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Informal Sector, Innovation and Growth

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Abstract

This paper applies the endogenous growth model with R&D in the presence of the informal sector. It establishes the existence of formal and informal sectors at the steady state, where the formal sector only can buy patented intermediate varieties. The patent for a finite period reduces the incentive to invest in R&D, thereby reducing growth. It further shows that the steady-state growth rate depends on the share of formal employment and vice versa. However, the extent to which the economy would grow depends on various country-specific factors, production-related characteristics and the cost of accessing production activities in the informal sector. As a country develops, we found that a drop in substitutability between formal and informal goods and a rise in formal wage rent with the development reduce the share of formal employment and growth rate. In contrast, improved formal productivity increases them. They together may produce a non-monotonic shape of growth and formal employment share with the level of development.

JEL Code: E26, O11

Key words: Informal Sector, R&D, Patent length, growth

1 Introduction

A significant share of establishments in a typical economy undertaking production activities outside the formal setting cannot afford to invest in R&D or may not be economically viable to do so. The investment in R&D definitely seems to affect the ability to grow the economy in the formal sector and absorb the employment therein. Conventionally argued that a firm having R&D activities would raise surplus and thereby manage to grow at a steady state depending on the level of innovation efforts exerted by the establishment. As a result, one would expect a monotonic relationship between innovation, surplus and growth rate.

The innovation that exploits and expands knowledge, experiences, and practices in the interrelationships between the sub-systems of work, its application, and adoption contributes to economic growth. The existing literature has modelled them to capture various factors behind the dynamics. The market competition, argued by Schumpeter's creative destruction, received huge importance in regard to the role of innovation in affecting economic growth in classical literature (Nicholas, 2003; Aghion and Howitt, 1992). The modern growth theories offered various alternative models that included R&D expenditure (Romer, 1994), rate of ideas expansion (Romer, 1990; Lucas Jr, 2009), level of human capital used in R&D sector (Lucas Jr, 1988), physical capital used in R&D sector (Funke and Strulik, 2000; Sequeira, 2011), the externality of infrastructure and public expenditures (Barro, 1990), the expansion of software sector that benefits freely to take advantage of knowledge outputs (Aghion et al., 2018), the interactions among the favourable institutions (Nelson, 2003) and between research and socio-economic institutions (Rodríguez-Pose and Crescenzi, 2008), knowledge spillover of technology transfer (Grossman and Helpman, 1994), optimum patent length (Aghion et al., 2001) and its breadth (Li, 2001; Sorek, 2011; Palokangas, 2011; Chu, 2022) and so on. Some of these works specifically focused on the level of innovation in producing differentiated goods, thereby contributing to growth. However, the existing literature ignored the existence of such heterogeneity in product and labour markets, and specifically the informal persistence in the developing world to a large extent.

The firms and workers who cannot find space in the formal sector crowd the informal sector for their survival in the developing world. This paper attempts to model the existence of formal and informal activities with differential productive capabilities and their implication on economic growth. A firm operating in the formal sector can have the technical, financial and legal capability to procure patented inputs and thereby creates an incentive to invest in R&D. The rate at which the patented varieties are used in the formal sector must influence the growth rate. On the other hand, the informal sector firms with less productivity lacks the technical, financial and legal capability is unable to procure the patented inputs and hence would not be able to contribute to the growth path. Moreover, one would easily presume that the degree of substitutability between formal and informal consumption goods, the productivity gaps, and the wage rent the formal workers enjoy would vary with the development of the country. They must have a differential impact on the formal employment and the growth rate. If they work in opposite directions, the formal sector employment and the growth rate will vary across the development of the economy. More specifically, this paper contributes to establishing a balanced growth path in terms of innovation in the presence of dual sectors. Second, it differs from the early models of R&D and Schumpeterian growth (Romer, 1990; Grossman and Helpman, 1994; Aghion et al., 2001) exhibited scale effects predicting that the growth rate of the economy is proportional to the size of the working population or human resources devoted to the R&D sector. Rather, this paper argues that the size of formal employment determines the level of growth. Finally, the growth rate and the share of formal employment may not necessarily rise monotonically with the level of development.

The classical theories of economic development believed that the informal sector that usually exists in a less developed economy would decline with the pace of modern sector development. And the informal sector, which uses traditional techniques and does not get support from the financial sector, cannot invest in R&D and offer a surplus to stimulate economic growth. It essentially suggests that an economy with a smaller informal sector would grow faster than others. The contemporary evidence reveals that the informal economy exists in almost all countries (if not all) to various degrees (Bacchetta and Bustamante, 2009; Schneider et al., 2010; Kuehn, 2014; Loayza, 2016; Medina and Schneider, 2018; Bonnet et al., 2019). In a typical developing economy, the sector contributes about 35% of the gross domestic product (GDP) and employs 70% of the labour force (Loayza, 2016). According to Bonnet et al. (2019), it employs 85.8% of the working population in Africa. The proportion is 68.2% in Asia and the Pacific, 68.6% in the Arab States, 40.0% in the Americas and 25.1% in Europe and Central Asia. Earlier, Jütting et al. (2009) also concluded that more than half of all jobs in the non-agricultural sectors of developing countries could be considered informal. Later, Schneider (2012) estimated that the share varies from 7.2% in the USA to 25.1% in Greece, with an unweighted average of 13.9% for 21 OECD countries. Such evidence strongly reveals that the presence of informal is much more prevalent in developing countries than that in developed countries.

On the other hand, the earlier empirical pieces of evidence show a negative and strong relationship between the growth rate and the size of the informal sector. Bonnet et al. (2019) showed empirically that informality is harmful to growth. Loayza (1996) also empirically shows that the informal sector hurts economic growth in Latin American countries. Eilat and Zinnes (2000) further showed that a decline in the informal sector is associated with a rise in the GDP growth rate. At the same time, a few recent empirical studies showed an inverted-U-shaped relationship between the size of the informal sector and the growth rate of the economy (Elgin and Birinci, 2016; Maiti and Bhattacharyya, 2020).

Only a few papers tried to analyse the interconnection between informality and growth theoretically. For example, Loayza (1996) argued that the informal sector lowers the availability of congestible public services to formal firms and thereby inversely impacts growth. Sarte (2000) theoretically argues that the informal sector associated with bureaucratic rent-seeking behaviour, in the presence of the high extra-legal cost with the environment of fragile property rights, lowers growth. Moreover, using an endogenous model, Nabi et al. (2009) showed that under certain conditions the informal sector can expand with growth. These papers have found a monotonic relationship between the growth rate and the informal sector. On the other hand, Maiti and Bhattacharyya (2020) considered the trade-off between taxation and enforcement for the existence of the informal sector and its impact on growth non-monotonically. This paper attempted to incorporate the inability of the informal sector to procure patented inputs. To the best of our knowledge, no such articles are available in the literature. This paper offers an alternative explanation for the inverse relationship between informality and growth rate. On the other hand, unlike traditional literature on endogenous growth, here the division of employment between formal and informal sectors also depends on the growth rate.

Our model is built on the literature that showed how the growth of the economy depends on the size of R&D and product variety (Romer, 1990). This literature did not recognise the existence of the informal sector. While the existence of an informal sector that economises the cost of production can accelerate economic growth, it may dampen innovation efforts and, thereby, the growth of the economy. The extent to which it may dampen depends on various country-specific and production-related characteristics and the cost of accessing the production activities in the informal sector. The paper extends the endogenous growth model with innovation in the presence of the informal sector. Some goods and services produced in the informal sector with competitive varieties can be an imperfect substitute for a few goods and services produced in the formal sector. Therefore, the informal sector can very much impact the profitability of formal firms, and thereby the return on their R&D expenditure and the incentive to spend on R&D [1]. So, the substitutability between the output produced in the formal and informal sectors can affect the R&D and, thereby, the economy's growth rate. Of course, the size of the informal sector would depend on a number of factors. For example, if the government

[1] Shekar (2021) established a favourable role of informal sector playing to boost innovation in the urban manufacturing sector in India.

makes labour laws more favourable to the workers' benefit, hiring a worker in the formal sector will be costlier. This would affect the labour allocation between the formal and informal sectors. Also, the government's tolerance towards informality affects the resource allocation between the formal and informal sectors. If the government is more tolerant towards the informal sector, then this sector can use more capital resources and produce more output. Moreover, the difference in production technology used by the formal and informal sectors also impacts the relative output produced by the formal and informal sectors and, thereby, the resultant economic growth. Moreover, in this paper, the equilibrium formal or informal employment share depends on the steady-state growth rate. If the economic growth rate is higher, then the rate at which new varieties are invented is also high. So the total number of patented varieties, which is nothing but an accumulation of newly invented varieties up to the sum of the earlier periods, will also be high. Since only the formal sector can procure patented inputs, the growth rate can impact the resource allocation between the formal and informal sectors and, thereby, the respective employment shares. To the best of our knowledge, the impact of the growth rate on the formal employment share is not discussed in the endogenous growth literature with R&D. In this paper, we mainly focus on these aspects and try to explain the relationship between informal employment and growth rate.

The paper is organised as follows. In section 2, we describe the theoretical model and the results. Section 3 attempts to calibrate the relationship using plausible model parameters. And, section 4 ends up with concluding remarks.

2 The Model

Let us assume that an economy consists of four sectors - households, formal and informal final goods-producing sectors and the intermediate inputs-producing sector. The economy produces two types of final goods; one in the formal and the other one in the informal sector. The final goods produced in the informal sector are entirely consumed and cannot be used for any other purpose. At the same time, the goods produced in the formal final good sector are used for consumption, R&D sector and production of intermediate inputs. The consumer may also save to finance the production of new intermediate goods with an incentive to receive rental or profit return. The balance in the trade-off between present and future consumptions here ensures the rate of the invention for new varieties, which essentially determines the growth rate of an economy.

