a lump-sum payment to the retailer to initiate or continue trading, most-favoured customer clauses, and exclusive supply arrangements. There are many definitions of buyer power. One approach is inverting the market power from seller side to buyer side and defining buyer power as ability of a buyer to maintain prices profitably below competitive levels. Anticompetitive conduct by powerful buyers can depress the prices of inputs they buy from sellers.

We first develop a benchmark model of rival supply chains with upstream first-mover pricing. We then compare this with four other types of vertical arrangements representing different modes of exploiting buyer power: downstream first mover pricing; Nash bargaining or Profit

¹ E-commerce giants like Amazon require a different theoretical framework of two-sided platforms with network externalities, which we do not attempt to model in this paper.
sharing, alternatively with linear and two-part tariffs; and vertical integration. In each case, we derive the market outcomes for different contracts, and show how they are affected by changes in the model’s exogenous parameters: degree of product differentiation, bargaining power, and production costs. We provide a complete unconditional ranking of wholesale and retail prices, consumer surplus and social welfare over four of these arrangements. We also derive a clear ranking of downstream firms’ profits which is conditional on the degree of product differentiation and the division of bargaining power between upstream and downstream firms. To the best of our knowledge, no earlier contribution has compared these vertical arrangements from this perspective, so we cite the relevant earlier literature separately for each model. We also contribute novel results to the literature on how powerful downstream firms can benefit consumers by exercising “countervailing power” against upstream firms. One further contribution of our paper is to draw attention to similar results in papers that have appeared in economics and management journals, which seldom cite each other. Some of these results emerge as special cases of our model.

2.1 Model structure
We assume two identical upstream agents selling a differentiated input to two identical downstream firms under exclusive bilateral contracts of different types. There is therefore no market for the input. This structure is illustrated in Fig. 1. By suitable choice of units, we assume that each downstream firm uses one unit supplied by an upstream agent to produce one unit of the final good. Downstream firms sell these horizontally differentiated products, but do not provide any retailing services (for example demonstration of the product). This allows us to assume that costs incurred in retailing are zero. It also abstracts from the problem of horizontal and vertical externalities arising from retailers’ sales efforts, allowing us to focus on

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2 The concept of countervailing power in this context was originally advanced by Galbraith (1952), but it has been formally modelled only since Dobson and Waterson (1997). Unlike them and later literature (e.g. Gaudin 2018 and other papers cited by him), we do not model increased buyer power as growing concentration arising from horizontal mergers of downstream firms, or as their polarization into a dominant retailer and a competitive fringe (Chen 2003). Chen et al (2016) analyse countervailing buyer power in the form of both increased concentration among retailers and greater bargaining power of a dominant retailer in an exclusive contract with a monopoly upstream supplier, while it competes with a fringe of price-taking small retailers. This market structure rules out the kind of strategic effects that play an important role in our model. Instead, assuming an unchanging market structure of symmetric duopoly at both levels, we compare outcomes across several different modes of exercising buyer power.
comparing the effects of buyer power in different kinds of relationships between upstream and downstream firms.\(^3\)

![Diagram of successive duopolies with exclusive trading](image)

**Figure 1. Successive duopolies with exclusive trading**

This market structure of successive duopolies with bilateral monopoly (exclusive trading) within each vertical pair, also referred to as bilateral duopolies or rival supply chains, is quite common in the literature, and has been motivated in different ways. One formulation treats the upstream agents as plant-specific labour unions whose objective function is to maximize their members’ wage bill (e.g. Horn and Wolinsky 1988, Symeonidis 2010). Workers at one plant cannot work at the other, due to the distance between plants, relationship-specific investments, or an agreement with the employing firm that no non-union member can be employed. However, our analysis includes two-part tariffs and vertical integration, which are not compatible with the interpretation of upstream agents as unions. A more reasonable justification for the structure is to interpret the upstream firms as manufacturers and downstream firms as retailers with scarce shelf space. Each retailer can stock the product of at most one upstream manufacturer. Scarce space can be allotted to the manufacturer who pays the highest ‘slotting allowance’ (Shaffer 1991). Alternatively, according to Lin (1990), a retailer who is selling the product of one upstream firm will not want to switch to the other

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\(^3\) For the same reason, we also do not deal with other issues that are prominent in the vertical contracting literature, such as raising rivals’ costs, foreclosure of entry, investment incentives, and horizontal merger at upstream or downstream levels.
supplier, because then it will be competing against the other retailer for the same product (intra- 
brand competition), which will result in the Bertrand Paradox with zero profits.

A different explanation of bilateral monopoly, in which downstream firms are manufacturers 
rather than retailers, is as follows. Each upstream manufacturer produces a different specialized 
intermediate input which is further processed or assembled by a particular downstream 
manufacturer that sells directly to consumers. Each vertical pair may specialize its technology 
or product with relationship-specific investments, so an upstream or downstream firm cannot 
switch to another buyer or supplier, respectively. Even if the downstream firm provides the 
upstream firm with equipment and raw materials to produce a good that is not relationship-
specific, the downstream firm would not like its supplier to sell it to a rival. It may then impose 
an exclusive supply contract.

Whatever is the underlying justification for assuming bilateral monopolies, we denote the 
upstream duopolist manufacturers as U_1 and U_2, and the downstream duopolists as D_1 and D_2. 
We assume that the terms of a contract involving the i^{th} downstream firm will specify w_i, the 
wholesale price. Because of buyer power, a slotting fee is also a possibility in our Nash 
Bargaining regime with a two-part tariff. This results in a contract of type (w, S) where S is the 
slotting allowance, a fixed amount independent of number of units bought from the 
manufacturer. It can be regarded as the mirror image of a franchise fee paid by the downstream 
firm to the manufacturer. We assume that the contracts of each vertical pair cannot be 
renegotiated, and are observable by the rival pair.\(^4\) Finally, downstream firms compete by 
simultaneously setting prices, i.e. as a Bertrand duopoly in differentiated products. The i^{th} 
downstream firm will sell its product to final consumers at a price of p_i. Cost and demand 
functions are common knowledge.

Consumers’ demand for the final good is linear, as in Singh and Vives (1984) with slight 
change in notations (Wang et al 2016) and denoted as:

\[
q_i = a - p_i + \gamma p_j \quad i, j = 1,2; \quad a > 0
\]  

\(^4\) This simplifies the analysis and also rules out the problem of post-contractual opportunism, whereby firms can 
reneg on unobservable exclusivity contracts, as pointed out by Hart and Tirole (1990) and Fumagalli and Motta 
(2001). (Some of the explanations for exclusivity discussed above could also make such opportunism 
unprofitable or technologically impossible.)
Here, \( q_i \) is the quantity of good \( i \) sold by downstream firm \( i \) at price \( p_i \), while \( p_j \) is the price of good sold by downstream firm \( j \). The coefficient of \( p_i \) is negative, confirming the inverse relationship between price of good \( i \) and quantity of good \( i \). The coefficient of \( p_j \) is positive, indicating that both goods are demand substitutes. In Appendix I we have shown how the above direct demand function can be obtained from an inverse demand function, which is in turn derived from a standard quasi-linear utility function. Appendix I also shows how the following interpretations and assumptions in regard to our direct demand specification correspond to the inverse demand functions.

The product differentiation and inter-brand substitutability is captured by parameter \( \gamma \) in the direct demand function. We assume \( \gamma \) lies between 0 and 1. When \( \gamma \) approaches 1 it implies products are close to perfect substitutes. However, results are not defined for values of \( \gamma = 1 \), therefore we are bounding \( \gamma \) strictly less than 1 in all our following derivations. When \( \gamma = 0 \), the inverse demand function reduces to \( p_1 = \alpha - \beta q_1 \), which shows that products are demand independent.

Each manufacturer is assumed to have constant and identical marginal costs, denoted by \( c \). Imposing the restriction \( c > 0 \) prevents the price of the goods from falling to zero, which would absurdly give the same result as the case of demand independence in equation (1) if \( p_j = 0 \).

In each vertical chain, trade between players is determined by the regime they choose. We first take a standard vertical oligopoly model with manufacturer first mover and linear pricing as our benchmark where downstream firms impose no restraints on manufacturers. We then compare its result with cases when downstream firms are the first movers; when downstream firms and their respective exclusive manufacturers share profits from Nash Bargaining with a linear or two-part tariff; and finally when upstream and downstream firms in each supply chain function as one unit in vertical integration.

2.2: Benchmark model

We begin with a benchmark linear pricing (B) model similar to McGuire & Staelin (1983), who studied the effect of product substitutability on Nash equilibrium distribution structures, when upstream manufacturers are first movers. Like our model, they also assumed a vertical
duopoly in which each manufacturer sells its good through a single exclusive retailer, with price competition and linear demand. They found that when products are very differentiated, manufacturers distribute through a company store (which is owned by it, i.e. vertically integrated) while when products are very similar, they prefer franchised outlets (decentralised or vertically separated structure). In the economics literature, this idea was developed independently by Lin (1988) and Bonanno and Vickers (1988), using different specifications of demand for differentiated products. They also worked with duopoly at upstream and downstream levels, in which each manufacturer exclusively deals with its downstream distributor, and fully extracts its profits via a franchise fee. (Manufacturers can do so by auctioning exclusive franchises to one out of many competing potential distributors.) These early papers showed that both manufacturers are better off with vertical separation. This is because a vertically integrated firm will maximize profits with respect to its upstream marginal costs, whereas separation induces the upstream firms to set their wholesale prices above marginal costs. This makes it optimal for the downstream firms to set higher prices that enable them to exploit the strategic complementarity of prices under Bertrand competition in the final goods market.\(^5\) That is, they strategically delegate pricing authority to their retailers in order to commit to a ‘fat cat’ strategy (Fudenberg and Tirole, 1984). In our study, we try to compare, apart from vertical separation and integration, the features of three other exogenous regimes that feature separated downstream firms which have buyer power.

The stages of the game in the benchmark case are as follows: In stage 1, each manufacturer contracts with a downstream firm over a wholesale price \(w\). In stage 2, downstream firms simultaneously set their strategic variable (retail price) after having observed the input prices faced by their competitors. We solve this model by backward induction, starting with stage 2. We first write downstream firm 1’s profit function as:

\[
\pi_{D1} = (p_1 - w_1)q_1
\]

When we differentiate this firm’s profit function with respect to \(p_1\), we get the following first order condition

\[
\frac{\partial \pi_{D1}}{\partial p_1} = q_1 + (p_1 - w_1)(-1) = 0
\]

\(^5\) Shaffer (1991) inverted this insight in a model in which differentiated retailers can extract the entire upstream profit by offering retail space to many competing manufacturers who produce a homogenous product.
On solving this we get,

\[ a - p_1 + \gamma p_2 - p_1 + w_1 = 0 \]
\[ \frac{a + \gamma p_2 + w_1}{2} = p_1 \]

Similarly, when we solve for downstream firm 2, we get following condition

\[ \frac{a + \gamma p_1 + w_2}{2} = p_2 \]

On substituting \( p_2 \) in \( p_1 \) we get,

\[ \frac{a(2 + \gamma) + 2w_1 + \gamma w_2}{4 - \gamma^2} = p_1 \]

In stage 1, we maximize upstream firm’s profit function given the prices and quantities from the above equations:

\[ \pi_{U1} = (w_1 - c)q_1 \]
\[ \frac{\partial \pi_{U1}}{\partial w_1} = \frac{\partial q_1}{\partial w_1} (w_1 - c) + q_1 = 0 \]
\[ = \frac{\partial q_1}{\partial p_1} \frac{\partial p_1}{\partial w_1} (w_1 - c) + q_1 \]
\[ \frac{\partial \pi_{U1}}{\partial w_1} = \frac{(\gamma^2 - 2)(w_1 - c)}{4 - \gamma^2} + \frac{2a + a\gamma - 2w_1 + \gamma^2 w_1 + \gamma w_2}{4 - \gamma^2} = 0 \]
\[ w_1 = \frac{a(2 + \gamma) + \gamma w_2 - c(\gamma^2 - 2)}{4 - 2\gamma^2} \]

Similarly, when we solve for upstream firm 2, we get the following condition

\[ w_2 = \frac{a(2 + \gamma) + \gamma w_1 - c(\gamma^2 - 2)}{4 - 2\gamma^2} \]

On substituting \( w_2 \) in \( w_1 \) we get,

\[ w_1^* = \frac{a(2 + \gamma) - c(\gamma^2 - 2)}{4 - 2\gamma^2 - \gamma}; \quad w_2^* = \frac{a(2 + \gamma) - c(\gamma^2 - 2)}{4 - 2\gamma^2 - \gamma} \]

When we substitute optimal wholesale price from the above equations, we get optimal prices and quantities as below:

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\(^6\) Even though with exclusive supply chains there is no market for the intermediate good, these functions give the wholesale price chosen by each upstream firm as its best response to the other upstream firm’s wholesale price. This is because the optimal wholesale prices are indirectly related through the downstream firms’ interaction in the final goods market.
\[ p_1^* = \frac{6a - 2a\gamma^2 - c(\gamma^2 - 2)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)}; \quad p_2^* = \frac{6a - 2a\gamma^2 - c(\gamma^2 - 2)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)} \]

\[ q_1^* = \frac{(2 - \gamma^2)(a - (1 - \gamma)c)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)}; \quad q_2^* = \frac{(2 - \gamma^2)(a - (1 - \gamma)c)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)} \]

We now proceed to model alternative ways of allowing for buyer power. The equilibrium values of the key endogenous variable for each model will be derived and then summarized in Table 1 below.

