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Privatization and Licensing under Public Budget Constraint[*]

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Abstract

We analyse the interplay of privatization and technology licensing under a public budget constraint, where a cost-disadvantaged public firm has to generate profits to pay for the license. In a mixed duopoly, we consider the licensing of a cost-reducing technology by an outsider innovator. The innovator chooses to license smaller sizes of innovation to both firms, whereas, larger innovation is licensed exclusively to the private firm. The public firm alone never gets the license. Thus, the public firm can never "catch up" with its more efficient private rival. We find the possibility of both partial and full privatization in our model. Additionally, from a social planner's perspective, it is always optimal to allocate licenses to both firms.

Keywords: mixed duopoly; technology licensing; privatization; budget constraint; welfare.

JEL codes: L32, L33, H42, O33, O38

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Privatization and Licensing under Public Budget Constraint

1. Introduction

Privatization is an important aspect of public policy across the world. Greece, Italy, Ireland, and Spain have privatized many of their unprofitable public enterprises since 2010; in India, the national carrier Indian Airlines was privatized in 2021. In industrial organization literature, privatization is a policy tool where the government divests an optimal percentage of shares in a public firm to maximize social welfare (Fershtman, 1990; Matsumura, 1998).

The focus of the early privatization literature was on the consequences of privatization (or alternatively, nationalization) on social welfare (Merrill and Schneider, 1966; DeFraja and Delbono, 1989). The analysis then extended to include foreign firms whose presence affects the outcomes, as the foreign firms' surplus is not considered by the public firm for welfare maximization (Fjell and Pal, 1996; Matsumura, 2003a). A branch of literature also analysed endogenous moves by the firms to determine whether a public firm leadership emerges in equilibrium (Pal, 1998; Matsumura, 2003a do so in quantity setting; Ogawa and Kato, 2006; Dastidar and Sinha, 2011 do so under price competition).

In a mixed oligopoly setting where both public and private firms compete in the product market, a rationale for privatization arises from the public firm's relative inefficiency compared to a private firm (Megginson and Netter, 2001). Here, welfare increases as privatization shifts production from the inefficient public firm to the more efficient private firm, and it is maximized at an optimal degree of privatization. A natural question is whether an inefficient public firm can reduce its production costs by licensing a new technology from an innovator.[1]

A growing set of studies examines the interplay between privatization and licensing in asymmetric cost situations.[2] The paper by Mukherjee and Sinha (2014) is the first one to show that in a constant marginal cost context, if technology licensing eliminates the cost asymmetry between the private and public firms, there is no rationale for privatizing the public firm. However, they also argue that partial privatization may be necessary due to the presence of structural rigidities, agency costs, and barriers to technology upgradation – such as the cost of technology installation and personnel training. Our paper demonstrates that another barrier to technology transfer is the public firm's budget constraint when it has to compete with another private firm for acquiring a new technology from an outsider innovator.

A crucial question that is not addressed in literature is how the public firm's licensing activity is funded when its objective is to maximize welfare. Studies make an implicit or explicit assumption that a public firm can acquire a license with government funding raised via a lump-sum tax on the public. In reality, the government may make budgetary provisions for the entire industry rather

[1] The optimal degree of privatization depends critically on the production technology, where marginal costs can be increasing or constant. In the increasing marginal cost case, partial privatization can improve welfare even if the private and public firms are equally efficient (De Fraja and Delbono,1989; Matsumura, 1998). However, in the constant marginal cost case, a public monopoly emerges in equilibrium if there is no cost asymmetry (Matsumura, 2003a; Pal, 1998; Fujiwara, 2007).

[2] There exists a vast literature on optimal licensing in a purely private oligopoly, which studies both the outsider and insider innovator context. For instance, in case of an outsider innovator, fixed fee licensing is found to be better than per-unit royalty licensing (see Kamien and Tauman, 1986; Katz and Shapiro, 1986; Kamien et al., 1992), but in case of an insider (i.e., a competitor), per-unit royalty licensing is preferred over fixed-fee (Wang, 1998, 2002; Wang and Yang, 1999; Kamien and Tauman, 2002).

than an individual firm, and such budgetary support may be inadequate. We, therefore, depart from this assumption by explicitly considering the possibility of technology licensing wherein the public firm acquires the license only by paying from its profits generated through privatization.[3] Thus, our paper investigates privatization and licensing under public budget constrain in a market setting. This is indeed a novel feature of our model.