2.1 Final Goods

Let us discuss the production of final goods. The final goods are produced respectively by the formal and the informal sectors in parallel, and the goods produced by them may be imperfect substitutes. The formal and informal sector firms use intermediate goods with different intensities. Two types of intermediate goods are produced in the economy - competitive and non-competitive. The price of non-competitive varieties would be higher because of their market power. Only the formal sector firms can afford to buy all varieties of intermediate goods (competitive and non-competitive varieties, denoted by N). On the other hand, the informal sector firms would be able to procure a fraction (say, γ) of competitively produced intermediate varieties (defined as N_c, where the subscript c denotes competitive varieties). Note that N_c must be a sub-set of N. Here, we further assume that the informal sector firms do not have the capabilities to procure the non-competitive varieties.

Let us assume that X_{iF} represents the amount of i-th intermediate variety used by the formal sector (whose output is denoted by Y_F) from N number of total available varieties in an economy (including competitive and non-competitive varieties). Here, subscript F denotes the formal sector. L represents the number of workers employed in the formal sector. We further assumed that i-th variety is invented by using η amount of Y_F. Producing one unit of variety requires only one unit of Y_F. Then, the production function of final goods produced in the formal sector can be specified as below:

$$Y_F = A \left[\int_0^N X_{iF}^{\alpha} \, di \right] L^{1-\alpha} \tag{1}$$

A represents the level of technology used by the formal sector firms.

Similarly, assume that the informal sector output (Y_I), (where I in the subscript denotes informal sector) uses some of the input varieties that have lost their patents and have become one of the competitive varieties, N_c. Only a γ fraction of N_c is used by the informal sector, where $\gamma(\in (0,1])$ is a parameter exogenously given to the system. In general, the use of patented varieties requires a high level of assets, technology, skill and knowledge, which are lacking in informal firms[2]. So, these firms cannot afford to procure the patented varieties. Given these financial and technical constraints, they use only competitive varieties invented long ago, and their usage has become public knowledge. This assumption is quite reasonable, as enough empirical evidence confirms that informal firms are relatively less capital-intensive to procure costly patents. As mentioned above, informal firms usually do not get bank financing or invest much in physical capital assets. Earlier, Thomas (1992) referred to a survey in 1983 of 10,000 households in Lima, where

[2]Only less than 5% informal establishments find external and formal finances during 2004-05 (NSSO, MOSPI, Govt of India, Report No. 519; 61/10/7)

almost half of the informal workers work with less than US$500 of capital per head. Whereas 90% of a comparable sample of formally employed workers work with more than US$6000 of capital per head. A survey by Söderbaum and Teal (2000), which collected data from Cameroon, Ghana, Kenya and Zimbabwe, pointed toward such differences. This study suggests that firms that employ more than 100 workers operate on an average at three to four times higher physical capital per worker than firms that employ fewer than six workers (mainly in the informal sector). All these evidences justify our assumption about the smaller capital usage of informal firms. This γ may vary across countries depending upon the level of development that represents the affordability of the informal sector or small-scale industries and how much the government allows the informal sector to thrive. If the government does not want the informal sector to prosper, it will impose stricter restrictions and implement them effectively. So, the informal sector will try to keep its size smaller and work with less capital to hide. Essentially, the γ is expected to be lower for the developed countries as more varieties should be available in the developed countries. So the fraction informal firms use gradually becomes lower. Moreover, the workers who do not find employment in the formal sector move to the informal sector, and, as a result, $(1-L)$ becomes informal employment. If X_{iIc} represents i-th variety of competitive inputs used by the informal sector, the production function of the informal sector can be written as follows:

$$Y_I = B\left[\int_0^{\gamma N_c} X_{iIc}^{\beta}\, di\right](1-L)^{1-\beta} \tag{2}$$

Here, B represents the level of technology that the informal sector firms use (where $A > B$). This assumption suggests that the formal sector has superior production technology compared to the informal sector. Moreover, it is assumed that $\alpha > \beta$, meaning that the formal sector is more capital intensive.

We assume that labourers are homogeneous. But, the workers who find employment in the formal sector receive higher wages because of better technology used in the formal sector and the favourable labour legislation applicable to the welfare of formal workers. So, the workers would prefer to find employment in the formal sector first. If they do not find in the formal sector crowd in the informal sector. Then, the relation between formal (W_F) and informal wages (W_I) can be represented as:

$$W_F = \phi W_I \tag{3}$$

Here, $\phi > 1$. And this captures the degree of wage dispersion or inequality between the formal and informal sectors. The higher the value of ϕ, the higher the wage gap. We consider that the formal wage is ϕ times bigger than the informal wage, depending on the legislative support given to formal employment. Hence, it is assumed to be an exogenously given. The favourable labour legislation of a state encourages labour unions, offers minimum wage and ensures employer's contribution to the social security benefit. Hence, the value of ϕ would be higher. In other words, the state can manipulate the level of ϕ by changing the labour legislation in a typical developing economy. It may also be the case that ϕ can be higher for developed countries than developing countries due to better social security arrangements for workers in the developed world. As the country develops, ϕ increases.

Since we assumed that the financial or technical capabilities of the informal sector have been too weak to procure non-competitive varieties, the profit expressions differ between the sectors. The profit of the formal sector can be expressed as follows:

$$\pi_F = Y_F - \int_0^N P_i X_{iF}\, di - W_F L \tag{4}$$

The price of formal final goods is assumed as numeraire and hence, equals to one. The price of i-th intermediate variety is defined as P_i. W_F is the formal wage rate. Note that the formal sector uses both types of intermediate goods - competitive and non-competitive varieties. The non-competitive varieties hold patents and sell with market power. They can also buy intermediate goods of competitive varieties without having patents.

If N_c represents the number of competitive varieties that expire the patent duration, the remaining varieties are N_m, which is equal to $(N - N_C)$. Here, N_m denotes the number of varieties with existing patent rights (where m in the subscript denotes monopoly varieties). As a result, these varieties enjoy some degree of market power. So, the profit expression of the formal sector represented in equation (4) can be re-written as follows:

$$\pi_F = Y_F - \int_0^{N_c} P_{ic} X_{iFc}\, di - \int_{N_c}^{N} P_{im} X_{iFm}\, di - W_F L \tag{5}$$

where, P_{ic} and P_{im} represent input prices of $i-$th competitive and non-competitive varieties respectively, and X_{iFc} and X_{iFm} denote their quantities respectively.

On the other hand, the profit expression of the informal sector can be written as follows:

$$\pi_I = P_I Y_I - \int_0^{\gamma N_c} P_{ic} X_{iIc}\, di - W_I(1 - L) \tag{6}$$

Where P_I represents the price of informal goods. With the help of these above-three profit expressions, one can solve the demand for inputs and labour in two sectors.

2.2 Inputs Demand

2.2.1 Labour and Competitive Input Varieties

The final goods sectors generate the demand for labour and intermediate goods. We assumed that workers are paid according to their marginal productivity. From the profit expression (4), we can derive the demand function for labours in the formal sector, which can be found as follows:

$$W_F = A(1-\alpha)L^{-\alpha} \int_0^N X_{iF}^{\alpha}\, di \tag{7}$$

Similarly, the demand function for labour in the informal sector can be obtained from equation (6) as follows:

$$W_I = P_I B \left[\int_0^{\gamma N_c} X_{iIc}^{\beta}\, di\right] (1-\beta)(1-L)^{-\beta} \tag{8}$$

The demand function of competitive varieties coming from the formal final good sector can be derived from equation (5) and expressed as follows:

$$P_{ic} = AL^{1-\alpha}\alpha X_{iFc}^{\alpha-1};\ \forall \text{ i} \in [0, N_c] \tag{9}$$

Since the intermediate varieties having active patents will be demanded by the formal sector, the demand function for such monopoly varieties having patents can be derived from equation (5) and written as below:

$$P_{im} = AL^{1-\alpha}\alpha X_{iFm}^{\alpha-1};\ \forall \text{ i} \in [N_c, N] \tag{10}$$

On the other hand, the demand function for competitive varieties raised by the informal sector can be derived from equation (6) and is found as follows:

$$P_{ic} = P_I B(1-L)^{1-\beta}\beta X_{iIc}^{\beta-1};\ \forall \text{ i} \in [0,\gamma N_c] \tag{11}$$

We further assume that one unit of Y_F is converted into one unit of an intermediate good once the blueprint has been discovered. So, when the patent of a variety expires, the price of it becomes competitive, P_{ic}, which will turn out to be one. Then, the demand for competitive varieties can be derived from the above-demand expressions, respectively, for the formal and informal sectors, as follows (from equations (9) and (11)):

$$X_{iFc}^{*} = [A\alpha]^{\frac{1}{1-\alpha}} L \tag{12}$$

$$X_{iIc}^{*} = [P_I B\beta]^{\frac{1}{1-\beta}} (1-L) \tag{13}$$

Lemma 1 *The demand for competitive varieties in the formal and informal sectors directly depends on their employment size.*

Lemma 2 *The relative demand for competitive varieties between formal and informal sectors is directly related to the technology gap, relative price and relative share of intermediate goods used to produce the final goods.*

2.2.2 R&D Firms and non-competitive input varieties

The intermediate goods-producing firms undertake R&D activities to expand new varieties and take funds from the household against the interest return. Therefore, the firms undertaking R&D and becoming successful in innovating new varieties enjoy monopoly power in selling innovative varieties, hence earning a positive surplus. Only the formal sector producing final goods can buy them before technical superiority and financial access. They need to solve two-stage decision problems. In the first stage, the firms decide whether to invest resources in inventing new varieties or designs. The firms can expand the resources if the present value of future expected profits exceeds the cost of R&D. In the second stage, the firms would determine the optimum price at which the invented products would be sold to the final goods sector. This price essentially determines the flow of profits. Therefore, the solution could be attained by applying the backward induction method.