2.3: Alternative contracts with buyer power

2.3.1: Downstream Firms’ First Mover (FM) Pricing Model

In this model we have two sequential steps, reversing the order of moves of the benchmark model. In stage 1, both downstream firms simultaneously announce that the retail price \( p_i \) will be a mark-up \( (m_i) \) over whatever wholesale price \( w_i \) \( (p_1 = w_1 + m_1) \) the manufacturer might subsequently charge. In Stage 2, given the retail price of \( p_i \), each manufacturer determines the optimal wholesale price \( w_i \). We find the equilibrium solutions by using backward induction.

Now the downstream firm is the leader and has first mover advantage while the manufacturer is the follower. We will solve for manufacturer’s reaction function first and then substitute it in the downstream firm’s profit maximization problem in stage 1 to get optimal retail prices of each product in the Nash equilibrium of the duopoly game in the final goods market. Thus, in this model each downstream firm sets its margin in stage 1 and remains committed to it after it receives goods from manufacturer in stage 2 because it is not profitable to deviate to any other retail price which maximizes his profits (Zhang et al 2012). Similar to Zhang et al (2012) and Wang et al (2016), we are studying the effect of an exclusive channel system. However they compare three cases where market power is equally shared between firms, there is seller power or buyer power in the supply chain. We are only focusing on buyer power under different vertical regimes.

Profit functions of manufacturer and downstream firm are as follows:
\[ \pi_{Di} = (p_i - w_i)q_i \quad i=1,2 \]  
\[ \pi_{ui} = (w_i - c)q_i \quad i=1,2 \]  

On maximizing manufacturer \( I \)'s profit with respect to wholesale price we get following first order condition:

\[ \frac{\partial \pi_{ui}}{\partial w_i} = (w_i - c)q_i \]

\[ \frac{\partial q_i}{\partial p_1} \frac{\partial p_1}{\partial w_i} + (\frac{\partial p_1}{\partial m_1} - \frac{\partial w_i}{\partial m_1})q_i = 0 \]

\[ \frac{\partial \pi_{ui}}{\partial w_i} = a - p_1 + \gamma p_2 - (w_i - c) = 0 \]

(since \( p_1 = w_1 + m_1 \) & \( q_1 = a - p_1 + \gamma p_2 \))

By rearranging terms, we get:

\[ w_1 = \frac{a - m_1 + \gamma m_2 + \gamma w_2 + c}{2} \]

Similarly, when we maximize profits for manufacturer 2, we get the following equation

\[ w_2 = \frac{a - m_2 + \gamma m_1 + \gamma w_1 + c}{2} \]

Downstream firm \( I \)'s profit function can be written as:

\[ \pi_{D1} = (p_1 - w_1)q_1 \]

When we differentiate this with respect to \( m_1 \), we get the following first order condition

\[ \frac{\partial \pi_{D1}}{\partial m_1} = (p_1 - w_1) \frac{\partial q_1}{\partial m_1} + (\frac{\partial p_1}{\partial m_1} - \frac{\partial w_1}{\partial m_1})q_1 = 0 \]

\[ = q_1 + m_1 \frac{\partial q_1}{\partial m_1} = 0 \]

As \( q_1 \) is function of \( m_1, m_2, c \) and \( \gamma \), when we substitute it in the above equation we get

\[ m_1 = \frac{a(2 + \gamma) + \gamma m_2 + c(2 + \gamma)(\gamma - 1)}{2(2 - \gamma^2)} \]

When we do the same exercise for downstream firm 2, it gives us following equation
When we substitute $m_2$ in $m_1$ and simplify we get $m_1$ as function of $c$ and $\gamma$.

$$m_1^* = \frac{(2 + \gamma)(a - (1 - \gamma)c)}{(4 - 2\gamma^2 - \gamma)} ; \quad m_2^* = \frac{(2 + \gamma)(a - (1 - \gamma)c)}{(4 - 2\gamma^2 - \gamma)}$$

$$q_1^* = \frac{(2+\gamma)(a-(1-\gamma)c)(2-\gamma^2)}{(4-2\gamma^2-\gamma)} ; \quad q_2^* = \frac{(2+\gamma)(a-(1-\gamma)c)(2-\gamma^2)}{(4-2\gamma^2-\gamma)}$$

$$p_1^* = \frac{6a - 2a\gamma^2 - c(\gamma^2 - 2)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)} ; \quad p_2^* = \frac{6a - 2a\gamma^2 - c(\gamma^2 - 2)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)}$$

It is interesting to note that equilibrium prices and quantities of final goods in this regime and in the benchmark model are same, while upstream firms’ equilibrium wholesale prices are different. This means that the difference in the two models’ optimal values is due to difference in margins. Comparison between the two regimes shows that margins are higher for the firms that are first movers. The following simple Lemma will be used to prove this result, and repeatedly thereafter:

- **Lemma 1**: $(a - (1 - \gamma)c) > 0$.

**Proof**: We must have $a > c$ for production to be viable. Since $0 \leq \gamma < 1$, by assumption $0 < (1 - \gamma) \leq 1$. Therefore, $(a - (1 - \gamma)c) > 0$. Q.E.D.

To prove that $w_{i,B}^{U*} > w_{i,FM}^{U*}$

**Proof**:

$$w_{i,B}^{U*} - w_{i,FM}^{U*} = \frac{a(2 + \gamma) - c(\gamma^2 - 2)}{4 - 2\gamma^2 - \gamma} - \frac{a(2 - \gamma^2) + c(6 - 4\gamma - 2\gamma^2 + \gamma^2)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma)}$$

$$= \frac{2(a - (1 - \gamma)c)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)}$$

Denominator of the above expression is positive, as $(2 - \gamma) > 0$ and $(4 - 2\gamma^2 - \gamma) > 0$ for $\gamma < 1$. In the numerator, $(a - (1 - \gamma)c) > 0$ by Lemma 1. So,
\[ w_{i,B}^U - w_{i,FM}^U > 0 \]
\[ \Rightarrow w_{i,B}^{U*} > w_{i,FM}^{U*} \]

Since the equilibrium prices of the final good remain unchanged, the reverse must hold for the margins of the downstream firms, i.e., \( m_{i,FM}^{D*} > m_{i,B}^{D*} \). Thus, the reversal in order of moves only changes the division of a given level of profits between the upstream and downstream firms, without affecting consumers (see Table 1). In this setting, buyer power for downstream firms does not countervail seller power of upstream firms as far as consumers are concerned.

**2.3.2: Nash Bargaining (NB) Models**

In the benchmark model explored in subsection 2.2.1 above, it was assumed that the more powerful upstream firms exploited their first mover advantage to impose “take it or leave it” offers on the weaker downstream firms. This has been a common modelling strategy in the literature. Some authors (e.g. Bonanno and Vickers 1988) additionally assume that the upstream firms can extract the entire producer surplus of the downstream firms through a fixed fee as part of a two-part tariff, which also maximizes the profits of the entire vertical chain by aligning the incentives of the upstream and downstream firms. Shaffer (1991) inverted this in a model in which differentiated retailers can extract the entire upstream profit by offering retail space to many competing manufacturers who produce a homogenous product.

A Nash Bargaining (NB) solution to the contractual terms between upstream and downstream firms in a vertical chain is more general, because it allows for different degrees of bargaining power, with “take it or leave it” offers as special cases when all bargaining power is upstream or downstream. Horn and Wolinsky (1988) were the first to apply NB to the determination of a linear wholesale price in this context. The more recent literature has extended it to bargaining between agents on both wholesale price and franchise fee, but it is concerned with issues such as incentives for investment (Wang et al 2010, Chen 2019, Aliprant and Petrakis 2022), horizontal mergers (Milliou and Petrakis 2007, Symeonidis 2010, Gaudin 2018), or the choice between price and quantity competition by downstream firms (Basak and Wang 2016, Aliprant and Petrakis, 2020). We also use Nash bargaining to determine the pricing contract between upstream and downstream firms, but we are focusing on how buyer power can alter profit allocation and consumer welfare. We also contrast the results when upstream and downstream
firms bargain over both the wholesale price and the lump-sum transfer (a two-part tariff), or just the wholesale price (a linear tariff). We also derive a subsidiary result about how the direction of the transfer in the former case depends on the parameters of the model.

**Linear Tariffs**

*Extensive form:* In stage 1, each manufacturer contracts with a downstream firm through simultaneous bilateral bargains where manufacturer and downstream firm $i$ bargain over wholesale price $w$. Relative bargaining power of manufacturer is $\mu$ and downstream firm is $1 - \mu$. Downstream firms have more bargaining power, as we assume $\mu$ lies between 0 and 0.5. In stage 2, downstream firms simultaneously set their strategic variable (retail price) after having observed the input prices faced by their competitors.\(^7\)

We begin with stage 2 in which downstream firms simultaneously set retail prices using consumers’ final demand, which was already worked out in the benchmark case. From their optimization problem we get prices as functions of wholesale prices which leads us to stage 1 of the game, where the equilibrium of bargaining between manufacturer and downstream firm is given with input price $w_i$ by the following maximization problem:

$$\argmax_{w_i} \left( \pi_{U_i} - \pi_{U_0} \right)^\mu (\pi_{D_i} - \pi_{D_0})^{1-\mu} \right)$$

(12)

For simplicity we have taken disagreement payoffs of both manufacturer and downstream firm equal to zero, which is also reasonable given our vertical structure in which neither has an alternative trading partner. First order conditions on maximizing (12) for $w_i$ gives:

$$\frac{\partial \pi_{D_i}}{\partial w_i} = \frac{\mu(p_i - w_i)}{(\mu - 1)(w_i - c)}$$

On simplifying we get,

\(^7\) Manasakis and Vlassis (2014) present only the results of a similar model, without deriving them, but with different notation, and upstream marginal costs assumed to be zero. We have confirmed that this special case of our more general results corresponds to theirs. The focus of their paper is very different, i.e. to compare the equilibrium choice of Bertrand vs Cournot competition in the final goods market.
\[
\frac{2 \left( \frac{\partial p_i}{\partial w_i} - 1 \right)}{(p_i - w_i) + (w_i - c) \left( \frac{\partial p_i}{\partial w_i} - 1 \right)} = \frac{\mu}{(\mu - 1)(w_i - c)}
\]

\[
w_1 = \frac{a\mu(2 + \gamma) + \mu\gamma w_2 + c(\gamma^2 - 2)(\mu + 2)}{2(\gamma^2 - 2)}
\]

\[
w_2 = \frac{a\mu(2 + \gamma) + \mu\gamma w_1 + c(\gamma^2 - 2)(\mu + 2)}{2(\gamma^2 - 2)}
\]

On substituting \(w_2\) in \(w_1\) we get,

\[
w_1^* = \frac{a(2 + \gamma)\mu + c(\gamma^2 - 2)(\mu - 2)}{2(2 - \gamma^2 - 2)\gamma\mu}; \quad w_2^* = \frac{a(2 + \gamma)\mu + c(\gamma^2 - 2)(\mu - 2)}{2(2 - \gamma^2 - \gamma\mu)}
\]

\[
p_1^* = \frac{a(2(2 + \mu - 2\gamma^2) - c(2 - \gamma^2)(-2 + \mu))}{(2 - \gamma)(4 - 2\gamma^2 - \gamma\mu)}; \quad p_2^* = \frac{a(2(2 + \mu - 2\gamma^2) - c(2 - \gamma^2)(-2 + \mu))}{(2 - \gamma)(4 - 2\gamma^2 - \gamma\mu)}
\]

\[
q_1^* = \frac{(\gamma^2 - 2)(a - c(1 - \gamma))(\mu - 2)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma\mu)}; \quad q_2^* = \frac{(\gamma^2 - 2)(a - c(1 - \gamma))(\mu - 2)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma\mu)}
\]

In equation (12) if we take \(\mu = 1\), the expression reduces to

\[
\arg\max_{w_i} \left( \pi_{U_i} - \pi_{U_0} \right)
\]

which is the same as the maximization problem of an upstream firm under the benchmark regime, and the resulting equilibrium prices and quantities are also the same. This confirms that the benchmark model corresponds to the NB model with a linear tariff when all bargaining power is with the upstream firm.