In our paper, we consider a licensing game in a mixed duopoly with an outsider innovator. In the first stage, the innovator chooses to license the new innovation to any one or to both firms depending on which option yields the highest payoff. In the next stage, given the licensing outcome, the public firm chooses its optimal degree of privatization to maximize welfare. In the final stage, the two firms compete in quantities and payoffs are realised.

Our main findings are based on the size of the innovation and on the size of the initial cost difference between the public and private firms. Under public budget constraint, the innovator optimally chooses to license smaller sizes of innovation to both firms, regardless of the initial cost asymmetry. On the contrary, for an intermediate range of initial cost asymmetry, larger innovation is licensed exclusively to the private firm. Under no circumstance does the innovator choose to license to the public firm alone. Thus, public budget constraint plays an important role in the innovator's licensing decision. Our results also imply that higher levels of innovation are not shared with the inefficient public firm, and thus its cost-difference with the private rival will aggravate. For smaller levels of innovation, both firms receive the technology, and as a result, their absolute cost difference is maintained. Therefore, it is obvious that market outcomes under certain

[3] A few papers have analysed privatization and budget issues (Wang *et al*, 2014; Niu, 2015) however, they do not consider licensing. Hence, they do not analyse the possibility of privatization that may be necessary to acquire a license for the public firm.

situations will not allow the public firm to "catch up" or even maintain the technology difference with its rival private firm.

Regarding the optimal degree of privatization in a market setting, we find that partial privatization is optimal for smaller initial cost difference. Full privatization is optimal for relatively large innovation, or larger initial cost difference.

We further investigate the problem of privatization and licensing in a social planner's setting where the social planner allocates the license/s to one or both firms. We find that it is always socially optimal to license to both firms simultaneously. Additionally, we also find that the optimal degree of privatization under the social planner's choice is always weakly less than the degree of privatization under the market determined outcomes.

A majority of studies that have handled privatization and licensing together in one framework, generally look at the two as disparate phenomena. While some papers study the impact of exogenous privatization on optimal licensing (see Gelves and Heywood, 2016), several others analyse endogenous privatization for exogenously given licensing regimes (see for example, Wang and Zeng, 2019; Wang *et al*., 2020). However, such an analysis of the two issues overlooks the fact that in many cases privatization is undertaken in order to access improved technology. Thus, in our model, we analyse the interdependence between the two by considering both optimal licensing and optimal privatization. In our paper, the public firm can use the profits it realises by privatizing to pay for a cost-reducing technology.

Our paper is closest to Chen *et al* (2014) and Wang and Zeng (2019). They both consider the case of an insider innovator in a mixed triopoly. Chen *et al* (2014) determine the optimal licensing outcomes when a private insider innovator faces a fully state owned firm and another private rival.

But they do not analyse privatization, or the effect of licensing on the optimal degree of privatization. On the other hand, Wang and Zeng (2019) consider exogenous licensing regimes, and analyse optimal privatization in their setting. Also, they consider the entry of a foreign firm, and its impact on optimal privatization. More specifically, Wang and Zeng (2019) consider exogenous licensing regimes with an insider innovator in a triopoly setting and show that licensing to the private (public) firm increases (decreases) the chances of privatization. Our paper differs from theirs in three crucial aspects. First, while they consider various exogenously given licensing outcomes with an insider innovator, we consider an explicit licensing game and arrive at the optimal licensing by an outsider innovator in a mixed duopoly. Second, in our paper, licensing is followed by privatization, unlike in their model where privatization occurs before licensing and as licensing is not optimally determined within their model, they do not consider the effect of privatization on licensing. Finally, in our model, a license can be acquired by the public firm only by using its own profits from privatization, but licensing to the public firm never occurs in equilibrium.