We shall decide the price of an invented variety. For monopolistically competitive intermediate varieties, the profit function at every time point is

$$\max_{X_{iFm}} \pi_i = P_{im}X_{iFm} - X_{iFm} \tag{14}$$

Incorporating the demand function for the monopolised varieties from equation (10) in (14), we get

$$X^*_{iFm} = \left[A\alpha^2\right]^{\frac{1}{1-\alpha}} L \tag{15}$$

Lemma 3 *As $\alpha < 1$, the demand from competitive varieties is higher than that of non-competitive varieties*

Since the sellers of non-competitive varieties charge a higher price, the demand for such varieties happens to be lower than the competitive varieties. One can derive the markup imposed by these sellers. By substituting (15) into (10), the optimum price of non-competitive varieties is found as follows:

$$P^*_{im} = AL^{1-\alpha}\alpha X^{*\alpha-1}_{iFm} = \frac{AL^{1-\alpha}\alpha}{AL^{1-\alpha}\alpha^2} = \frac{1}{\alpha} > 1 \tag{16}$$

Lemma 4 *The price of a non-competitive variety is greater than one. The competition from competitive varieties does not affect the mark-up of non-competitive input sellers.*

It reveals that the optimum price of non-competitive varieties will always be greater than the final goods price produced in the formal sector.

Let us now decide whether the firm could invest in innovation efforts from the profits earned out of the invention. If invested, what would be the growth rate of variety expansion? The maximum profit earned by the non-competitive intermediate input sellers can be obtained from equations (14), (15) and (16) as follows:

$$\pi_i^* = [A\alpha]^{\frac{1}{1-\alpha}}(1-\alpha)L \tag{17}$$

Lemma 5 *The equilibrium profit of a non-competitive variety depends on the technology and labour employed in the formal final good sector.*

Since a patent allows the non-competitive variety to sell with a markup, it offers a profit. Assume that the invented intermediate varieties are patented, and the patent expires after τ periods. On the other hand, the invention process is not costless and requires η amount of final goods produced by the formal sector, which is fixed. It is further assumed that the final goods produced in the informal sector cannot be used in the innovation process due to their inferior varieties. If η is the cost of R&D incurred by an innovative firm, the discounted present profit has to be equal to η in the steady state at a finite growth rate of varieties. We consider an institutional setup in which the inventor of variety i retains a perpetual monopoly power over the production and sale of the good, X_i, that uses his or her design. The flow of monopoly rentals will then provide the incentive for invention. The monopoly rights could be enforced through explicit patent protection laws. It would, in either case, be realistic to assume that the inventor's monopoly position lasted only for a finite time or eroded gradually over time once the patents expired periodically. Therefore, the present value of the returns from discovering the i-th intermediate good is given by

$$\eta = \int_t^{t+\tau} \pi_i^*(\omega).e^{-\int_t^{\omega} r(v)\,dv}\, d\omega = V(t,t) \tag{18}$$

If $r(v)$ denotes the instantaneous real interest rate, the total return for a period from t to ω becomes $\int_t^{\omega} r(v)dv$. Then, V(t,t) is the discounted present value of future profits for a variety, which is invented and received a patent at the time point t, and the value is measured at the time point t. After τ periods, the patent expires, the variety becomes competitive, and its profits turn zero.

2.3 Household

2.3.1 Consumption

Household consumes the final goods produced in the formal and informal sectors. While the whole amount of informal goods is consumed, a part of the formal goods is saved for intermediate production. The saving goes to the R&D sector for the production of new varieties that earns a surplus. The surplus passes on to the household in return for the saving. Essentially, the household will choose the consumption goods and saving to maximise the lifetime utility subject to the income earned from the formal and informal labour and the rental return. Assume that the composite consumption of the representative household consists of the consumption of formal and informal final goods as follows:

$$C = [aC_F^{\sigma} + (1-a)C_I^{\sigma}]^{\frac{1}{\sigma}} \tag{19}$$

The elasticity of substitution between formal and informal goods is $\dfrac{1}{1-\sigma}; 1 > \sigma > -\infty$. The representative household maximises the discounted present value of instantaneous utilities with respect to consumption, C

$$U = \int_0^{\infty} \frac{C^{1-\theta} - 1}{1-\theta} e^{-\rho t}\, dt \tag{20}$$

This is maximised subject to the inter-temporal budget constraint mentioned in the form of asset accumulation. The rate of asset accumulation depends on the surplus available from the earnings (formal and informal wages received and profits from non-competitive varieties) after meeting the consumption expenditures. This appears as an inter-temporal budget constraint for utility maximisation.

Before optimising them, let us define the price of composite consumption as P. Note that the price of informal goods is P_I, and the price of formal goods is assumed as numeraire. If P is the minimum price of C, the consumption of C_F and C_I are chosen optimally as follows:

$$\mathrm{C}_F + P_I C_I = PC \tag{21}$$

From equations (19) and (21), we get $\left(\dfrac{C_F}{C_I}\right)^{\sigma-1} = \dfrac{(1-a)}{aP_I}$.

$$C_F = C_I \left(\frac{(1-a)}{aP_I}\right)^{\frac{1}{\sigma-1}} \tag{22}$$

This gives the trade-off between the consumption of formal and informal goods. From equations (21) and (22), we obtain that $C_I \left\{\dfrac{(1-a)}{aP_I}\right\}^{\frac{1}{\sigma-1}} + P_I C_I = PC$. This would help us to eliminate C_F from (22).

$$C_I \left\{ P_I + \left[\frac{(1-a)}{aP_I}\right]^{\frac{1}{\sigma-1}} \right\} = PC \tag{23}$$

Using equations (19) and (22), we obtain $C = [aC_F^{\sigma} + (1-a)C_I^{\sigma}]^{\frac{1}{\sigma}}$

$$C = \left[aC_I^{\sigma} \left\{\frac{(1-a)}{aP_I}\right\}^{\frac{\sigma}{\sigma-1}} + (1-a)C_I^{\sigma} \right]^{\frac{1}{\sigma}}$$

$$C = C_I[a^{\frac{1}{1-\sigma}}(1-a)^{\frac{\sigma}{\sigma-1}}P_I^{\frac{\sigma}{1-\sigma}} + (1-a)]^{\frac{1}{\sigma}} \tag{24}$$

Substituting (24) into (23), one can represent the composite price as follows (see Appendix for derivation):

$$P = \left\{(1-a)^{\frac{1}{1-\sigma}}P_I^{\frac{\sigma}{\sigma-1}} + a^{\frac{1}{1-\sigma}}\right\}^{\frac{\sigma-1}{\sigma}} \tag{25}$$

From the utility maximisation (given by equation (20)) subject to the intertemporal budget constraint (described in the next subsection), we can get the path of optimum composite consumption, $C(t)$. Given the value of $C(t)$ for each time point, and P_I (as consumers are price takers), the optimum division of C into C_I and C_F from equations (22) and (24) can be derived.

Lemma 6 *The composite price, P, is monotonically related to the final goods price produced in the informal sector, P_I.*

2.3.2 Asset Accumulation

The non-competitive firms in the intermediate sector are engaged in innovation, and the product produced out of these innovation efforts are patented. This allows the firms to generate a surplus. Households own all firms in the economy, and all other firms make zero profits due to perfectly competitive market structure. Only the firms selling patented varieties earn positive profits. So, the market value of those firms is the only asset of the households at an aggregate level. The appreciated market value of varieties represents asset accumulation. If $V(\omega, t)$ is the value of the variety at time point t, which was invented at time point ω, the asset at any point of time, t, can be represented as follows:

$$Assets(t) = \int_{t-\tau}^{t} V(\omega, t).\dot{N}(\omega)\, d\omega; \tau > t - \omega \tag{26}$$

This is so because patents of varieties invented before $(t-\tau)$ time point have already expired and are now produced competitively. Hence, only varieties invented after $(t-\tau)$ periods have effective patent protection. The market value of such firms adds to the asset accummulation. Since $N(\omega)$ denotes the number of varieties available at time point ω, and $\dot{N}(\omega)$ denotes the change in the number of varieties at the time point ω. So, the number of new varieties invented and patented at the time point ω is also denoted by $\dot{N}(\omega)$. So, at time point t, the market value of all varieties invented at time point ω is $V(\omega, t).\dot{N}(\omega)$. Integrating all such market values over the period $(t-\tau)$ to t, we get the value of assets at time point t. Now, a variety, invented at ω, has a valid patent up to $(\omega+\tau)$. So, its value at the time point t (where, $\tau > t-\omega > \omega$) is the discounted present value of future profits generated up to time point $(\omega+\tau)$.

Hence, $V(\omega,t) = \int_t^{\omega+\tau} \pi_i^*(v) e^{-\int_t^v r(z)\,dz}\,dv$. So, the change in $Assets(t)$ with respect to t will be expressed as follows:

$$\frac{dAssets(t)}{dt} = V(t,t).\dot{N}(t) - V(t-\tau,t).\dot{N}(t-\tau) + \int_{t-\tau}^{t} \frac{dV(\omega,t)}{dt}.\dot{N}(\omega)\,d\omega \tag{27}$$

The first term of this expression captures the value of the newly invented variety. The second term shows the loss of value from the varieties expired for the period $(t-\tau)$. And, the last term represents that value change from varieties invented during $(t-\tau)$ to t. In the household's dynamic optimisation problem, C is the control variable, and $Assets$ is the state variable. The household also allocates its composite consumption between the consumption of formal and informal final goods at each point in time (discussed in the previous section). We cannot solve the dynamic optimisation in this general setting. Since this is an endogenous growth model like the 'Lab-equipment type of Research and Development', an equilibrium will always exist. We solve the model for an economy in a steady state. Next, we shall also show that only one steady state will exist.

2.4 The Steady State

At the steady state, the major macroeconomic variables grow at a constant rate, and the formal employment share would come to a fixed level. As a result, π_i^* (given by equation (17)) does not change. The firm producing new varieties takes the finance from the consumers against a return. Now, η is assumed to be a constant but independent of time. So, the interest rate $r(v)$ has to be constant. If r shows an upward or downward trend, then $V(t,t)$ cannot be equal to the η for the same time period τ and the same amount of profit per period. Therefore, $r(v) = r; \forall v$. So, equation (18) becomes

$$\eta = \pi_i^* \int_t^{t+\tau} e^{-\int_t^{\omega} r\,dv}\,d\omega \tag{28}$$

After integrating this expression (see Appendix A2 for the derivation), we find the expression as follows:

$$\eta = \frac{\pi_i^*}{r}[1 - e^{-r\tau}] \tag{29}$$

In usual models, if $\tau \to \infty$, then $\eta = \dfrac{\pi_i^*}{r}$.