We henceforth use NB1 for the Nash Bargaining case with a linear tariff, to distinguish it from the case with a two part tariff which we shall derive below and denote as NB2. On differentiating the \(p_{i,NB1}\) derived above with respect to \(\mu\) we get a positive relationship:

\[
\frac{\partial p_{i,NB1}}{\partial \mu} = \frac{2(2 + \gamma)(a - c + c\gamma)(2 - \gamma^2)}{(2 - \gamma)(-4 + 2\gamma^2 + \gamma\mu)^2}
\]
In the numerator $(2 + \gamma) > 0; (a - c + c\gamma) > 0$ by Lemma 1; and $(2 - \gamma^2) > 0$. In the denominator $(2 - \gamma) > 0$ and the other term is a squared term. Hence shown. Along with the preceding result, this means that beginning with the benchmark case, final goods prices vary inversely with downstream bargaining power. This confirms the existence of countervailing power exercised by downstream firms in the NB1 regime.

Non-Linear Tariffs

Extensive form: We study a two stage game. In stage 1, each manufacturer contracts with a downstream firms through simultaneous bilateral bargains where manufacturer and downstream firm i bargain over wholesale price $w$ and slotting allowance $S$. As before, relative bargaining power of manufacturer is $\mu$ and downstream firm is $1-\mu$, with $\mu$ lying between 0 and 0.5. In stage 2, downstream firms simultaneously set their strategic variable (retail price) after having observed the input prices faced by their competitors.

We begin with stage 2 in which downstream firms simultaneously set retail prices using consumer’s final demand. From their optimization problem we get prices as function of wholesale prices, which leads us to stage 1 of the game where the equilibrium of bargaining between manufacturer and downstream firm is given with input price $w_i$ and slotting allowance $S$ by the following maximization problem:

$$\arg\max_{w_i, S} \{ (\pi_{Ui} - \pi_{U0})^\mu (\pi_{Di} - \pi_{D0})^{1-\mu} \}$$  \hspace{1cm} (12a)

In this case also, we have taken disagreement payoffs of both manufacturer and downstream firm equal to zero. First order conditions on maximizing (12a) for $w_i$ and $S$ give (see Appendix II for proofs):

$$\frac{\partial \pi_{Di}}{\partial w_i} + \frac{\partial \pi_{Ui}}{\partial w_i} = 0$$  \hspace{1cm} (13)

$$S = (1 - \mu) \pi_{Ua} - \mu \pi_{Da}$$  \hspace{1cm} (14)
Where, \( \pi_{Ua} \) is profit of manufacturer excluding slotting allowance and similarly \( \pi_{Da} \) is downstream firm’s profit excluding slotting allowance. \( \pi_{Ua} - S \) gives us \( \pi_{Ui} \), manufacturer’s total profit and \( \pi_{Da} + S \) equals to \( \pi_{Di} \), which is downstream firm’s total profit. Thus,

\[
\pi_{Ua} = (w - c)q; \quad \pi_{Da} = (p - w)q
\]

\[
\pi_{Di} = (p_l - w_l)q_i + S \quad \pi_{Ui} = (w_l - c)q_i - S \quad i=1,2
\]

When we solve the above first order conditions for optimal wholesale price, we find

\[
w^* = \frac{a\gamma^2 - c(\gamma^2 - 2)(2 - \gamma)}{4 - 2\gamma - \gamma^2}
\]

On substituting this in prices and quantities we get following optimal values:

\[
p_1^* = \frac{2a - c(\gamma^2 - 2)}{4 - 2\gamma - \gamma^2}
\]

\[
p_2^* = \frac{2a - c(\gamma^2 - 2)}{4 - 2\gamma - \gamma^2}
\]

\[
q_1^* = \frac{(2 - \gamma^2)(a - c(1 - \gamma))}{4 - 2\gamma - \gamma^2}
\]

\[
q_2^* = \frac{(2 - \gamma^2)(a - c(1 - \gamma))}{4 - 2\gamma - \gamma^2}
\]

\[
S^* = \frac{(\gamma^2 - 2\mu)(2 - \gamma^2)(a - c(1 - \gamma))^2}{(4 - 2\gamma - \gamma^2)^2}
\]

We shall show below that this regime gives lower consumer prices than either the benchmark or downstream first mover regimes, so it creates countervailing power. However, as is usual in such models, the final goods prices are independent of the bargaining parameter \( \mu \), which only affects the redistribution of maximized profits via \( S^* \). So, greater buyer power in the NB2 model does not translate into greater countervailing power on behalf of consumers.

Under Nash bargaining with a two-part tariff, profits of manufacturer and downstream firm are as follows:

\[
\pi_{NB2}^* = \frac{2(1 - \mu)(2 - \gamma^2)(a - c(1 - \gamma))^2}{(4 - \gamma^2 - 2\gamma)^2}
\]
\[ \frac{U_{NB2}^{U}}{\gamma} = \frac{2(\mu)(2 - \gamma^2)(a - c(1 - \gamma))^2}{(4 - \gamma^2 - 2\gamma)^2} \]

These profit expressions also include the slotting allowance derived from the optimisation process. The equilibrium wholesale price charged by the upstream firms is more than their marginal costs (proof in Appendix III), so downstream firms are induced to exploit the strategic complementarity of prices, as in the benchmark model.

We now determine conditions under which the sign of the lump sum transfer \( S^* \) is positive or negative, and in the following section we compare the prices and profits of downstream firms under Nash Bargaining with other vertical arrangements.

**Proposition 1:** \( S^* > 0 \) for all values of \( \gamma \in (0, 1) \), \( c \in [0, 0.5) \) & \( (\gamma^2 - 2\mu) > 0 \)

\[ S^* = \frac{(\gamma^2 - 2\mu)(2 - \gamma^2)(a - c(1 - \gamma))^2}{(4 - \gamma^2 - 2\gamma)^2} \]

The denominator in the above expression is a squared term which will always be positive. In the numerator, \((a - c(1 - \gamma))^2 > 0\) and \((2 - \gamma^2) > 0\). This implies \( S^* \) is positive as long as \((\gamma^2 - 2\mu) > 0\), and negative otherwise. This inequality is graphed in \( \mu, \gamma \) space in Fig. 2 below:

![Graph](image)

**Figure 2:** Values of \( \gamma \) and \( \mu \) for which \( S^* > 0 \)
The area of parameter space compatible with positive slotting allowances is larger, the greater is downstream market power (lower $\mu$) and the less the degree of product differentiation (higher $\gamma$). The condition $(\gamma^2 - 2\mu) > 0$ implies that bargaining power lies in the range $0 < \mu < 0.5$.

**Proof:** Since by assumption $0 < \gamma < 1 \Rightarrow 0 < \gamma^2 < \gamma$. Along with our condition $2\mu < \gamma^2$, this gives $0 < 2\mu < \gamma^2 < \gamma < 1 \Rightarrow 0 < \mu < 0.5$.

Hence, slotting allowance will be positive only if $\mu < 0.5$, as is obvious from the graph. However, this condition is not sufficient. We can have negative values of $S^*$, i.e. a franchise fee payable to the manufacturer, even if the latter has low bargaining power $\mu$. Product differentiation takes place at manufacturer’s level, so if there is higher product differentiation, the manufacturer is more likely to sell with a franchise fee than otherwise (unshaded region). Product differentiation can thus offset lower upstream bargaining power. This analysis shows how the fixed component of a two-part tariff gets redistributed, either in form of slotting allowance or as franchise fee, between upstream and downstream firms on the basis of bargaining power and degree of product differentiation.

On differentiating $S^*$ with respect to $\mu$, we find that as $\mu$ falls $S^*$ rises.

\[
\frac{\partial S^*}{\partial \mu} = \frac{2(\gamma^2 - 2)(a - c(1 - \gamma))^2}{(\gamma^2 + 2\gamma - 4)^2} < 0
\]

In the above expression, the denominator is positive as it is a squared term. In the numerator, the second term is squared and hence positive while $(\gamma^2 - 2) < 0$ for all values of $\gamma \in (0,1)$. This shows that, as expected, greater bargaining power with downstream firms monotonically reduces the franchise fee and turns it into a slotting allowance. In contrast, $S^*$ behaves non-monotonically with respect to $\gamma$. The switch from negative to positive $S^*$ as $\gamma$ increases is well defined as in Figure 2, but we show in Appendix IV that $S^*$ again approaches zero as $\gamma \to 1$ (i.e. as products become almost homogenous and we get the Bertrand Paradox).

**2.3.3: Vertical Integration Model**

In vertical integration, upstream and downstream firms integrate to form a single entity. Integrated firm’s profits are divided between shareholders of the erstwhile upstream and
downstream firms according to their relative bargaining power $\mu$. The profit function of a vertically integrated firm is as below:

$$\pi_1 = (p_1 - c)q_1$$  \hspace{1cm} (15)$$

Differentiating its profit function with respect to its price. For firm 1,

$$\frac{\partial \pi_{VI1}}{\partial p_1} = (p_1 - c)\frac{\partial q_1}{\partial p_1} + q_1 = 0$$

$$= (p_1 - c)(-1) + a - p_1 + \gamma p_2 = 0$$

On simplifying the above equation, we get,

$$p_1 = \frac{a + c + \gamma p_2}{2} \hspace{1cm} (16)$$

When we repeat same exercise for integrated firm 2, we get

$$p_2 = \frac{a + c + \gamma p_1}{2} \hspace{1cm} (17)$$

When we substitute $p_2$ in $p_1$ we get following optimal retail quantities and prices of each product.

$$p_1^* = \frac{a + c}{(2 - \gamma)}; p_2^* = \frac{a + c}{(2 - \gamma)}$$

$$q_1^* = \frac{(a - c(1 - \gamma))}{(2 - \gamma)}; q_2^* = \frac{(a - c(1 - \gamma))}{(2 - \gamma)}$$