Equation (29) can be written as

$$\frac{\eta r}{\pi_i^*} = [1 - e^{-r\tau}] \tag{30}$$

This equation shows a unique interest rate (say, r^*) under the steady state. $\frac{\eta r}{\pi_i^*}$ in the LHS is showing an upward-sloping straight line passing through the origin against r-axis (see

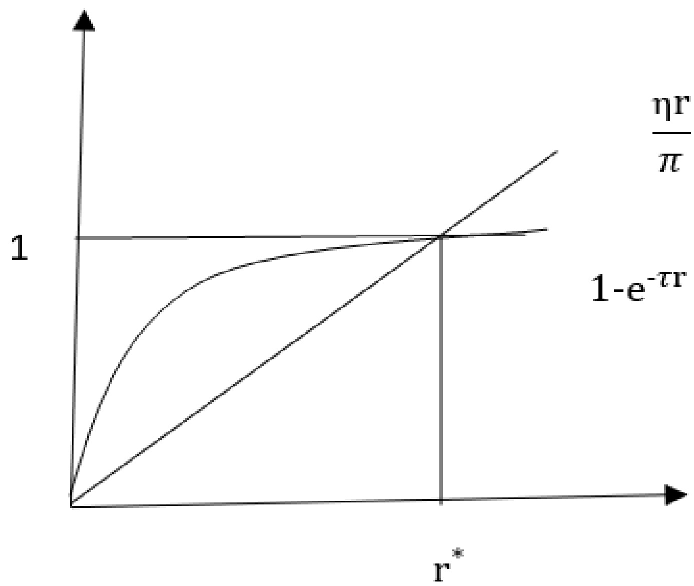
Figure 1). And, $[1-e^{-r\tau}]$ in the RHS also represents a positive slope against the same. Then, we need to find whether they intersect each other and offer a unique equilibrium.

At $r \to 0$, slope of $[1-e^{-r\tau}]$ is $[\tau e^{-r\tau}]_{r=0} = \tau$. Now, the slope of $\frac{r\eta}{\pi_i^*}$ line is always $\frac{\eta}{\pi_i^*}$. As η is equal to the discounted present value of π_i^* for τ time period, it means that $\eta < \tau.\pi_i^*$. As a result, $\frac{\eta}{\pi_i^*} < \tau$. So, the slope of $[1-e^{-r\tau}]$ is higher than $\frac{r\eta}{\pi_i^*}$ at $r \to 0$. Since $[1-e^{-r\tau}]$ cannot exceed 1, these two curves will definitely intersect each other as both curves are continuous and LHS will exceed 1 for some finite $r > \frac{\pi_i^*}{\eta}$. Moreover, the slope of $[1-e^{-r\tau}]$ falls with the increase in r, and so they will intersect only once.

$$\frac{d^2[1-e^{-r\tau}]}{dr^2} = \frac{d[\tau e^{-r\tau}]}{dr} = -\tau^2 e^{-r\tau} < 0.$$

The intersection of $\frac{\eta r}{\pi_i^*}$ curve and $[1-e^{-r\tau}]$ curve gives us the equilibrium interest rate, (r^*). Then, r^* is unique. As π_i^* increases, the slope of $\frac{\eta r}{\pi_i^*}$ decreases. Therefore, r^* increases. The determination of equilibrium interest rate (r^*) has been presented in Figure 1.

Figure 1: Equilibrium interest rate (r^*)



Lemma 7 *At steady state, we find that* $\eta = \frac{\pi_i^*}{r}[1-e^{-r\tau}]$. *This offers a unique interest rate,* r^*.

Proposition 1 *Given* r *and* η, *profit declines with the length of patent periods. For a finite period,* τ, *it becomes* $\pi^* = \frac{\eta r}{1-e^{-r\tau}}$. *When* $\tau \to \infty$, *we find that* $\pi^* \to \eta r$.

Given a fixed rate of earning from the market, a firm would be interested in investing in R&D or buying a patent if the return from this investment gets bigger than the market interest rate. For a fixed R&D cost, η, the profit has to be higher for a shorter period of patent. The profit can be lower to meet the R&D cost if the length of the patent rises. For the infinite horizon, the π would be equalised to ηr, which is similar to (Romer, 1990). The profit starts rising with the lower length of τ, and it rises at $\frac{1}{1-e^{-r\tau}}$.

Since the equilibrium interest rate is constant in a steady state, we can express the market value of varieties simply. Substituting r^* under steady state (see Appendix A2 for the derivation), $V(\omega, t)$ becomes

$$V(\omega, t) = \frac{\pi_i^*}{r}\left[1 - e^{-r[\tau-(t-\omega)]}\right] \tag{31}$$

Note that the value of a variety invented at time-point t can be expressed as $V(t,t) = \dfrac{\pi_i^*}{r}\left[1 - e^{-r\tau}\right] = \eta$ from equation (30). And, $V(\omega, t) = 0$ where $\tau \leq t - \omega$.

Proposition 2 *At the steady state, the interest rate* (r^*) *is rising with the formal employment* (L)*.*

If the rate of return from the investment on R&D rises, the profit goes up. This would raise the demand for formal employment. On the other hand, since formal employment increases the demand for monopoly varieties, this raises the profit of monopolistically competitive variety-producing firms. As a result, the rate of return rises.

2.4.1 Innovation and Asset Progression

The return would encourage investment in innovation. The innovative firms produce new varieties and hence can generate a surplus from the monopoly rent for a finite period of patent length. The present discounted profit value during the patented period builds the assets of the firms. Note that the firms that have already exhausted some period could accumulate the rest of the patent duration. Now, $\dot{N}(\omega)$ is the change of new varieties invented at time point ω. There is a depreciation in the value of existing assets as time passes. Because the existing varieties become closer to their respective dates of patent expiry. As time passes, the new investment engaged in innovation for new varieties would add to the asset accumulation. Substituting (31) into (26), we get

$$Assets(t) = \frac{\pi_i^*}{r}\int_{t-\tau}^{t} [1 - e^{-r(\tau-(t-\omega))}]\dot{N}(\omega)\, d\omega \tag{32}$$

This expression expresses the present discounted value of profits earned by all the firms invented during the period from $t-\tau$ to t. Note that the firms invented earlier would add a lower amount of present value.

From the above equation, we get the change in assets over time as follows:

$$\frac{dAssets(t)}{dt} = \frac{\pi_i^*}{r}[1 - e^{-r\tau}]\dot{N}(t) - \pi_i^*\int_{t-\tau}^{t}\left[e^{-r(\tau-(t-\omega))}\right]\dot{N}(\omega)\, d\omega \tag{33}$$

The first term of (33) is the gross investment in the form of profits earned by newly invented varieties in the period t, which is equal to $\dot{N}(t).\eta$. The second term is the loss of assets due to depreciation in value for continuous patent expiry and monopoly rent. Earlier the invention (i.e., lower the ω) shorter the patent length (i.e., $\tau - (t-\omega)$) lower the profits adding to the discounted present value of assets.

If $\delta(v)$ is the growth rate of N at the time point v, then $N(t) = N(\omega)e^{\int_\omega^t \delta(v)\,dv}$ or $N(\omega) = N(t)e^{-\int_\omega^t \delta(v)\,dv}$. At the steady state, $\delta(v)$ should not vary with time. Therefore, we assume that $\delta(v) = \delta$ for all v. Then, it can be expressed as follows:

$$\dot{N}(t) = \delta.N(t) \text{ and } \dot{N}(\omega) = \delta.N(\omega)$$
$$\text{So, } N(\omega) = \frac{\dot{N}(\omega)}{\delta} \text{ and } N(t) = \frac{\dot{N}(t)}{\delta}$$

Incorporating these in the above equation, we get $N(t) = \dfrac{\dot{N}(t)}{\delta} = N(\omega)e^{\int_\omega^t \delta.\,dv} = \dfrac{\dot{N}(\omega)}{\delta}e^{\int_\omega^t \delta.\,dv}$. Comparing these equalities, one can express the rate of new inventions as $\dot{N}(t) = \dot{N}(\omega)e^{\int_\omega^t \delta.\,dv}$. Taking the inverse of the exponential term, we express it as follows:

$$\dot{N}(\omega) = \dot{N}(t)e^{-\int_\omega^t \delta.\,dv} \tag{34}$$

Equation (34) shows that $\dot{N}$ grows at the rate δ. If $\dot{N}(t)$ is discounted by δ rate, the rate in the past period, ω, can be found. Using (32) and (34), the asset expression can be expressed as follows:

$$Assets(t) = \frac{\pi_i^*}{r}\dot{N}(t)\int_{t-\tau}^{t}[1 - e^{-r(\tau-(t-\omega))}]e^{-\int_\omega^t \delta.\,dv}\,d\omega$$

Substituting $\dfrac{\pi_i^*}{r} = \dfrac{\eta}{[1-e^{-r\tau}]}$ from equation (29), and rearranging the terms in the above expression, we find

$$\eta\dot{N}(t) = \frac{Assets(t)}{H_1} \tag{35}$$

$$\text{Where, } H_1 = \frac{\int_{t-\tau}^{t}[1 - e^{-r(\tau-(t-\omega))}]e^{-\int_\omega^t \delta.\,dv}\,d\omega}{[1-e^{-r\tau}]} = \frac{\int_{t-\tau}^{t}[1 - e^{-r(\tau-(t-\omega))}]e^{-\delta.(t-\omega)}\,d\omega}{[1-e^{-r\tau}]} \tag{36}$$

Equation (35) shows that H_1 captures the ratio of the current value of a unit asset and the current value of the unit investment. The denominator shows that accumulation of earnings from the varieties grows at δ rate during the period from $(t-\omega)$ to t. And the numerator indicates the accumulation of varieties at the t -th period. Since $Assets(t)$ is the state variable in the household's optimisation problem. This expression would help to express current investment in terms of $Assets(t)$. As r and δ are constants in the steady state, H_1 would also be constant in the steady state.

Similarly, the expression of asset accumulation, given in (33), can further be simplified with the help of (29). Substituting (34) into (33), we find the expression as follows:

$$\frac{dAssets(t)}{dt} = \frac{\pi_i^*}{r}[1 - e^{-r\tau}]\dot{N}(t) - \pi_i^* \int_{t-\tau}^{t} \left[e^{-r(\tau-(t-\omega))}\right] \dot{N}(t)e^{-\int_\omega^t \delta.\, dv}\, d\omega \tag{37}$$

Further substituting π_i^* from equation (29), and then using equation (35) in the above equation

$$\frac{dAssets(t)}{dt} = \eta\dot{N}(t) - \frac{r.Assets(t)}{H_1[1 - e^{-r\tau}]} \int_{t-\tau}^{t} \left[e^{-r(\tau-(t-\omega))}\right] e^{-\delta(t-\omega)}\, d\omega$$

Incorporating equation (35) in the above equation, we get

$$\frac{dAssets(t)}{dt} = \eta\dot{N}(t) + \frac{r.Assets(t)}{H_1}H_2 \tag{38}$$

where,

$$H_2 = -\frac{\int_{t-\tau}^{t} \left[e^{-r(\tau-(t-\omega))}\right] e^{-\delta(t-\omega)}\, d\omega}{[1 - e^{-r\tau}]}$$

Equation (37) captures the change in the value of *Assets* with respect to time. It changes for two reasons. The first term on the RHS of this equation measures one such reason. As a household makes savings, that is spent for investment to discover new intermediate product varieties. The investment gives dividends or profit to the household. The second term on the RHS captures the loss in the value of existing *Assets* as time passes because existing varieties come closer to patent expiry. This loss of return from the patent expiry depreciates the asset. In the equation, H_2 indicates the loss from the patent expiry during the period from $(t - \omega)$ to t compared to the return for the entire patent length. Again, for the fixed values of δ and r at the steady state, H_2 would also be constant.

We assume that the household saves to receive a return from the investment on the R&D to produce new varieties. As the income left after consumption is saved and invested, and in this economy, investment is made only to develop new blueprints of varieties. So, the trade-off of spending between present and future consumption depends on the return from the investment in new varieties. Therefore, the budget constraint represents that the investment in R&D must be equalised to the surplus income after the consumption in the current period. A household earns from three sources - wage income from formal sector employment, wage received from informal sector employment and the return from investment in R&D sector producing new input varieties. Hence, the budget constraint can be expressed as follows:

$$\eta\dot{N}(t) = W_F L + W_I(1 - L) + \pi_i^* \int_{t-\tau}^{t} \dot{N}(\omega)\, d\omega - PC \tag{39}$$

Using equation (34) in the above equation, we get

$$\eta\dot{N}(t) = W_F L + W_I(1 - L) + \pi_i^* \int_{t-\tau}^{t} \dot{N}(t)e^{-\delta(t-\omega)}\, d\omega - PC \tag{40}$$

Substituting π_i^* from equation (29) and using equation (35) in the above equation, we obtain

$$\eta\dot{N}(t) = W_F L + W_I(1-L) + \frac{r.Assets(t)}{H_1} H_3 - PC \tag{41}$$

where, $H_3 = \dfrac{\int_{t-\tau}^{t} e^{-\delta(t-\omega)}\, d\omega}{[1-e^{-r\tau}]}$. H_3 captures the accumulated patent varieties in their returns or investment upto period t. This links the total earning from patented varieties with the current investment level. Again, this will be a constant given the fixed level of δ and r at the steady state. Substituting (38) into (41) and rearranging the terms, we can express the equation of motion as follows:

$$\frac{dAssets(t)}{dt} = W_F L + W_I(1-L) + \frac{r.Assets(t)}{H_1}(H_2 + H_3) - PC \tag{42}$$

This expression captures the residue income after the consumption adds to the path of asset accumulations. As already discussed, the current value of $Assets$ has two impacts on the change in the value of $Assets$. On the one hand, it generates income, which is used for accumulating $Assets$; on the other hand, it reduces $Assets$ due to the fall of its valuation with the passing of time. This essentially helps us to find a steady and optimum consumption path. So, the representative household Maximises (20) subjected to (42).

2.4.2 Balanced Growth Path

The optimisation problem of the representative household can be solved using the Hamiltonian approach. Here, the asset would be the state variable and the consumption is the control variable. Therefore, the present value Hamiltonian function can be represented as follows:

$$H(t) = \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t}$$

$$+\lambda(t)\left(W_F L + W_I(1-L) + \frac{r.Assets(t)}{H_1}(H_2 + H_3) - PC\right) \tag{43}$$

Here, $\lambda(t)$ represents constraint multiplier for period t. The first-order condition with respect to the state variable is represented as follows:

$$\dot{\lambda}(t) = -\frac{\partial H(t)}{\partial Assets(t)} = -\lambda(t) r \frac{H_3}{H_1} - \lambda(t) r \frac{H_2}{H_1}$$

$$\frac{\dot{\lambda(t)}}{\lambda(t)} = -\frac{r}{H_1}(H_3 + H_2) \tag{44}$$

The transversality condition of the optimisation problem can be found as follows:

$$\lim_{t\to\infty} \lambda(t).Assets(t) = 0 \tag{45}$$

If the initial value of the asset is assumed as $assets(0)$, then the expression of the condition can be written as $\lim_{t\to\infty} \lambda(0).Assets(0)e^{-[\frac{rH_3}{H_1}+\frac{rH_2}{H_1}]t}.e^{\overline{g}(t).t}$; where $\overline{g}(t)$ is the average growth rate of assets between 0 and t time point.

Lemma 8 *The transversality condition requires as $t \to \infty$,*

$$\overline{g}(t) < \frac{r}{H_1}(H_2 + H_3) \tag{46}$$

As the time approaches infinity, asset accumulation cannot grow faster than the return from the assets.

Once the transversality condition is satisfied, we can find a unique solution for consumption and growth rate. We have already discussed that P is a price index constructed as the minimum spending for C, where C_F and C_I are chosen optimally. The first-order condition of the control variable can be written as follows:

$$C^{-\theta}e^{-\rho t} = \lambda P \tag{47}$$

Taking the derivative of this with respect to time, we get

$$\frac{\dot{C}}{C} = \frac{1}{\theta}\left(-\rho - \frac{\dot{\lambda}}{\lambda} - \frac{\dot{P}}{P}\right) \tag{48}$$

Using (44) into (48), we get

$$\frac{\dot{C}}{C} = \frac{1}{\theta}\left(\frac{r}{H_1}(H_2 + H_3) - \rho - \frac{\dot{P}}{P}\right) \tag{49}$$

This gives us the key expression for finding the growth path. To establish stable growth of C, it is required to show a stable value of RHS expression. Note that H_1, H_2 and H_3 are constants. One can portray the growth rate if we can identify the $\dot{P}/P$ path.

Asset accumulation, the key driver of growth in this model, depends on the rate of newly invented varieties. We assumed that the total varieties are the sum of monopoly and competitive varieties and the change in total varieties would be the sum of the respective varieties, i.e., $N(t) = N_c(t) + N_m(t)$ and $\dot{N}(t) = \dot{N}_c(t) + \dot{N}_m(t)$. At the steady state, $\frac{\dot{N}(t)}{N(t)}$ must be a constant. All newly invented varieties at time point t will lose their patent protection after the τ time period, and there will make a new addition to the competitive variety after τ periods. So, for the stability we need that $\dot{N}_c(t+\tau) = \dot{N}(t)$ and $N_c(t+\tau) = N(t)$.

At the steady state, $\frac{\dot{N}}{N} = \frac{\dot{N}(t)}{N(t)} = \frac{\dot{N}_c(t+\tau)}{N_c(t+\tau)} = \frac{\dot{N}_c}{N_c} = \delta$. Using algebraic manipulation[3] , we can express as follows:

$$\frac{\dot{N} - \dot{N}_c}{N - N_c} = \frac{\dot{N}_m}{N_m} = \frac{\dot{N}}{N} = \frac{\dot{N}_c}{N_c} = \delta \tag{50}$$

[3]If $\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$, then $\frac{Z_1 - Z_3}{Z_2 - Z_4} = \frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$

In the steady state, N, N_m and N_c are growing at the same rate, δ. If the change of monopoly varieties is δ, the rate of varieties getting the expiry of projects would also be the same. As a result, the rise of competitive varieties would be the same.

If the new varieties that need formal employment grow at the same rate as the patent-expired varieties, there would not be any change in formal and informal employment. So, in a steady state, the formal and informal sector employment must be constant. This suggests that the formal employment would be fixed at $L = L^*$. Using this employment, we can further derive the sectoral outputs. From (1), (15) and (12), the formal output is $Y_F = A[N_c X_{iFc}^{*\alpha} + (N - N_c) X_{iFm}^{*\alpha}] L^{*(1-\alpha)}$. Substituting the equilibrium input demands and simplifying, we get

$$Y_F = A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} N_c \left[1 + \frac{N_m}{N_c} \alpha^{\frac{\alpha}{1-\alpha}} \right] L^* \tag{51}$$

Taking the derivative with respect to time, we find that $\frac{\dot{Y}_F}{Y_F} = \frac{\dot{N}_c}{N_c} = \delta$. This suggests that the new varieties are used in the formal sector, and the formal sector output would essentially grow at δ rate.