The results of all the regimes are summarized in the following table. Profits in the VI case are allocated to upstream and downstream firms according to the Nash bargaining parameter $\mu$. The logic is that the terms of any merger are worked out so as to compensate their shareholders accordingly, either by a cash buyout or via the swap ratio for shares in the merged firm.
<table>
<thead>
<tr>
<th>W1</th>
<th>Linear Pricing (Benchmark Case)</th>
<th>Downstream Firm First Mover Pricing Model</th>
<th>Vertical Integration</th>
<th>Nash Bargaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>w1</td>
<td>( a(2 + \gamma) - c(y^2 - 2) ) (\frac{4 - 2y^2 - \gamma}{4 - 2y^2 - \gamma})</td>
<td>( a(2 - y^2) + c(6 - 4y - 2y^2 + y^2) ) (\frac{2(2 - y^2 - y\mu)}{2(2 - y^2 - y\mu)})</td>
<td>c</td>
<td>( a(2 + \gamma)\mu + c(y^2 - 2)(\mu - 2) ) (\frac{2(2 - y^2 - y\mu)}{2(2 - y^2 - y\mu)})</td>
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</table>

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<tr>
<th>W2</th>
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<tbody>
<tr>
<td>w2</td>
<td>( a(2 + \gamma) - c(y^2 - 2) ) (\frac{4 - 2y^2 - \gamma}{4 - 2y^2 - \gamma})</td>
<td>( a(2 - y^2) + c(6 - 4y - 2y^2 + y^2) ) (\frac{2(2 - y^2 - y\mu)}{2(2 - y^2 - y\mu)})</td>
<td>c</td>
<td>( a(2 + \gamma)\mu + c(y^2 - 2)(\mu - 2) ) (\frac{2(2 - y^2 - y\mu)}{2(2 - y^2 - y\mu)})</td>
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</tr>
</thead>
<tbody>
<tr>
<td>q1</td>
<td>( (2 - y^2)(\alpha - (1 - \gamma)c) ) (\frac{4 - 2y^2 - \gamma}{4 - 2y^2 - \gamma})</td>
<td>( (a - (1 - \gamma)c)(2 - y^2) ) (\frac{2(2 - y^2 - y\mu)}{2(2 - y^2 - y\mu)})</td>
<td>( (a - (1 - \gamma)c) )</td>
<td>( (y^2 - 2)(a - (1 - \gamma)c)(\mu - 2) ) (\frac{2(2 - y^2 - y\mu)}{2(2 - y^2 - y\mu)})</td>
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<tbody>
<tr>
<td>q2</td>
<td>( (2 - y^2)(\alpha - (1 - \gamma)c) ) (\frac{4 - 2y^2 - \gamma}{4 - 2y^2 - \gamma})</td>
<td>( (a - (1 - \gamma)c)(2 - y^2) ) (\frac{2(2 - y^2 - y\mu)}{2(2 - y^2 - y\mu)})</td>
<td>( (a - (1 - \gamma)c) )</td>
<td>( (y^2 - 2)(a - (1 - \gamma)c)(\mu - 2) ) (\frac{2(2 - y^2 - y\mu)}{2(2 - y^2 - y\mu)})</td>
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</thead>
<tbody>
<tr>
<td>p1</td>
<td>( 6a - 2a\gamma - c(y^2 - 2) ) (\frac{4 - 2y^2 - \gamma}{4 - 2y^2 - \gamma})</td>
<td>( 6a - 2a\gamma - c(y^2 - 2) ) (\frac{2(2 - y^2 - y\mu)}{2(2 - y^2 - y\mu)})</td>
<td>( a + c )</td>
<td>( a(2 + \mu - 2\gamma^2) - c(2 - y^2)(\mu - 2) ) (\frac{2(2 - y^2 - y\mu)}{2(2 - y^2 - y\mu)})</td>
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<td>( 6a - 2a\gamma - c(y^2 - 2) ) (\frac{4 - 2y^2 - \gamma}{4 - 2y^2 - \gamma})</td>
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<td>( a + c )</td>
<td>( a(2 + \mu - 2\gamma^2) - c(2 - y^2)(\mu - 2) ) (\frac{2(2 - y^2 - y\mu)}{2(2 - y^2 - y\mu)})</td>
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<th>Vertical Integration</th>
<th>Nash Bargaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>pjoint</td>
<td>( \frac{2(a - c + c\gamma^2)(2 - y^2)(3 - y^2)}{(2 - y^2)(4 - 2y^2 - y\gamma)^2} )</td>
<td>( \frac{2(a - (1 - \gamma)c)(2 - y^2)(3 - y^2)}{(2 - y^2)(4 - 2y^2 - y\gamma)^2} )</td>
<td>( \frac{(a - (1 - \gamma)c)^2}{(y - 2)^2} )</td>
<td>( \frac{2(2 - y^2)(a - (1 - \gamma)c)(\mu - 2)((2 + \mu - 2\gamma^2)\gamma^2 - 4 + y\mu)^2}{(2 - y^2 - y\mu)^2} )</td>
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<table>
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<th>N0</th>
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<th>Vertical Integration</th>
<th>Nash Bargaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>n0</td>
<td>( (\gamma^2 - 2)^2(2 - y^2)(2 - y^2) ) (\frac{(a - (1 - \gamma)c^2(2 - y^2)(2 + y)}{(2 - y^2)(4 - 2y^2 - y\gamma)^2} )</td>
<td>( \frac{(a - (1 - \gamma)c^2(2 - y^2)(2 + y)}{(2 - y^2)(4 - 2y^2 - y\gamma)^2} )</td>
<td>( \frac{(1 - \mu)(a - (1 - \gamma)c^2}{(y - 2)^2} )</td>
<td>( \frac{2(1 - \mu)(2 - y^2)(a - (1 - \gamma)c^2(\mu - 2)}{(4 - 2y^2 - y\gamma)^2} )</td>
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<td>n1</td>
<td>( (\gamma^2 - 2)^2(2 - y^2)(2 - y^2) ) (\frac{(a - (1 - \gamma)c^2(2 - y^2)(2 + y)}{(4 - 2y^2 - y\gamma)^2} )</td>
<td>( \frac{(a - (1 - \gamma)c^2(2 - y^2)(2 + y)}{(2 - y^2)(4 - 2y^2 - y\gamma)^2} )</td>
<td>( \frac{\mu(a - (1 - \gamma)c^2}{(y - 2)^2} )</td>
<td>( \frac{2(2 + \gamma)(2 - y^2)(a - (1 - \gamma)c^2(2 - \mu)}{(2 - y)(2y^2 + y\mu - 4)^2} )</td>
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<th>Linear Pricing (Benchmark Case)</th>
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<tbody>
<tr>
<td>s*</td>
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</table>

\( \frac{(\gamma^2 - 2\mu)(2 - y^2)(a - (1 - \gamma)c^2}{(4 - 2y^2 - y\gamma)^2} \)
Section 3: Comparisons across vertical regimes

Let $\pi_{i,j}^k$ represent optimal profit for agents denoted by $k$ which can be U (manufacturer) or D (downstream firm). ‘i’ can be equal to 1 referring to number 1 firm or 2 referring to number 2 firm. ‘j’ defines regime type, which can be B (Benchmark model), NB1 (Nash Bargaining with linear tariff), NB2 (Nash Bargaining with two-part tariff), FM (First Mover pricing model) or VI (Vertical Integration model). In Appendix III we prove that the equilibrium quantities and profits in all the regimes are non-negative. Here we first examine the effect of the exogenous parameters on the endogenous variables (wholesale and retail prices and downstream profits) within each of the vertical regimes, and then we rank the endogenous variables under the different regimes for given levels of the exogenous parameters.

**Proposition 2:** Partial derivatives for all values of $\gamma \in (0, 1), \mu \in (0, 0.5)$ & $c \in [0, 0.5)$

- We have already shown above that $\frac{\partial p_i^1}{\partial \mu} > 0$ for NB1 and $\frac{\partial p_i^1}{\partial \mu} = 0$ for NB2.
- $\frac{\partial w_{i,j}^1}{\partial c} > 0$ for all j regimes
- $\frac{\partial w_{i,j}^1}{\partial \gamma} \leq 0$ for all j regimes; for NB2 only for values of $\gamma > 0.78$
- $\frac{\partial p_i^1}{\partial c} > 0$; $\frac{\partial p_i^1}{\partial \gamma} < 0$ for all j regimes
- $\frac{\partial \pi_{i,j}^D}{\partial \mu} \leq 0$; $\frac{\partial \pi_{i,j}^D}{\partial \gamma} < 0$ for all j regimes

The above partial derivatives show that under all regimes, as marginal cost increases the wholesale and retail prices increase. Under all regimes in which bargaining power is a parameter (NB1, NB2 and VI), the negative partial derivative of profits with respect to $\mu$ shows that as manufacturer’s bargaining power ($\mu$) increases, the profits of downstream firms decline. To get the partial derivatives with respect to $\gamma$, as shown in Appendix IV, we substitute $a = a(1 - \gamma)$ from the inverse demand specification. Since the expressions are to the 3rd and 4th power of $\gamma$, the signs of these expressions are verified by plotting. The effect of increased product substitutability is negative on wholesale and retail prices for all the vertical regimes except for wholesale prices under Nash Bargaining contract with two-part tariff where the relationship is non-monotonic, as shown below.
\[
\frac{\partial w_{NB}^*}{\partial \gamma} = \frac{(\alpha - c)\gamma(8 - 14\gamma + 4\gamma^2 + \gamma^3)}{(-4 + 2\gamma + \gamma^2)^2}
\]

In the above expression, the denominator is positive. In the numerator \(\alpha - c > 0\), while for the second term \((8 - 14\gamma + 4\gamma^2 + \gamma^3)\) we get the following graph:

![Graph](image)

Figure 3: Values of \(\gamma\) for which under Nash Bargaining two-part tariff contract, the first order sign for wholesale prices changes from positive to negative.

From Table 1, we see that when the products are demand independent \((\gamma = 0)\), the wholesale prices under the contract are set equal to upstream marginal costs. As the products become more similar, the strategic effect becomes more important to soften competition, which initially requires an increase in wholesale prices to induce downstream firms to raise their prices to exploit strategic complementarity. However, as the goods come close to being perfect substitutes, competition increases, which leads to fall in wholesale and retail prices. We show in Appendix IV that as products become more homogenous \((\gamma \to 1)\) the wholesale and retail prices fall to marginal cost. This reduces profits of upstream and downstream firms to zero in all the regimes, confirming the existence of the Bertrand paradox in the model. The slotting allowance \(S^*\) also converges to zero as \(\gamma \to 1\). We now proceed to compare the equilibrium values of the endogenous variables across the vertical regimes.
Proposition 3: For all values of $\gamma \in (0, 1)$ and $c \in [0, 0.5)$

1. $p_{i,VI}^* \leq p_{i,NB2}^* < p_{i,NB1}^* < p_{i,FM}^* = p_{i,B}^*$ when $\gamma^2 < 2\mu$
2. $p_{i,VI}^* < p_{i,NB1}^* < p_{i,NB2}^* < p_{i,FM}^* = p_{i,B}^*$ when $\gamma^2 > 2\mu$
3. $p_{i,VI}^* < p_{i,NB1}^* = p_{i,NB2}^* < p_{i,FM}^* = p_{i,B}^*$ when $\gamma^2 = 2\mu$

We have already shown above that prices are the same in the FM and benchmark cases. Now we prove the three inequalities successively.

1. $p_{i,VI}^* \leq p_{i,NB1}^*$

\[
p_{i,NB1}^* = \frac{a(2(2 + \mu) - 2\gamma^2) - c(2 - \gamma^2)(\mu - 2)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma\mu)}, \quad p_{i,VI}^* = \frac{a + c}{(2 - \gamma)}
\]

\[
p_{i,NB1}^* - p_{i,VI}^* = \frac{a(2(2 + \mu) - 2\gamma^2) - c(2 - \gamma^2)(\mu - 2)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma\mu)} - \frac{a + c}{(2 - \gamma)}
\]

\[
= \frac{(a + c(-1 + \gamma))(2 + \gamma)\mu}{(\gamma - 2)(2\gamma^2 + \gamma\mu - 4)} \geq 0
\]

In the above expression, in the numerator $(a + c(-1 + \gamma)) > 0$ by Lemma 1, $(2 + \gamma) > 0$. So, the numerator is positive. In the denominator, $(\gamma - 2) < 0$ and $(2\gamma^2 + \gamma\mu - 4) < 0$ for all values of $\gamma$ and $\mu$ in given ranges (the last expression is non-factorizable and checked in Mathematica for the direction of its signs). Thus, $p_{i,VI}^* \leq p_{i,NB1}^*$. It turns out that when all bargaining power is with the downstream firm ($\mu = 0$), then the equilibrium price of Nash bargaining with linear tariff will be same as for vertical integration. A slotting fee is not possible under this contract, but the downstream firm can enforce marginal cost pricing on its supplier to eliminate double marginalization and extract the entire channel profits. This amounts to a contract of ‘wholesale price maintenance’, which is the mirror image of the familiar retail price maintenance imposed by powerful upstream firms on powerless retailers.

2. $p_{i,VI}^* \leq p_{i,NB2}^*$

\[
p_{i,NB2}^* = \frac{2a - c(\gamma^2 - 2)}{(4 - 2\gamma - \gamma^2)}, \quad p_{i,VI}^* = \frac{a + c}{(2 - \gamma)}
\]

\[
p_{i,NB2}^* - p_{i,VI}^* = \frac{2a - c(\gamma^2 - 2)}{(4 - 2\gamma - \gamma^2)} - \frac{a + c}{(2 - \gamma)}
\]
The denominator of above expression is positive as $(2 - \gamma) > 0, (4 - 2\gamma^2 - \gamma) > 0$. In the numerator, $(a - (1 - \gamma)c) > 0$ and the expression, $\gamma^2 \geq 0$ for all values of $\gamma$ between 0 and 1. This shows that $p^*_i, VI \leq p^*_i, NB2$ holds true for all relevant values of $\gamma$ and c. We can further say that when $\gamma$ approaches 0, $p^*_i, NB2 - p^*_i, VI = 0$. The Nash Bargaining price again converges to the price under vertical integration, but the reason is different. As noted above, when the products are demand independent, the strategic motive for raising final goods prices is absent. The channel partners’ interest is to maximize their joint profits by setting wholesale price equal to marginal cost to avoid double marginalization, as in the vertically integrated solution for independent monopolists. However, here they remain vertically separated and share the profits via the franchise fee, depending on their relative bargaining power.

3. $p^*_i, NB1 > p^*_i, NB2$ when $\gamma^2 < 2\mu$
$p^*_i, NB1 < p^*_i, NB2$ when $\gamma^2 > 2\mu$
$p^*_i, NB1 = p^*_i, NB2$ when $\gamma^2 = 2\mu$

When we compare prices for Nash bargaining with linear tariff vs non-linear tariff we find that $p^*_i, NB1 > p^*_i, NB2$ is true only for the region shown in figure below.

Proof:

$$p^*_i, NB2 = \frac{2a - c(\gamma^2 - 2)}{(4 - 2\gamma - \gamma^2)}, p^*_i, NB1 = \frac{a(2(2 + \mu) - 2\gamma^2) - c(2 - \gamma^2)(\mu - 2)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma\mu)}$$

$$p^*_i, NB1 - p^*_i, NB2 = \frac{a(2(2 + \mu) - 2\gamma^2) - c(2 - \gamma^2)(\mu - 2)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma\mu)} - \frac{2a - c(\gamma^2 - 2)}{(4 - 2\gamma - \gamma^2)}$$

$$= \frac{2(a + c(-1 + \gamma))(2 - \gamma^2)(\gamma^2 - 2\mu)}{(-2 + \gamma)(-4 + 2\gamma + \gamma^2)(-4 + 2\gamma^2 + \gamma\mu)}$$

Note that $(a + c(-1 + \gamma)) > 0$ by Lemma 1; the second term in the numerator is also positive; and all three terms in the denominator are strictly negative. So the sign of the entire expression
depends only on the sign of $\gamma^2 - 2\mu$. This turns out to be the same condition which determined the sign of $S^*$ in Proposition 1. When plotted in Mathematica software, this gave us a region in Figure 4, for which $p_{i,\text{NB1}}^* > p_{i,\text{NB2}}^*$. This turns out to be the complement of the zone consistent with a franchise fee in Fig. 2.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{gamma_mu_plot.png}
\caption{Values of \(\gamma\) and \(\mu\) for which \(p_{i,\text{NB1}}^* > p_{i,\text{NB2}}^*\)}
\end{figure}

In a model with a similar structure of bilateral duopoly, Gal-Or (1991) showed that $p_{i,\text{NB1}}^* > p_{i,\text{NB2}}^*$ when the manufacturers make ‘take it or leave it’ offers to their retailers. This corresponds to our benchmark case, or to the NB1 case with $\mu = 1$, which would lie along a vertical axis further to the right of this figure. Her result is thus a special case of ours. It does not hold for combinations of high buyer power and substitutability between the goods in the upper part of Figure 4. Our earlier result that $P_{i,\text{NB1}}$ is decreasing in the degree of buyer power (i.e., lower $\mu$) can be visualized as a horizontal leftward movement across this figure for a given level of product differentiation ($\gamma$), ultimately reversing the inequality since $P_{i,\text{NB2}}$ remains invariant with respect to $\mu$. This result shows that the ranking of final goods prices across these two vertical regimes is altered when we introduce varying degrees of upstream bargaining power. Combined with Proposition 1, this result also shows that the switch from franchise fee to slotting fee as $\mu$ decreases or $\gamma$ increases coincides with the parameter values that reverse
the ranking of \( p_{i,NB1}^* \) and \( p_{i,NB2}^* \). This illustrates the difference in mechanisms for profit sharing in the two types of Nash Bargaining contracts.

4. \( p_{i,NB1}^* < p_{i,FM}^* \)

\[
p_{i,NB1}^* = \frac{a(2(2 + \mu) - 2\gamma^2) - c(2 - \gamma^2)(\mu - 2)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma\mu)}
\]

\[
p_{i,FM}^* = \frac{6a - 2a\gamma^2 - c(\gamma^2 - 2)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)}
\]

\[
p_{i,FM}^* - p_{i,NB1}^* = \frac{6a - 2a\gamma^2 - c(\gamma^2 - 2)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)} - \frac{a(2(2 + \mu) - 2\gamma^2) - c(2 - \gamma^2)(\mu - 2)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma\mu)}
\]

\[
= \frac{2(a + c(-1 + \gamma))(2 + \gamma)(\gamma^2 - 2)(\mu - 1)}{(2 - \gamma)(2\gamma^2 + \gamma - 4)(2\gamma^2 + \gamma\mu - 4)} > 0
\]

In the above expression, in the numerator \((a + c(-1 + \gamma)) > 0\) by Lemma 1, \((\mu - 1) < 0, (\gamma^2 - 2) < 0\). So, the numerator is positive. In the denominator, \((2 - \gamma) > 0, (2\gamma^2 + \gamma - 4) < 0\) and \((2\gamma^2 + \gamma\mu - 4) < 0\) for all values of \(\gamma\) and \(\mu\) in given ranges (the last two expressions are non-factorizable and checked in Mathematica for the direction of their signs). Thus, \(p_{i,FM}^* > p_{i,NB1}^*\).

5. \( p_{i,NB2}^* < p_{i,FM}^* \)

\[
p_{i,NB2}^* = \frac{2a - c(\gamma^2 - 2)}{(4 - 2\gamma - \gamma^2)} ; \quad p_{i,FM}^* = \frac{6a - 2a\gamma^2 - c(\gamma^2 - 2)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)}
\]

\[
p_{i,FM}^* - p_{i,NB2}^* = \frac{6a - 2a\gamma^2 - c(\gamma^2 - 2)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)} - \frac{2a - c(\gamma^2 - 2)}{(4 - 2\gamma - \gamma^2)}
\]

\[
= \frac{2(\gamma^2 - 2)(a - (1 - \gamma)c)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)(4 - 2\gamma - \gamma^2)} > 0
\]
The denominator of the above expression is positive as \((2 - \gamma) > 0\); \((4 - 2\gamma - \gamma^2) > 0\) and \((4 - 2\gamma^2 - \gamma) > 0\) for \(\gamma < 1\). In the numerator, \((a - (1 - \gamma)c) \geq 0\) as shown in Lemma 1, and the expression, \(2(\gamma^2 - 2)^2 \geq 0\) for all values of \(\gamma\) between 0 and 1.

Combining results 1-5 above gives us a ranking of final goods prices across the five regimes:

\[ p_{iVI} \leq p_{iNB2} < p_{iNB1} < p_{iFM} = p_{iB} \] when \(\gamma^2 < 2\mu\)

\[ p_{iVI} \leq p_{iNB1} < p_{iNB2} < p_{iFM} = p_{iB} \] when \(\gamma^2 > 2\mu\)

\[ p_{iVI} \leq p_{iNB1} = p_{iNB2} < p_{iFM} = p_{iB} \] when \(\gamma^2 = 2\mu\)

This proves Proposition 3, which establishes that buyer power in some form is weakly beneficial for consumers, relative to the benchmark case.

However, comparison of equilibrium values of the remaining endogenous variables across all five regimes is more complicated, and gives no clear rankings if we include both NB1 and NB2. Therefore, the NB contract type will henceforth be represented only by NB2. This gives us unambiguous rankings for most variables, and clear zones of the parameter space for ranking downstream profits under the four remaining regimes.

**Proposition 4:**

\[ w_{i,F} > w_{i,FM} > w_{i,NB2} \geq w_{i,VI} \] for all values of \(\gamma \in (0, 1)\) and \(c \in [0, 0.5)\)

On comparing wholesale prices in the other four types of regimes, we find that wholesale price will be lowest for vertical integration as upstream and downstream firms maximize their integrated profit behaving as single entity, setting wholesale price equal to upstream firm’s marginal cost. We prove the inequalities successively.

We have already proved in subsection 2.3.1 that \(w_{i,B} > w_{i,FM}\) for all values of \(\gamma \in (0, 1)\) and \(c \in [0, 0.5)\).

1. \(w_{i,FM} > w_{i,NB2}\)

\[
\begin{align*}
    w_{i,FM} - w_{i,NB2} &= \frac{a(2 - \gamma^2) + c(6 - 4\gamma - 2\gamma^2 + \gamma^3)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma)} - \frac{a\gamma^2 - c((\gamma^2 - 2)(2 - \gamma))}{(4 - 2\gamma - \gamma^2)} \\
    &= \frac{(2 - \gamma)(4 - 2\gamma^2 - \gamma)(a - c(\gamma^2 - 2))}{(4 - 2\gamma - \gamma^2)}
\end{align*}
\]
\[ w_{i,FM}^* - w_{i,NB2}^* = \frac{2(4 - 2\gamma - 7\gamma^2 + 4\gamma^3 + 2\gamma^4 - \gamma^5)(a - (1 - \gamma)c)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma)(4 - 2\gamma - \gamma^2)} \]

The denominator of above expression is positive as \((2 - \gamma) > 0; (4 - 2\gamma - \gamma^2) > 0 \text{ and } (4 - 2\gamma^2 - \gamma) > 0\) for \(\gamma < 1\). In the numerator, \((a - (1 - \gamma)c) > 0\) as shown in above Lemma 1, and the expression, \(4 - 2\gamma - 7\gamma^2 + 4\gamma^3 + 2\gamma^4 - \gamma^5\) can be factorized to \((1 - \gamma)(4 + 2\gamma - 5\gamma^2 - \gamma^3 + \gamma^4)\) which is greater than 0 for all values of \(\gamma\) between 0 and 1.

So,

\[ \Rightarrow w_{i,FM}^* - w_{i,NB2}^* > 0 \]

\[ w_{i,FM}^* > w_{i,NB2}^* \]

2. \(w_{i,NB2}^* \geq w_{i,VI}^*\)

\[ w_{i,NB2}^* - w_{i,VI}^* = \frac{a\gamma - c((\gamma^2 - 2)(1 - \gamma))}{4 - 2\gamma - \gamma^2} - c \]

\[ w_{i,NB2}^* - w_{i,VI}^* = \frac{\gamma^2(a - c(1 - \gamma))}{4 - 2\gamma - \gamma^2} \]

The denominator of the above expression is positive as \((4 - 2\gamma - \gamma^2) > 0\). In the numerator, \((a - (1 - \gamma)c) > 0\) by Lemma 1. When \(\gamma\) approaches 0, \(w_{i,NB2}^* - w_{i,VI}^*\) approaches 0, confirming our earlier result that the NB2 equilibrium converges to the VI equilibrium when products are demand independent. Thus, \(w_{i,NB2}^* \geq w_{i,VI}^*\) holds true for all relevant values of \(\gamma\) and \(c\).