Proposition 3 *Since $\alpha^{\frac{\alpha}{1-\alpha}} < 1$, Y_F increases with N, L^*, A and N_C (for given N). As $\frac{N_m}{N_c}$ is constant in steady state, both grow at the same rate, i.e., $\frac{\dot{Y}_F}{Y_F} = \frac{\dot{N}_c}{N_c} = \delta$.*

Similarly, from (2) and (13), one can derive the informal sector output. $Y_I = B\gamma N_c X_{iIc}^{*\beta} (1 - L)^{1-\beta}$. Substituting the equilibrium input demand from equation (13), we get

$$Y_I = B\gamma N_c [P_I B \beta]^{\frac{\beta}{1-\beta}} (1 - L^*) \tag{52}$$

If $\dot{P} = 0$, we can show that $\frac{\dot{Y}_I}{Y_I} = \frac{\dot{N}_c}{N_c} = \delta$. Because the informal sector firm would use the varieties that expired the patent length, increasing the informal production at the same rate.

Lemma 9 *If P_I is constant at the steady state, then $\frac{\dot{Y}_I}{Y_I} = \frac{\dot{N}_c}{N_c}$*

Moreover, since $W_F = (1-\alpha)\frac{Y_F}{L^*}$ from (7) and $W_I = P_I(1-\beta)\frac{Y_I}{(1-L^*)}$ from (8), W_F and W_I also grow at the same rate. In order to claim that P_I is constant, one needs to establish $\dot{P} = 0$ at the steady state.

Lemma 10 *If P_I is constant at the steady state, then $\frac{\dot{W}_F}{W_F} = \frac{\dot{W}_I}{W_I} = \frac{\dot{N}_c}{N_c}$*

Let us now derive P_I. It depends on H_1, H_2 and H_3. Note that r is the common factor of these components. Since r is fixed at the steady state, we can show that these components are also fixed (see appendix A3, A4 and A5 for proofs).

Integrating the expression of (36), we find that (see appendix A3):

$$H_1 = \frac{1}{\delta} - \frac{re^{-r\tau}[1 - e^{(r-\delta)\tau}]}{\delta(\delta - r)[1 - e^{-r\tau}]} \tag{53}$$

As H_1 is a constant. So, from equation (35), $Assets(t)$ grows at the same rate of $\dot{N}(t)$ in the steady state.

Similarly, integrating this expression (38), we further find (see Appendix for derivation):

$$H_2 = -\frac{e^{-r\tau}[1 - e^{(r-\delta)\tau}]}{(\delta - r)[1 - e^{-r\tau}]} \tag{54}$$

Similarly, from (41)

$$H_3 = \frac{\int_{t-\tau}^{t} e^{-\delta(t-\omega)}\, d\omega}{[1 - e^{-r\tau}]} = \frac{e^{-\delta t}}{[1 - e^{-r\tau}]}\left[\frac{e^{\delta(t)} - e^{\delta(t-\tau)}}{\delta}\right] = \frac{1 - e^{-\delta\tau}}{\delta[1 - e^{-r\tau}]} \tag{55}$$

Given the constant values of H_1 and H_2, we can infer the growth rate of assets. From (37), we find:

$$\frac{\frac{dAssets(t)}{dt}}{Assets(t)} = \eta\frac{\dot{N}(t)}{Assets(t)} + \frac{rH_2}{H_1} \tag{56}$$

Since H_1 and H_2 are constants, and Assets(t) grows at the growth rate of $\dot{N}(t)$, therefore, Assets(t) grows at a constant rate. Moreover, the parameters affecting P_I would further the distribution of formal and informal employment and the resultant growth at the steady state.

Now, we can determine the equilibrium growth rate, where P_I^* and L^* are determined constant. We will get those solutions from the labour market and both formal and informal goods market equilibrium conditions. As there are three markets, and if the labour and informal markets are in equilibrium, the formal market must also be in equilibrium by Walras law. Given other things, the labour allocation that satisfies equation (3) gives the equilibrium employment distribution. Substituting the value of Y_F and Y_I into wages, the equilibrium condition for the formal labour market can be written as follows: (See Appendix for derivation)

$$P_I = \frac{(1-\alpha)^{1-\beta} A^{\frac{1-\beta}{1-\alpha}} \alpha^{\frac{\alpha(1-\beta)}{1-\alpha}} [1 + (e^{\delta\tau} - 1)\alpha^{\frac{\alpha}{1-\alpha}}]^{1-\beta}}{B\beta^{\beta}\phi^{1-\beta}(1-\beta)^{1-\beta}\gamma^{1-\beta}} \tag{57}$$

The above equation shows that the relative price of the informal sector, P_I, depends on the growth rate, δ, which essentially depends on many parameters. Since δ is constant in the steady state. P_I should also be constant. Equation (25) shows that if P_I is constant, then price index P is also constant. The rise of wages raises the cost of production, but the entry of competitive varieties at the same rate. They cancel each other to maintain the price of the informal goods at the same level.

Lemma 11 *As δ is constant in the steady state. P_I and P should also be constant.*

It is assumed that informal sector goods cannot be used for production in the intermediate goods sector. So, the equilibrium condition of the informal sector is as follows:

$$C_I = Y_I \tag{58}$$

This suggests that the sectoral production will be the same once the C_I is determined at the steady state. On the other hand, the formal final goods will be used for final consumption and the rest will be for R&D sector and for intermediate goods production in the competitive and non-competitive sectors of the formal sector and competitive informal sector. In equilibrium, the demand and supply of the formal final goods market can be expressed as follows:

$$Y_F = C_F + \eta\dot{N} + \int_0^{N_c} X^*_{iFc}\, di + \int_{N_c}^{N} X^*_{iFm}\, di + \int_0^{\gamma N_c} X^*_{iIc}\, di \tag{59}$$

Using the relation between C_F and C_I given by equation (22), informal market equilibrium and optimum quantities of varieties, we get

$$A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}.e^{-\delta\tau}[1+(e^{\delta\tau}-1)\alpha^{\frac{\alpha}{1-\alpha}}]L^*$$
$$= \left\{\frac{(1-a)}{aP_I}\right\}^{\frac{1}{6-1}} B\gamma e^{-\delta\tau}[P_I B\beta]^{\frac{\beta}{1-\beta}}(1-L^*) + \eta\delta + e^{-\delta\tau}A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}L^*$$
$$+[1-e^{-\delta\tau}]A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}L^* + \gamma.e^{-\delta\tau}[P_I B\beta]^{\frac{1}{1-\beta}}(1-L^*) \tag{60}$$

Substituting P_I from equation (57) in the above equation, we get an equation capturing a relationship between δ and L^*.

Another equation with a relationship between δ and L^* can be obtained from the consumption growth expressed in equation (49). The consumption also grows at the rate δ^* at the steady state for the following reason. Equation (52) shows that as P_I is constant, $Y_I = C_I$ grows at the rate of δ^* at the steady state. From equation (24), C also grows at that rate. So, the steady state, which exists, is unique. All variables grow at the rate δ^*, and the employments in formal and informal sectors are constant. As it is already shown that P is fixed in the steady state, so the $\frac{\dot{P}}{P} = 0$. Putting $\frac{\dot{P}}{P} = 0$ in that equation, we get

$$\delta = \frac{\dot{c}}{c} = \frac{r\frac{H_3+H_2}{H_1} - \rho}{\theta}$$

Using equations (53), (54) and (55), we get $\frac{H_3+H_2}{H_1} = 1$, So, we get the usual Euler equation given by

$$\delta = \frac{\dot{c}}{c} = \frac{r-\rho}{\theta} \tag{61}$$

Using equations (17) and (29) in the above equations, we get another equation relating δ^* and L^*.

Therefore, these two equations (60 and 61) with δ^* and L^* will provide us with a unique set of solutions for equilibrium growth rate (δ^*) and formal employment (L^*). Putting that δ^* in equation (57), we can get equilibrium P_I^*. As it is obvious, the values of δ^* and L^* depend on the values of different parameters in the model. In this analysis, we are focusing only on the steady state and not on the transitional dynamics. Hence we assume that the initial value of the ratio of the number of patented varieties and the number of competitive varieties, i.e., $\dfrac{N_m(0)}{N_c(0)} = (e^{\delta^*\tau} - 1)$. So, the economy starts from a steady state and stays there forever.

Proposition 4 *The steady-state growth rate depends on the size of formal employment and vice versa. They are determined simultaneously in the equilibrium.*

We have seen that the growth depends on the surplus raised by the formal firms that procure the patented inputs. The higher the formal employment greater the profit and higher the growth. Therefore, the size of formal employment determines the level of growth. This differs from the standard literature on growth with scale effect that highlights the size of total employment.

Unlike the existing literature on endogenous growth with R&D, in this model, the growth rate affects the division of employment between formal and informal sectors. In the balanced growth equilibrium, the economy's growth rate also denotes the growth rate at which new varieties are invented and patented. So, for a given time period of patent validity τ, a higher number of patented varieties will be accumulated with a higher growth rate. This implies that the ratio of the number of patented varieties and the total number of varieties will be higher (see equations (A.5) and (A.6) in the Appendix). Since only the formal final good sector uses the patented varieties, so a relatively higher proportion of patented varieties will alter the amount of resource allocation between formal and informal final good sectors and hence corresponding employment.

Proposition 5 *The formal employment share and growth rate exhibit a non-monotonic relation across the economies with the level of development.*

One would expect both ϕ and A to rise with the development. In the developed world, wage difference would be more due to higher productive technology used in the formal sector and better protection of formal workers' rights. Similarly, the technology of formal firms tends to be far improved than those of the less developed world. Better technology suggests a greater surplus in the formal sector. As a result, that would raise both L and

r monotonically. However, higher ϕ makes labour more expensive for the formal sector and should reduce formal employment and hence r.

We expect that the substitutability of final goods produced in formal and informal sectors will fall with the development of the nation. The technology used in the informal sector should be improved in the developed world, and hence the technology gap would be lower. So, σ may fall with the development. We cannot comment about its impact on formal employment share as we do not know its impact on P_I a priori, which is determined from the interaction of both the relative demand and supply of informal final output relative to formal final output. Similarly, with development, as the number of varieties available increases, we can expect that the proportion used by the informal sector will become smaller and smaller. One would expect a rise in formal employment or a fall in informal employment as the informal sector uses proportionately fewer resources. If we combine them, we will find that both L and r would reveal a non-monotonic path of growth with the development.