If we combine the above results we can say that,

\[ w_{i,B}^* > w_{i,FM}^* > w_{i,NB2}^* \geq w_{i,VI}^* \]

This proves Proposition 4.

With symmetric firms whose products enter symmetrically into consumer demand, prices are inversely related to consumer surplus and social welfare, so from Proposition 3 we can conclude
Vertical integration is welfare enhancing because under vertical separation, a downstream firm determines retail price by imposing a margin over and above wholesale price charged by its upstream firm, while under vertical integration upstream and downstream firms behave as a single entity, eliminating double marginalization in the vertical structure. If integration is not feasible, then given that downstream firms are exercising buyer power, it is better for welfare if they do so by Nash bargaining rather than first mover pricing.

**On comparing profits under different regimes for downstream firms**

Our comparison of wholesale and retail prices gave us unambiguous ranking across the four vertical regimes. In comparing downstream profits, however, the binary comparisons often turn out to be parameter-dependent. This can be shown by comparing a representative downstream firm’s profit under alternative pairs of vertical regimes, and then obtaining zones of the parameter space that is consistent with rankings across all four regimes. Since firms are symmetric at both levels, and we exogenously assume that both supply chains adopt the same kind of vertical regime, we can compare profits for a representative downstream firm. There are six possible binary comparisons among the four regimes.

1. Comparing profits from Vertical integration and Nash bargaining contract type with two-part tariff, i.e.,

\[
\pi_{1,\text{NB2}}^{D^*} - \pi_{1,\text{VI}}^{D^*} = \frac{2(1-\mu)(2-\gamma^2)(1-c(1-\gamma))^2}{(4-\gamma^2-2\gamma)^2} - \frac{(1-\mu)(1-c+\gamma c)^2}{(-2+\gamma)^2}
\]

\[
= \frac{\gamma^3(1-\mu)(4-3\gamma)(1-c(1-\gamma))^2}{(4-\gamma^2-2\gamma)^2(-2+\gamma)^2}
\]

The denominator of the above expression is positive as \((\gamma - 2)^2 > 0, (4-\gamma^2 - 2\gamma) > 0\). In the numerator, \((1 - (1 - \gamma)c) > 0, (1 - \mu) > 0, (4 - 3\gamma) > 0\); and \(\gamma^3 > 0\) for all values of \(\gamma\) strictly between 0 and 1. This shows that \(\pi_{1,\text{VI}}^{D^*} < \pi_{1,\text{NB2}}^{D^*}\) holds true for all relevant values of \(\gamma\) and \(c\). Bonanno and Vickers (1988) showed that upstream firms which can extract the

---

8 We have derived similar results for upstream firms’ profits, which are not included because of the length of the paper and its focus on buyer power.
entire downstream profits through a franchise fee get higher profits from the benchmark case as compared to VI, because the upstream firms set $w > c$, which raises the prices charged by retailers. This commitment to higher prices exploits the strategic complementarity in prices and softens competition between the chains. Here we showed that the same holds for the downstream firms with any positive bargaining power, that is $(1 - \mu) > 0$, when a lump-sum transfer is possible, and for even slightly substitutable products (infinitesimally small $\gamma$).

2. Comparing profits from First mover pricing model and Linear Pricing (benchmark) model, i.e., $(\pi_{1,FM}^{D^*}, \pi_{2,FM}^{D^*}) > (\pi_{1,B}^{D^*}, \pi_{2,B}^{D^*})$

This result was already implied by our earlier finding that the only difference between the two regimes is that wholesale prices are lower, and therefore downstream margins are higher, in the FM case. However, this can be confirmed explicitly by comparing the profit expressions as follows:

$$\pi_{1,FM}^{D^*} - \pi_{1,B}^{D^*} = \frac{(1-c+c\gamma)^2(2-\gamma^2)(2+\gamma)}{(2-\gamma)(4-2\gamma^2-\gamma)^2} - \frac{(-2+\gamma^2)^2(1+(\gamma-1)c)^2}{(4-2\gamma^2-\gamma)(2-\gamma)^2}$$

$$= \frac{2(2-\gamma^2)(1-c(1-\gamma))^2}{(4-2\gamma^2-\gamma)^2(-2+\gamma^2)}$$

The denominator of above expression is positive as $(\gamma - 2)^2 > 0, (4-2\gamma^2 - \gamma) > 0$. In the numerator, $(1 - (1 - \gamma)c) > 0 ; (2 - \gamma^2) > 0$ for all values of $\gamma$ between 0 and 1. This shows that $\pi_{1,FM}^{D^*} > \pi_{1,B}^{D^*}$ holds true for all relevant values of $\gamma$ and $c$.

3. Comparing profits from Nash Bargaining with two-part tariff and Linear Pricing (benchmark),

$$\pi_{1,NB2}^{D^*} - \pi_{1,B}^{D^*} = \frac{2(1-\mu)(2-\gamma^2)(1-c(1-\gamma))^2}{(4-\gamma^2-\gamma)^2} - \frac{(-2+\gamma^2)^2(1+(\gamma-1)c)^2}{((4-2\gamma^2-\gamma)(2-\gamma))^2}$$

$$= \frac{(\gamma^2-2)(1-c(1-\gamma))^2(4\mu-1)(4-2\gamma^2-\gamma)(2-\gamma)^2-(\gamma^2-2)(4-\gamma^2-2\gamma)^2)}{(4-2\gamma^2-\gamma)^2(-2+\gamma^2)(4-\gamma^2-2\gamma)^2}$$

30
The denominator of the above expression is positive as \((\gamma - 2)^2 > 0, (4 - 2\gamma^2 - \gamma) > 0, (4 - \gamma^2 - 2\gamma) > 0\). In the numerator, \((1 - (1 - \gamma)c) > 0, (\gamma^2 - 2) < 0\) for all values of \(\gamma\) between 0 and 1. The expression \((2(\mu - 1)((4 - 2\gamma^2 - \gamma)(2 - \gamma)^2) - (\gamma^2 - 2)(4 - \gamma^2 - 2\gamma)^2)\) can be shown by numerical simulation to be positive for all relevant values of \(\mu\) and \(\gamma\). This shows that \(\pi_{1, NB2}^{D^*} > \pi_{1, B}^{D^*}\) holds true for all relevant values of \(\gamma, \mu\) and \(c\). We now show how the remaining comparisons are conditional on parameter values, but they can still give us an overall ranking of outcomes.

4. Comparing profits from vertical integration and downstream first mover model:

\[
\pi_{1, VI}^{D^*} - \pi_{1, FM}^{D^*} = \frac{(1-\mu)(1-c+\gamma c)^2}{(-2+\gamma)^2} - \frac{(1-c+\gamma c)^2(2-\gamma^2)(2+\gamma)}{(2-\gamma)(4-2\gamma^2-\gamma)^2}
\]

\[
= \frac{(1-c+\gamma c)^2}{(-2+\gamma)^2(4-2\gamma^2-\gamma)^2}[(4 - 2\gamma^2 - \gamma)^2(1 - \mu) - (2 - \gamma^2)(4 - \gamma^2)]
\]

In the above equality, \(c\) comes in the \((1 - c + \gamma c)^2\) term which is non-negative for all values of \(c\) between 0 and 0.5. So, if we plot the above graph in \(\mu\) & \(\gamma\) space we get the following graph:

Figure 5: Values of \(\gamma\) and \(\mu\) for which \((\pi_{1, VI}^{D^*}, \pi_{2, VI}^{D^*}) > (\pi_{1, FM}^{D^*}, \pi_{2, FM}^{D^*})\)
5. Comparing profits from downstream first mover contract and Nash bargaining contract with a two-part tariff, i.e.,

\[
\pi_{1,\text{NB}2}^D - \pi_{1,\text{FM}}^D = \frac{2(1-\mu)(2-\gamma^2)(1-c(1-\gamma))^2}{(4-\gamma^2-2\gamma)^2} - \frac{(1-c+\gamma)^2(2-\gamma^2)(2+\gamma)}{(2-\gamma)(4-2\gamma^2-\gamma)^2}
\]

\[
= (2 - \gamma^2)(1 - c(1 - \gamma))^2 \left( \frac{2(1-\mu)}{(4-\gamma^2-2\gamma)^2} - \frac{(2+\gamma)}{(2-\gamma)(4-2\gamma^2-\gamma)^2} \right)
\]

In the above equality, \(c\) appears only in the \((1 - c + \gamma c)^2\) term which is non-negative for all values of \(c\) between 0 and 0.5. So, we can plot the above relationship in \((\mu, \gamma)\) space to get the following graph:

![Figure 6: Values of \(\gamma\) and \(\mu\) for which \((\pi_{1,\text{FM}}^D, \pi_{2,\text{FM}}^D) < (\pi_{1,\text{NB}2}^D, \pi_{2,\text{NB}2}^D)\)](image)

6. Comparing profits from Vertical integration and Benchmark model i.e.,

\[
\pi_{1,\text{VI}}^D - \pi_{1,\text{B}}^D = \frac{(1-\mu)(1-c+\gamma c)^2}{(-2+\gamma)^2} - \frac{(-2+\gamma)^2(1+(\gamma-1)c)^2}{((4-2\gamma^2-\gamma)(2-\gamma))^2}
\]
Once again, $c$ comes in $(1 - c + \gamma c)^2$ term which is non-negative for all values of $c$ between 0 and 0.5. So, if we plot the above relationship in $\mu$ & $\gamma$ space we get the following graph:

![Graph showing values of $\gamma$ and $\mu$ for which $(\pi_{1,VI}D^*, \pi_{2,VI}D^*) > (\pi_{1,B}D^*, \pi_{2,B}D^*)$](image)

**Figure 7:** Values of $\gamma$ and $\mu$ for which $(\pi_{1,VI}D^*, \pi_{2,VI}D^*) > (\pi_{1,B}D^*, \pi_{2,B}D^*)$

We restate the six inequalities analysed above:

1. $(\pi_{1,NB2}D^*, \pi_{2,NB2}D^*) > (\pi_{1,VI}D^*, \pi_{2,VI}D^*)$
2. $(\pi_{1,FM}D^*, \pi_{2,FM}D^*) > (\pi_{1,B}D^*, \pi_{2,B}D^*)$
3. $(\pi_{1,NB2}D^*, \pi_{2,NB2}D^*) > (\pi_{1,B}D^*, \pi_{2,B}D^*)$
4. $(\pi_{1,VI}D^*, \pi_{2,VI}D^*) > (\pi_{1,FM}D^*, \pi_{2,FM}D^*)$
5. $(\pi_{1,NB2}D^*, \pi_{2,NB2}D^*) > (\pi_{1,FM}D^*, \pi_{2,FM}D^*)$
6. $(\pi_{1,VI}D^*, \pi_{2,VI}D^*) > (\pi_{1,B}D^*, \pi_{2,B}D^*)$

In the above inequalities the first three are unconditional while the others are conditional on $\mu$ and $\gamma$ values. Out of the $4! = 24$ possible orderings of profits under the 4 regimes, 19 can be ruled out because they violate the unconditional inequalities 1-3. This leaves the following possible rankings for downstream profits under the four regimes:
1. ZONE 1: \((\pi_{1,NB2}^{D^*}, \pi_{2,NB2}^{D^*}) > (\pi_{1,VI}^{D^*}, \pi_{2,VI}^{D^*}) > (\pi_{1,FM}^{D^*}, \pi_{2,FM}^{D^*}) > (\pi_{1,B}^{D^*}, \pi_{2,B}^{D^*})\)

2. ZONE 2: \((\pi_{1,NB2}^{D^*}, \pi_{2,NB2}^{D^*}) > (\pi_{1,FM}^{D^*}, \pi_{2,FM}^{D^*}) > (\pi_{1,VI}^{D^*}, \pi_{2,VI}^{D^*}) > (\pi_{1,B}^{D^*}, \pi_{2,B}^{D^*})\)

3. ZONE 3: \((\pi_{1,NB2}^{D^*}, \pi_{2,NB2}^{D^*}) > (\pi_{1,FM}^{D^*}, \pi_{2,FM}^{D^*}) > (\pi_{1,B}^{D^*}, \pi_{2,B}^{D^*}) > (\pi_{1,VI}^{D^*}, \pi_{2,VI}^{D^*})\)

4. ZONE 4: \((\pi_{1,FM}^{D^*}, \pi_{2,FM}^{D^*}) > (\pi_{1,NB2}^{D^*}, \pi_{2,NB2}^{D^*}) > (\pi_{1,VI}^{D^*}, \pi_{2,VI}^{D^*}) > (\pi_{1,B}^{D^*}, \pi_{2,B}^{D^*})\)

5. ZONE 5: \((\pi_{1,FM}^{D^*}, \pi_{2,FM}^{D^*}) > (\pi_{1,NB2}^{D^*}, \pi_{2,NB2}^{D^*}) > (\pi_{1,B}^{D^*}, \pi_{2,B}^{D^*}) > (\pi_{1,VI}^{D^*}, \pi_{2,VI}^{D})\)

For case 3 there is no common region for which the inequality hold true. The remaining cases hold in the respective shaded regions in Figure 8.