Proposition 6 *Formal employment is positive and monotonically related to patent duration.*

3 Calibration

We found a steady state of growth that involves a number of factors. In order to visualise the alleged relationship between innovation, the informal sector and growth, an attempt has been made to calibrate them using specific values of parameters. We are interested in the behaviour of r and L. Since the growth rate varies symmetrically with r, so we are focusing on r to understand the impact on growth. Our main interest is to simulate the results for the parameters that change with economic development. A and B representing the levels of technology used in formal and informal sectors, respectively must be different. Some studies (e.g., Marjit and Maiti (2009); WorldBank (2018)) seem to suggest that formal firms are three to five times more productive than informal firms. So, the base values A and B are assumed to be 100 and 30. However, the results have been repeated for the range of B from 30 to 70 for robustness checking. Further, since the informal wage contains neither any social security benefit nor the union rent, we assume that the formal wage is 1.2 times higher than the informal wage. Andrabi et al. (2009) have estimated that the union wage is 1.1-1.3 times higher than the reservation wage in the European market. Of course, the difference may be significantly differ in the developing world. But, we kept the baseline value of ϕ as 1.2. The degree of substitutability between the formal and informal goods, captured by σ would definitely vary across countries depending upon the development (Amaral and Quintin, 2006). Individuals in the developed world may prefer to consume goods produced in the formal sector more because of better quality. So, their substitutability will decline with the development of country (Foellmi and Zweimüller, 2011). The baseline value of σ is assumed as 0.1. It ranges from -0.4 to 0.4. The baseline values of the rest parameters are similar to the standard literature (see Table 1).

Table 1: Baseline values of parameters

Parameters	Baseline
A	100
B	30 to 70
α	0.4
β	0.3
ϕ	1.2
γ	0.75
σ	0.1 (-0.4 to 0.4)
a	0.5
ρ	0.02
τ	20
η	1000

We are interested in simulating the effect of parameters representing the level of development on formal employment and interest rate (a key determinant of economic growth). For a rise in σ, we find that both L and r increase (see Figures 1 and 2). On the other hand, they rise with a fall of ϕ. If σ falls and ϕ rises with the development, we find that they lead to both formal employment and growth rate falling.

On the other hand, γ would fall, and A would rise with the development. They result in a monotonic rise of both L and r. Combining them will find that both L and r would reveal non-monotonic relation with the development if we combine them. The results do not change much for different values of Bs and σs.

Figure 2: Formal Employment (L)

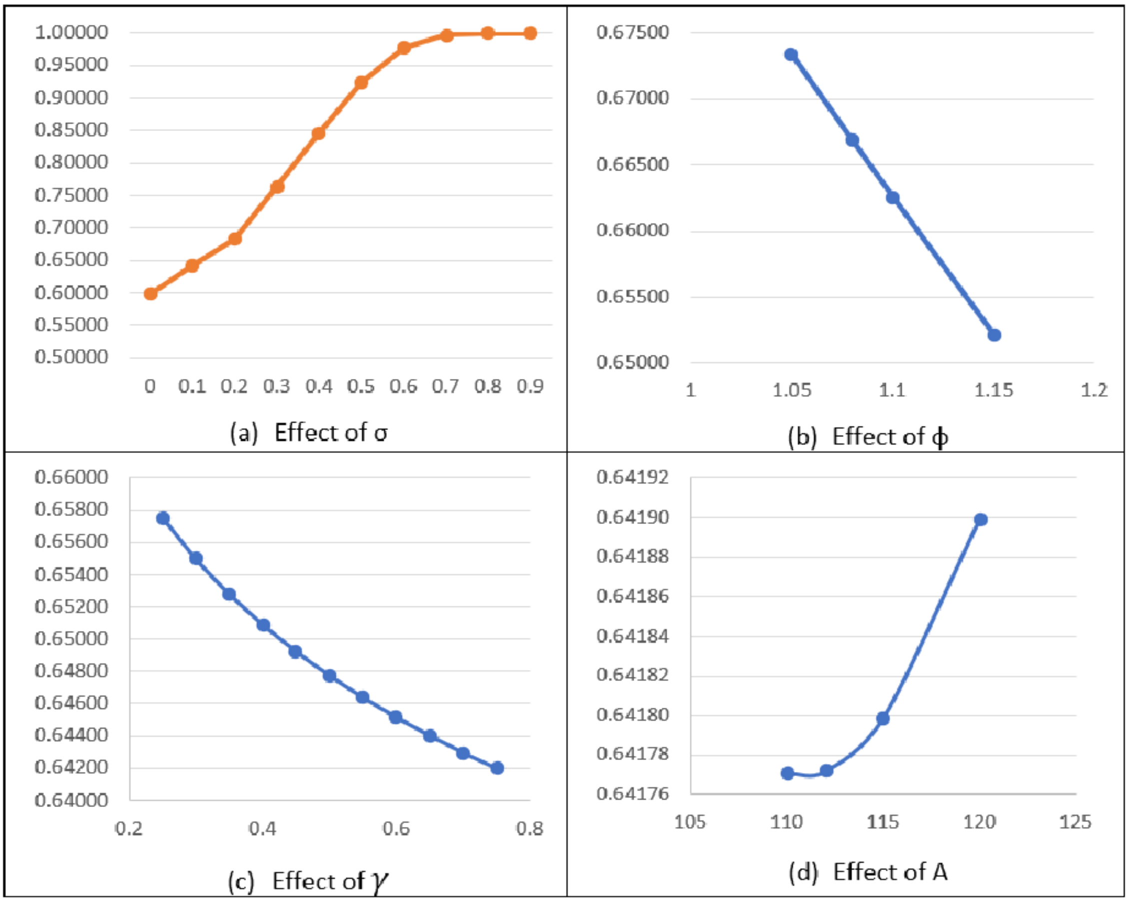
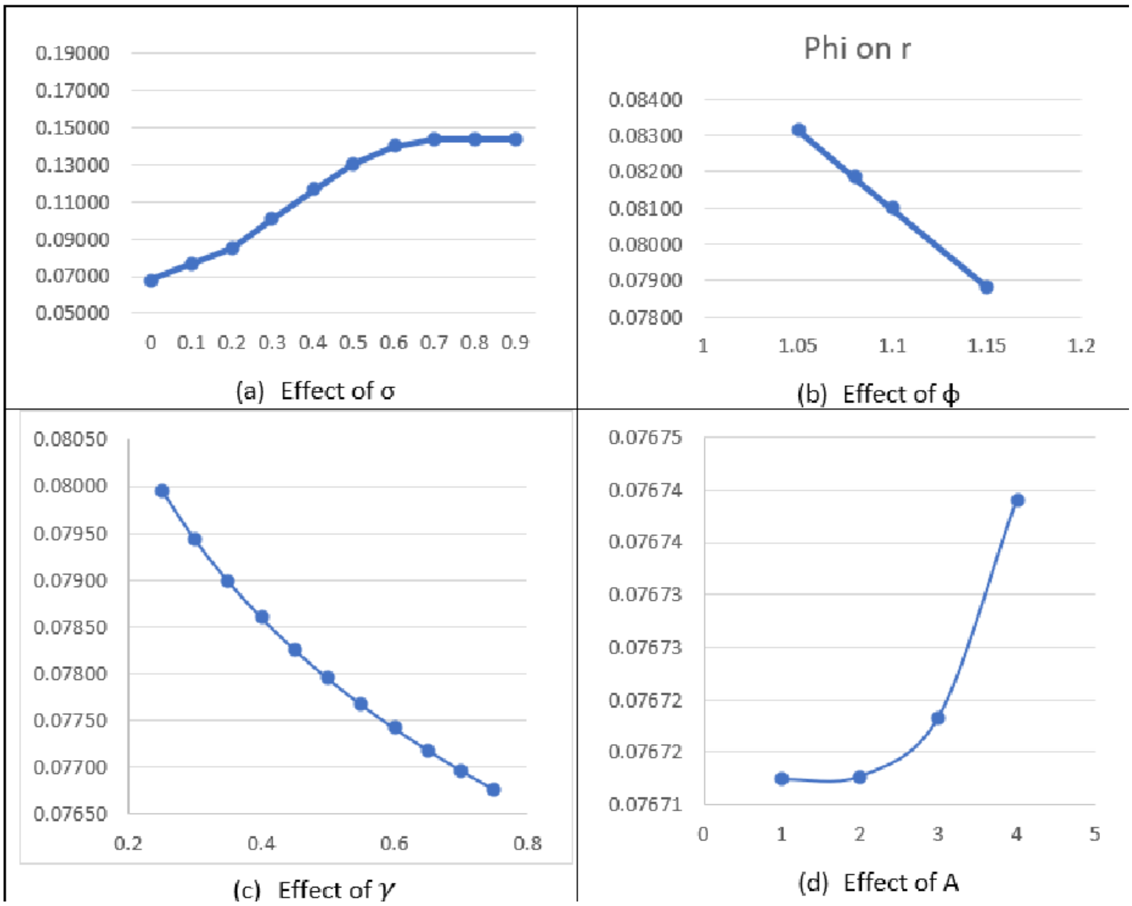


Figure 3: Interest Rate (r)



4 Concluding Remarks

The paper offers a theoretical growth model with innovation in the presence of the informal sector. A typical firm producing in the informal sector cannot afford to buy patented non-competitive varieties but rely on competitive varieties. The goods and services produced in the informal sector can act as an imperfect substitute for some of the goods and services produced in the formal sector with better technology. The formal sector firms procure the patented varieties of inputs, which are invented by investing in R&D. It is observed that the size of formal employment expands the non-competitive varieties, affecting the return from R&D that raises the economic growth. So, we find that the growth is monotonically and positively related to the size of formal employment. On the other hand, the growth rate also impacts the formal employment share by changing the allocation of resources between the formal and informal final good sectors. However, growth and formal employment may not monotonically rise with the development.