Figure 8: Zones for different values of \(\gamma\) and \(\mu\) consistent with different rankings of downstream profits under the four vertical regimes.

From the graph it is clear that Zones 1 and 2, with Nash bargaining, are the best for downstream firms when they have more bargaining power and products are more differentiated. When downstream firms have less bargaining power and/or products are
more similar, they prefer first-mover pricing (Zones 4 and 5). These two regimes are always better for them than vertical integration, which does not allow them to exploit strategic complementarity of prices, and the benchmark case, in which downstream firms have no buyer power.

**Section 4: Conclusion**

For this study we set up a model of two competing supply chains producing and selling differentiated products. We compared a standard benchmark case in which upstream firms are first movers against four alternative vertical regimes representing different modes of exercising buyer power: downstream first movers, Nash bargaining with linear and two-part tariffs, and vertical integration. We found that reversing the order of moves only affects the firms’ margins in favour of the downstream firms, without affecting the price of the final good. Greater buyer power in the Nash bargaining solution to a linear wholesale pricing contract does depress the final goods’ price. But if bargaining takes place over the components of a two-part tariff contract, greater bargaining power with downstream firms, or less product differentiation, reduces the franchise fee and turns it into a slotting allowance, showing that buyer power is used to make upstream firms pay a fixed amount for the right to sell to downstream firms in the supply chain. Higher product differentiation implies less competition at downstream level, which leads to higher wholesale and retail prices, as well as higher downstream firms’ profits. Increase in the marginal cost of production leads to increase in wholesale as well as retail prices.

We then ranked the equilibrium values of the endogenous variables across the different vertical regimes. Standard results from earlier literature emerged as special cases of our model. We showed that downstream firms will do better under Nash Bargaining or first-mover pricing, depending on their bargaining power and the degree of product differentiation, while the benchmark regime without buyer power obviously gives them the worst outcomes. When we compare consumer surplus or social welfare, vertical integration is the best, while Nash Bargaining is ranked second. The benchmark regime is again (weakly) the least preferred in the consumer surplus and social welfare comparisons. Buyer power in some form is therefore beneficial not only for the downstream firms, but also (weakly) for social welfare. This finding supports the countervailing power hypothesis. But
there remains a conflict between the regimes that are best for consumers and for the downstream firms.

One limitation of this study is that it was not feasible to work out endogenous choice of contract type, because simultaneous choice from among our five different vertical regimes would give us a 5x5 payoff matrix for the downstream firms alone. Determining the Nash equilibria would be prohibitively complicated. However, in future work we hope to endogenize the decision to integrate, by posing it pairwise as an alternative to each of our vertically separated structures. The objective would be to find out whether unilateral, simultaneous, or sequential vertical integration are Nash equilibrium outcomes.
Appendix I: Derivation of the demand function

Following Singh and Vives (1984), we assume a representative consumer’s utility function as:

\[ U(q_0, q_1, q_2) = \alpha q_1 + \alpha q_2 - \frac{\beta q_1^2 + \beta q_2^2 + 2\lambda q_1 q_2}{2} + q_0 \]

Here \( q_i \) is quantity produced by manufacturer i. \( q_0 \) is a Hicksian composite commodity consisting of all other goods outside the market of interest. Since we are working with real prices we are normalizing price one unit of this basket equal to 1. We assume \( \alpha > 0, \beta > 0 \). To derive the demand function we maximise this utility function for \( q_0, q_1 \) and \( q_2 \):

\[
\max_{q_0, q_1, q_2} \{ U(q_1, q_2) = \alpha q_1 + \alpha q_2 - \frac{\beta q_1^2 + \beta q_2^2 + 2\lambda q_1 q_2}{2} + q_0 \}
\]

Subject to the budget constraint: \( Y = q_0 + p_1 q_1 + p_2 q_2 \)

On maximisation we get the following inverse demand function

\[ p_1 = \alpha - \beta q_1 + \lambda q_2 \]
\[ p_2 = \alpha - \beta q_2 + \lambda q_1 \]

On rearranging terms we get direct demand functions as

\[ q_1 = a - bp_1 + \gamma p_2 \]

Where,

\[ a = \alpha(\beta - \lambda)/\delta \]
\[ b = \beta/\delta \]
\[ \gamma = \frac{\lambda}{\delta} \]

where \( \delta \equiv (\beta^2 - \lambda^2) \)

- When \( \gamma/b \) approaches 1 it implies \( \frac{\gamma}{b} = \frac{\lambda/\delta}{\beta/\delta} = \frac{\lambda}{\beta} \rightarrow 1 \), which implies that \( \delta \rightarrow 0 \), where the demands are undefined. Therefore, we assume \( \gamma < b \). We have adapted this restriction for the case where \( b=1 \), which is used to derive the results. \( b=1 \) implies \( \frac{\beta}{\delta} = 1 \).

- Most papers in economics journals follow Singh and Vives (1984) by substituting the parameters of the inverse demand function into the \( a, b \) and \( \gamma \) parameters of the direct
demand function before proceeding with the firms’ profit-maximization exercise. The direct demand specification without substitution was used in the early papers on vertical relationships with downstream competition in prices, e.g. Lin (1990) and O’Brien and Shaffer (1993). It was actually first used by McGuire and Staelin (1983), and continues to be used extensively in the literature on marketing and operations research (although it is not derived from maximizing a utility function). See Wang et al (2016), Li et al (2020) and many other papers cited there. However, it creates a problem of discontinuity as $\gamma/b$ approaches 1.

- Another problem with using the direct demand specification, which does not seem to have been noticed by earlier authors, is that it gives the same 'monopoly' results when either $\gamma = 0$ (signifying independent demands and no competition), or $p_j = 0$ (signifying intense competition). This can be averted by bounding prices above zero by assuming positive marginal costs.

- We assume $\alpha \geq c$, as the manufacturer is assumed to have constant marginal cost of production equal to $c$ and since price will never fall below marginal cost.

**Appendix II: First order conditions for Nash Bargaining model**

Manufacturer’s profit can be written as

$$\pi_{U1} = (w_1 - c)q_1 - S_1$$

Where,

$w_1$: input price

$S$: slotting allowance

Downstream firm’s profit can be written as

$$\pi_{D1} = (p_1 - w_1) q_1 + S_1$$ (A1)

Define the Nash product of upstream and downstream profits as:

$$N = (\pi_{D1})^{1-\mu} (\pi_{U1})^\mu$$ (A2)

On differentiating for fixed fee $S$, we get
\[ \frac{\partial N}{\partial S} = (1-\mu) (\pi_D)^\mu (\pi_U)^\mu + (-\mu) (\pi_D)^{1-\mu} (\pi_U)^{1-\mu} = 0 \]  
(A3)

When we solve above first order condition for \( \pi_D \), we get

\[ \frac{(1-\mu)\pi_U}{\mu} = \pi_D \]

(A4)

When we substitute the profit functions of manufacturer and downstream firm in equation A4, we get

\[ \frac{(1-\mu)}{\mu} [(w_1-c)q_1 - S_1] = (p_1-w_1) q_1 + S_1 \]

On rearranging the terms on both side for variable \( S \), we get:

\[ S^* = (1-\mu) [(w_1-c)q_1] - \mu [(p_1-w_1) q_1] \]

On differentiating \( N \) for wholesale price \( w_1 \), we get

\[ \frac{\partial N}{\partial w_1} = (1-\mu) (\pi_D)^\mu (\pi_U)^\mu \frac{\partial \pi_D}{\partial w_1} + \mu (\pi_D)^{1-\mu} (\pi_U)^{1-\mu} \frac{\partial \pi_U}{\partial w_1} = 0 \]

\[ \Rightarrow \frac{(1-\mu)\pi_U}{\mu} \frac{\partial \pi_D}{\partial w_1} = -\pi_D \frac{\partial \pi_U}{\partial w_1} \]

Substituting equation A4, we get

\[ \frac{\partial \pi_D}{\partial w_1} + \frac{\partial \pi_U}{\partial w_1} = 0 \]

Appendix III: Proof that equilibrium quantities and profits are non-negative

We will now confirm that signs of optimum wholesale prices, retail prices, quantities and profits under all regimes are non-negative for all values of \( \gamma \) between 0 & 1 and \( c \) between 0 & 0.5. This assures that our equilibrium profits will also be non-negative.

To show:
1. \[ q_{1,VI}^* = \frac{a-c+\gamma c}{(2-\gamma)} \geq 0 \]

As we know that \((2 - \gamma) > 0\) , given Lemma 1 this implies that optimal quantity will be non-negative under Vertical Integration regime.

2. \[ w_{1,VI}^* = c \]

3. \[ P_{1,VI}^* = \frac{a+c}{(2-\gamma)} \geq 0 \]

As, \((2 - \gamma) > 0 \& (a + c) > 0\) this implies that optimal retail price of downstream firms will be non-negative under Vertical Integration regime.

4. \[ \pi_{1,VI}^{D^*} = \frac{(1-\mu)(a-c+\gamma c)^2}{(2-\gamma)^2} \geq 0 \]

As, \((2 - \gamma) > 0, (1 - \mu) > 0\) , given Lemma 1 this implies that optimal profits of downstream firms will be non-negative under Vertical Integration regime.

5. \[ q_{1,FM}^* = \frac{(a-c+c)(2-\gamma^2)}{(2-\gamma)(4-2\gamma^2)} \geq 0 \]

Given our assumption that \(\gamma\) will lie between 0 and 1 and given Lemma 1, optimal quantity will be non-negative under First mover pricing model.

6. \[ w_{1,FM}^* \geq c \]

\[ w_{1,FM}^* - c = \frac{a(2b^2 - \gamma^2) + c(6b^3 - 4b^2\gamma - 2b\gamma^2 + \gamma^3)}{(2b - \gamma)(4b^2 - 2\gamma^2 - \gamma b)} - c \]

\[ = \frac{(a + c(-1 + \gamma))(2 - \gamma^2)}{8 - 6\gamma - 3\gamma^2 + 2\gamma^3} \]

Since,\((2 - \gamma^2) > 0 \) and \(8 - 6\gamma - 3\gamma^2 + 2\gamma^3 > 0\). So given Lemma 1, optimal wholesale prices will be more than marginal cost ‘c’ under First Mover pricing regime.
7. \[ P_{1,FM}^* = \frac{6a-2a\gamma^2-c(\gamma^2-2)}{(4-2\gamma^2-\gamma)(2-\gamma)} \geq 0 \]

Since we assumed \( \gamma \) will lie between 0 and 1, hence optimal downstream firm’s prices will be non-negative under First mover pricing model.

8. \[ \pi_{D,FM}^* = \frac{(a-c+c\gamma)(2-\gamma^2)(2+\gamma)}{(2-\gamma)(4-2\gamma-\gamma^2)^2} \geq 0 \]

Optimal downstream firm’s profits will be non-negative under First mover pricing model (FM) as both numerator and denominator are square terms.

9. \[ q_{1,NB}^* = \frac{(2-\gamma^2)(a-c(1-\gamma))}{(4-2\gamma-\gamma^2)} \geq 0 \]

Since, \((2 - \gamma^2) > 0\) and \((4 - 2\gamma - \gamma^2) > 0\). So given Lemma 1, optimal quantity will be non-negative under Nash bargaining contract type (NB)

10. \[ w_{1,NB}^* \geq c \]

\[ w_{1,NB}^* - c = a\gamma^2(2 + \gamma) - c(4 - \gamma^2)(\gamma^2 - 2) \]

\[ = \gamma^2(a + c(-1 + \gamma)) \]

\[ = \frac{\gamma^2(a + c(-1 + \gamma))}{b(4 - 2\gamma - \gamma^2)} \]

Since, \((4 - 2\gamma - \gamma^2) > 0\). So given Lemma 1, optimal wholesale prices will be more than marginal cost under Nash bargaining contract type (NB2)

11. \[ \pi_{NB}^* = \frac{2(1 - \mu)(2 - \gamma^2)(1 - c(1 - \gamma))^2}{(4 - \gamma^2 - 2\gamma)^2} + \frac{(\gamma^2 - 2\mu)(2 - \gamma^2)(1 - c(1 - \gamma^2))^2}{(4 - 2\gamma - \gamma^2)^2} \geq 0 \]

Since, \((2 - \gamma^2) > 0\), \((1 - \mu) > 0\), and \((4 - 2\gamma - \gamma^2) > 0\). So given Lemma 1, optimal quantity will be non-negative under Nash bargaining contract type (NB2)

12. \[ w_{1,B}^* \geq c \]
\[ w_{i,B} - c = \frac{a(2 + \gamma) - c(\gamma^2 - 2)}{4 - 2\gamma^2 - \gamma} - c = \frac{(2 + \gamma)(a + c(-1 + \gamma))}{(4 - \gamma - 2\gamma^2)} \]

Since \((4 - \gamma - 2\gamma^2) > 0\). So given Lemma 1, optimal wholesale prices will be more than marginal cost under benchmark regime (B).
Appendix IV: Tables of first order and second order derivatives of endogenous variables with respect to parameters

To show:

1. $w_{i,FM}^* \rightarrow c$ as $\gamma \rightarrow 1$

\[ w_{i,FM}^* = \frac{a(2 - \gamma^2) + c(6 - 4\gamma - 2\gamma^2 + \gamma^3)}{(2 - \gamma)(4 - 2\gamma^2 - \gamma)} \]

From the inverse demand function (Appendix I), $a = \alpha(1 - \gamma)$. This implies that as $\gamma \rightarrow 1$, $a$ will tend to 0 and the first expression in the numerator will tend to 0. The second expression $c(6 - 4\gamma - 2\gamma^2 + \gamma^3) \rightarrow 1$ as $\gamma \rightarrow 1$. This confirms the numerator equals to $c$. The denominator also converges to 1 as $\gamma \rightarrow 1$. Hence shown.

Similarly we can show for wholesale price of all other regimes that as $\gamma \rightarrow 1$ wholesale price converges to marginal cost.

2. $p_{i,NB2}^* \rightarrow c$ as $\gamma \rightarrow 1$

\[ p_{i,NB2}^* = \frac{2a - c(\gamma^2 - 2)}{(4 - 2\gamma - \gamma^2)} \]

$a = \alpha(1 - \gamma)$ implies that as $\gamma \rightarrow 1$, $a$ will tend to 0 and the first expression in the numerator will tend to 0. The second expression $c(\gamma^2 - 2) \rightarrow -1$ as $\gamma \rightarrow 1$. This confirms the numerator equals to $c$. The denominator also converges to 1 as $\gamma \rightarrow 1$. Hence shown.

Similarly we can show for retail price of all other regimes that as $\gamma \rightarrow 1$ retail price converges to marginal cost.
3. \( S^* \to 0 \) as \( \gamma \to 1 \)

\[
S^* = \frac{(\gamma^2 - 2\mu)(2 - \gamma^2)(a - c(1 - \gamma))^2}{(4 - \gamma^2 - 2\gamma)^2}
\]

The numerator of \( S^* \) reduces to \( a(1 - 2\mu) \) as \( \gamma \to 1 \). Since \( a = \alpha(1 - \gamma) \), it reduces to 0 as products become more homogenous.

4. \( \pi_{1,B}^D \to 0 \) as \( \gamma \to 1 \)

\[
\pi_{1,B}^D = \frac{(-2 + \gamma^2)(1 + (\gamma - 1)c)^2}{((4 - 2\gamma^2 - \gamma)(2 - \gamma))^2}
\]

Once again, as \( \gamma \to 1 \), \( a \) will tend to 0 and the first expression in numerator will tend to 0. This confirms numerator equals to 0. Hence shown.

Similarly we can show for profits of downstream firms of all other regimes that as \( \gamma \to 1 \) downstream firm’s profits converges to 0.
### Vertical Integration

<table>
<thead>
<tr>
<th>Prices</th>
<th>$\frac{\partial p_{1,VI}^*}{\partial c}$</th>
<th>$\frac{\partial^2 p_{1,VI}^*}{\partial c^2}$</th>
<th>$\frac{\partial p_{1,VI}^*}{\partial \gamma}$</th>
<th>$\frac{\partial^2 p_{1,VI}^*}{\partial \gamma^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prices</td>
<td>$\frac{a}{(2 - \gamma)} &gt; 0$</td>
<td>$0$</td>
<td>$\frac{c - \alpha}{(\gamma - 2)^2} &lt; 0$</td>
<td>$\frac{2(\alpha - c)}{(\gamma - 2)^3} &lt; 0$</td>
</tr>
</tbody>
</table>

### Nash Bargaining (with Two-part tariff)

<table>
<thead>
<tr>
<th>Prices</th>
<th>$\frac{\partial p_{1,NB}^*}{\partial c}$</th>
<th>$\frac{\partial^2 p_{1,NB}^*}{\partial c^2}$</th>
<th>$\frac{\partial p_{1,NB}^*}{\partial \gamma}$</th>
<th>$\frac{\partial^2 p_{1,NB}^*}{\partial \gamma^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prices</td>
<td>$\frac{-(\gamma^2 - 2)}{(4 - 2\gamma - \gamma^2)} &gt; 0$</td>
<td>$0$</td>
<td>$\frac{2(\alpha - \gamma)(2 - 2\gamma + \gamma^2)}{(-4 + 2\gamma + \gamma^2)^2} &lt; 0$</td>
<td>$\frac{4(\alpha - c)\gamma(6 - 3\gamma + \gamma^2)}{(-4 + 2\gamma + \gamma^2)^3} \leq 0$</td>
</tr>
</tbody>
</table>

### First mover pricing

<table>
<thead>
<tr>
<th>Prices</th>
<th>$\frac{\partial p_{1,FM}^*}{\partial c}$</th>
<th>$\frac{\partial^2 p_{1,FM}^*}{\partial c^2}$</th>
<th>$\frac{\partial p_{1,FM}^*}{\partial \gamma}$</th>
<th>$\frac{\partial^2 p_{1,FM}^*}{\partial \gamma^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prices</td>
<td>$\frac{-(\gamma^2 - 2)}{(4 - 2\gamma^2 - \gamma)(2 - \gamma)} &gt; 0$</td>
<td>$0$</td>
<td>$\frac{2(\alpha - \gamma)(6 - 2\gamma - 3\gamma^2 + \gamma^4)}{(-2 + \gamma)^2(-4 + \gamma + 2\gamma^2)^2} &lt; 0$</td>
<td>$\frac{4(\alpha - c)(-28 - 6\gamma + 45\gamma^2 - 17\gamma^3 - 6\gamma^4 + 2\gamma^6)}{(-2 + \gamma)^3(-4 + \gamma + 2\gamma^2)^3} &lt; 0$</td>
</tr>
</tbody>
</table>
### Wholesale Prices

#### Vertical Integration

\[
\frac{\partial w_{1,VI}^*}{\partial c} = 1 > 0 \quad \frac{\partial w_{1,VI}^*}{\partial \gamma} = 0
\]

#### Nash Bargaining (with two-part tariff)

\[
\frac{\partial w_{1,NB2}^*}{\partial c} = \frac{-(\gamma^2 - 2)(2 - \gamma)}{(4 - 2\gamma - \gamma^2)} > 0 \quad \frac{\partial w_{1,NB2}^*}{\partial \gamma} = \frac{(\alpha - c)\gamma(8 - 14\gamma + 4\gamma^2 + \gamma^3)}{(-4 + 2\gamma + \gamma^2)^2} < 0 \text{ conditional on certain values of } \gamma \text{ as shown in figure 3}
\]

#### First mover pricing

\[
\frac{\partial w_{1,FM}^*}{\partial c} = \frac{6 - 4\gamma + \gamma^3 - 2\gamma^2}{(\gamma - 2)(2\gamma^2 + \gamma - 4)} > 0 \quad \frac{\partial w_{1,FM}^*}{\partial \gamma} = \frac{(c - \alpha)(4 + 4\gamma - 12\gamma^2 + 4\gamma^3 + \gamma^4)}{(-2 + \gamma)^2(-4 + \gamma + 2\gamma^2)^2} < 0
\]

#### Benchmark pricing

\[
\frac{\partial w_{1,B}^*}{\partial c} = \frac{\gamma^2 - 2}{(2\gamma^2 + \gamma - 4)} > 0 \quad \frac{\partial w_{1,B}^*}{\partial \gamma} = \frac{(c - \alpha)(2 + \gamma^2)}{(-4 + \gamma + 2\gamma^2)^2} < 0
\]
### Downstream firm’s Profits

#### Vertical Integration

\[
\frac{\partial \pi_{1,VI}^{D^*}}{\partial \mu} = \frac{-(a - c + \gamma c)^2}{(-2 + \gamma)^2} < 0
\]
\[
\frac{\partial \pi_{1,VI}^{D^*}}{\partial \gamma} = \frac{2(c - \alpha)^2(1 - \gamma)(1 - \mu)}{(\gamma - 2)^3} < 0
\]

#### Nash Bargaining

\[
\frac{\partial \pi_{1,NB}^{D^*}}{\partial \mu} = \frac{-2(2-\gamma^2)(a-c(1-\gamma))^2}{(4-\gamma^2-2\gamma)^2} < 0
\]
\[
\frac{\partial \pi_{1,NB}^{D^*}}{\partial \gamma} = \frac{4(c-\alpha)^2(-4+4\gamma+6\gamma^2-9\gamma^3+3\gamma^4)(1+\mu)}{(-4+2\gamma+\gamma^2)^3} < 0
\]

#### First mover pricing

\[
\frac{\partial \pi_{1,FM}^{D^*}}{\partial \mu} = 0
\]
\[
\frac{\partial \pi_{1,FM}^{D^*}}{\partial \gamma} = -\frac{2(c - \alpha)^2(-8 + 4\gamma + 14\gamma^2 - 8\gamma^3 - 4\gamma^4 + \gamma^5 + \gamma^6)}{(\gamma - 2)^2(2\gamma^2 + \gamma - 4)^3} < 0
\]

#### Benchmark pricing

\[
\frac{\partial \pi_{1,B}^{D^*}}{\partial \mu} = 0
\]
\[
\frac{\partial \pi_{1,B}^{D^*}}{\partial \gamma} = \frac{2(c - \alpha)^2(8 - 36\gamma^2 + 32\gamma^3 + 10\gamma^4 - 18\gamma^5 + 3\gamma^6 + \gamma^7)}{(2 - \gamma)^3(2\gamma^2 + \gamma - 4)^3} < 0
\]
References