Appendix

A1: Derivation of composite Price

From (23) and (24), we get

$$\frac{\left\{P_I + (1-a)^{\frac{1}{\sigma-1}} a^{\frac{1}{1-\sigma}} P_I^{\frac{1}{1-\sigma}}\right\}}{P} = [a^{\frac{1}{1-\sigma}}(1-a)^{\frac{\sigma}{\sigma-1}} P_I^{\frac{\sigma}{1-\sigma}} + (1-a)]^{\frac{1}{\sigma}}$$

$$\Rightarrow \frac{P_I}{P}\left\{1 + (1-a)^{\frac{1}{\sigma-1}} a^{\frac{1}{1-\sigma}} P_I^{\frac{\sigma}{1-\sigma}}\right\} = (1-a)^{\frac{1}{\sigma}}\left\{1 + a^{\frac{1}{1-\sigma}} P_I^{\frac{\sigma}{1-\sigma}} (1-a)^{\frac{1}{\sigma-1}}\right\}^{\frac{1}{\sigma}}$$

$$\Rightarrow \frac{P_I}{P} = (1-a)^{\frac{1}{\sigma}}\left\{1 + a^{\frac{1}{1-\sigma}} P_I^{\frac{\sigma}{1-\sigma}} (1-a)^{\frac{1}{\sigma-1}}\right\}^{\frac{1-\sigma}{\sigma}}$$

$$\Rightarrow \frac{P_I}{P} = \left\{(1-a)^{\frac{1}{1-\sigma}} + a^{\frac{1}{1-\sigma}} P_I^{\frac{\sigma}{1-\sigma}}\right\}^{\frac{1-\sigma}{\sigma}}$$

$$\Rightarrow \frac{P_I}{P} = P_I\left\{(1-a)^{\frac{1}{1-\sigma}} P_I^{\frac{\sigma}{\sigma-1}} + a^{\frac{1}{1-\sigma}}\right\}^{\frac{1-\sigma}{\sigma}}$$

$$P = \left\{(1-a)^{\frac{1}{1-\sigma}} P_I^{\frac{\sigma}{\sigma-1}} + a^{\frac{1}{1-\sigma}}\right\}^{\frac{\sigma-1}{\sigma}} \tag{A.1}$$

A2: Derivation of equation (29)

$$\eta = \pi_i^* \int_t^{t+\tau} e^{-\int_t^{\omega} r\, dv}\, d\omega.$$

$$\Rightarrow \eta = \pi_i^* \int_t^{t+\tau} e^{-r[\omega - t]}\, d\omega$$

$$= \pi_i^* e^{rt}\left[\frac{e^{-r\omega}}{-r}\right]_t^{t+\tau} = \pi_i^* e^{rt}\left[\frac{e^{-r(t+\tau)} - e^{-rt}}{-r}\right]$$

$$= \pi_i^* e^{rt}\left[\frac{e^{-rt} - e^{-r(t+\tau)}}{r}\right] = \frac{\pi_i^*}{r}[1 - e^{-r\tau}] \tag{A.2}$$

Simplification of market value of varieties expression under steady-state.

$$V(\omega, t) = \int_t^{\omega+\tau} \pi_i^* e^{-\int_t^v r\, dz}\, dv$$

$$V(\omega, t) = \int_t^{\omega+\tau} \pi_i^* e^{-r(v-t)}\, dv = \pi_i^* e^{rt}\int_t^{\omega+\tau} e^{-rv}\, dv$$

$$= \pi_i^* e^{rt}\left[\frac{e^{-rv}}{-r}\right]_t^{\omega+\tau} = \pi_i^* e^{rt}\left[\frac{e^{-rt} - e^{-r(\omega+\tau)}}{r}\right]$$

$$V(\omega, t) = \frac{\pi_i^*}{r}\left[1 - e^{-r[(\omega+\tau)-t]}\right]$$

A3: Derivation of H1

$$\text{Since } \int_{t-\tau}^{t} v\frac{du}{\delta} = \left[\frac{uv}{\delta}\right]_{t-\tau}^{t} - \frac{1}{\delta}\int_{t-\tau}^{t} u\frac{dv}{d\omega}\, d\omega$$

Take $v = [1 - e^{-r(\omega+\tau-t)}]$, $u = e^{\delta\omega}$ and $\dfrac{du}{d\omega} = \delta e^{\delta\omega}$

$$\Rightarrow \frac{du}{\delta} = e^{\delta\omega} d\omega$$

So putting values of v and $\dfrac{du}{\delta}$, we get

$$H_1 = \left[\frac{e^{\delta\omega}[1-e^{-r(\omega+\tau-t)}]}{\delta}\right]_{t-\tau}^{t} . \frac{e^{-\delta t}}{[1-e^{-r\tau}]} - \frac{1}{\delta.[1-e^{-r\tau}]}\int_{t-\tau}^{t} e^{\delta\omega}e^{-\delta t}[re^{-r\omega}e^{-r(\tau-t)}]\, d\omega$$

$$= \frac{e^{-\delta t}}{\delta.[1-e^{-r\tau}]}\left\{e^{\delta t}[1-e^{-r\tau}] - e^{\delta(t-\tau)}[1-e^{0}]\right\} - \frac{e^{-\delta t}e^{-r(\tau-t)}.r}{\delta.[1-e^{-r\tau}]}\left[\frac{e^{\omega(\delta-r)}}{\delta-r}\right]_{t-\tau}^{t}$$

$$= \frac{[1-e^{-r\tau}]}{\delta.[1-e^{-r\tau}]} - \frac{e^{-\delta t}r[e^{(\delta-r)t} - e^{(\delta-r)(t-\tau)}]e^{-r(\tau-t)}}{\delta(\delta-r).[1-e^{-r\tau}]}$$

$$H_1 = \frac{1}{\delta} - \frac{re^{-r\tau}[1-e^{(r-\delta)\tau}]}{\delta(\delta-r).[1-e^{-r\tau}]} \tag{A.3}$$

A4: Derivation of H2

$$H_2 = -\frac{\int_{t-\tau}^{t}[e^{rt}e^{-r\tau}]e^{-r\omega}e^{-\delta(t-\omega)}\, d\omega}{[1-e^{-r\tau}]}$$

Integrating this expression, we find :

$$H_2 = -\frac{e^{r(t-\tau)}e^{-\delta t}}{[1-e^{-r\tau}]}\int_{t-\tau}^{t} e^{(\delta-r)\omega}\, d\omega$$

$$= -e^{r(t-\tau)}e^{-\delta t}\frac{e^{(\delta-r)t} - e^{(\delta-r)(t-\tau)}}{(\delta-r)[1-e^{-r\tau}]}$$

$$H_2 = -\frac{e^{-r\tau}[1-e^{(r-\delta)\tau}]}{(\delta-r)[1-e^{-r\tau}]}$$

A4: Derivation of P_I

To get labour market equilibrium, we use equations (1), (2), (3), (7), (8), (51) and (52) and they give the following relation:

$$W_F = (1-\alpha)\frac{Y_F}{L} = \phi W_I = \phi(1-\beta)\frac{Y_I P_I}{(1-L)}$$

Substituting the value of Y_F and Y_I, we can rewrite as follows:

$$(1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}N_c\left[1 + \frac{N_m}{N_c}\alpha^{\frac{\alpha}{1-\alpha}}\right] = P_I\phi(1-\beta)B\gamma N_c[P_I B\beta]^{\frac{\beta}{1-\beta}} \tag{A.4}$$

This expression contain P_I. Since $N_c(t)$ grows at a rate δ in the steady rate, then one can write that $N_c(t+\tau) = N(t) = N_c(t).e^{\delta\tau}$. This can be rewritten as follows:

$$N_c(t) = N(t).e^{-\delta\tau} \tag{A.5}$$

Since $N_m(t) = N(t) - N_c(t)$, we can further rewrite as $N_m(t) = N(t) - N_c(t) = N(t)[1 - e^{-\delta\tau}]$. This can be expressed in the form of ratio as below:

$$\frac{N_m(t)}{N_c(t)} = \frac{1-e^{-\delta\tau}}{e^{-\delta\tau}} = \frac{1}{e^{-\delta\tau}} - 1 = e^{\delta\tau} - 1 \tag{A.6}$$

Substituting (A.6) into (A.4), we have

$$(1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}[1+(e^{\delta\tau}-1)\alpha^{\frac{\alpha}{1-\alpha}}] = P_I\phi(1-\beta)B\gamma[P_IB\beta]^{\frac{\beta}{1-\beta}} \tag{A.7}$$

A5: Derivation of equation (60)

The equilibrium condition for the formal final goods market can be written as follows:

$$Y_F = C_F + \eta\dot{N} + \int_0^{N_c} X^*_{iFc}\,di + \int_{N_c}^{N} X^*_{iFm}\,di + \int_0^{\gamma N_c} X^*_{iIc}\,di \tag{A.8}$$

We use equations (51), (A.5), and (A.6) to substitute for Y_F, equations (22), (52), (58), and (A.5) to substitute for C_F and equation (50) to substitute for $\eta\dot{N}$. For substituting quantities of intermediate inputs, we use equations (12), (13), (15), and (A.5). So we get

$$A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}.N(t).e^{-\delta\tau}[1+(e^{\delta\tau}-1)\alpha^{\frac{\alpha}{1-\alpha}}]L^*$$

$$= \left\{\frac{(1-a)}{aP_I}\right\}^{\frac{1}{6-1}} B\gamma N(t)e^{-\delta\tau}[P_IB\beta]^{\frac{\beta}{1-\beta}}(1-L^*) + \eta\delta N(t) + N(t)e^{-\delta\tau}A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}L^*$$

$$+N(t)[1-e^{-\delta\tau}]A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}L^* + \gamma N(t).e^{-\delta\tau}[P_IB\beta]^{\frac{1}{1-\beta}}(1-L^*) \tag{A.9}$$

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