In an interesting study, Dadpay *et al.* (2022) consider an international mixed duopoly with a partially privatized domestic firm and a foreign private firm. In their model, they find that the optimal licensing contract varies with whether the innovator is a Cournot or Stackelberg competitor. However, in their paper, the degree of privatization is exogenous. Haraguchi and Matsumura (2020) and Cho et al (2022) both consider the case of *free* licensing followed by optimal privatization; therefore, in their models paying for the license fee is not a concern to the public firm, as is the case in ours.[4] Wang *et al* (2014) consider a public firm's budget constraint in the presence of efficiency improving cross-ownership. They find that full privatization is optimal whether or not the budget constraint is imposed on the public firm. But, they do not consider licensing of the technology, as we do in our paper. With the assumption of a convex cost structure, Wang *et al* (2020) study the optimal degree of privatization and its impact on licensing outcomes in an international mixed triopoly. In their model, they also focus on the case where the public firm is an innovator. However, in their paper, licensing to the public firm is funded by the government.

The rest of the paper is organized as follows. In Section 2 we set up a three stage game of licensing, privatization and quantity competition in a mixed duopoly model. We solve it in a market setting and characterize the players' subgame perfect equilibrium choices. In Section 3, we compare the market outcomes with the solution of the social planner's problem. Finally, we conclude our discussions and mention some future research possibilities in Section 4.

2. Model

We consider a mixed duopoly with a public firm and a profit maximizing private firm ($i = 0,1$ respectively). They compete *a la* Cournot with homogeneous product by choosing quantities (q_i). The inverse demand function is given by $p(Q) = a - Q$, where $Q = q_0 + q_1$. We assume that the two firms have constant, but asymmetric marginal costs. The private firm's marginal cost is c (with $c < a$), and the public firm's marginal cost is given by $c + t$, where $t > 0$ represents the

[4] Haraguchi and Matsumura (2020) find that privatization prompts a foreign innovator to voluntarily transfer technology to the domestic private firm, but their model is restricted to the case of a cost-free knowledge transfer to the private firm.

initial cost difference between private and public firm. Thus to begin with, the private firm is more efficient than the public firm.

There is an outsider innovator which does not produce the good or compete in the market (such as an R&D institution). It develops a technology (a common innovation) that can reduce a firm's cost of production by $\varepsilon > 0$.[5] Thus, if the public (alternatively, private) firm gets the technology, its marginal cost of production becomes $c + t - \varepsilon$ (alternatively, $c - \varepsilon$). Importantly, if both firms receive the technology, both their costs are reduced by an amount ε but their relative cost difference, t remains unchanged. For the rest of the paper, we define two new variables as $x = \frac{t}{a-c}$ and $y = \frac{\varepsilon}{a-c}$ where x relates to the initial cost asymmetry and y relates to the size of innovation, normalized by the factor $(a - c)$, which is the difference between the demand intercept and the marginal cost of the private firm. Note that $x, y > 0$ as $t, \varepsilon > 0$ and $a > c$. This change of variables allows us to present a general characterisation of results without much complexity of algebra. Further, in our model, we make the following assumptions to get a strictly positive degree of privatization under different regimes. These assumptions are also sufficient to ensure that the innovation size is non-drastic.[6]

Assumption 1: $0 < y < \frac{1-4x}{3}$ and

Assumption 2: $0 < y < x$.

We consider the following three stage game of technology licensing, privatization, and Cournot competition.

[5] See Sinha (2016) for the definition of common innovation as opposed to a new technology innovation.
[6] Non-drastic in our model requires that the output of firm i is positive when firm j is the only recipient of the new technology, $i \neq j$.

Stage 1: The licensing stage

The outsider innovator chooses between three alternative licensing options: to license to the public firm only ($L0$), to license to the private firm only ($L1$), or to license to both public and private firms ($L2$) by charging appropriate fixed fees. The licensing offer constitutes an offer of the technology in return for a fixed fee to the innovator. To implement these three licensing regimes and to extract the maximum fee from the licensee(s), the innovator can design three licensing game structures as follows. For implementing the public (private) licensing regime $L0$ ($L1$) the innovator designs the game $G0$ ($G1$), where it first makes an offer of licensing to the public (private) firm and then the public (private) firm decides whether to accept or reject the offer. If the offer is accepted then the licensing stage is over. If the offer is rejected, then the innovator makes an offer to the private (public) firm, which can then decide whether to accept or reject. The design of the single firm licensing games $G0$ and $G1$ is such that it uses the threat of the alternative licensing option that is available to the innovator, and thus extracts the maximum licensing fee from the public firm in $G0$ and from the private firm in $G1$. The extensive forms of the two games are depicted in Figure 1.

To implement the regime of licensing to both firms ($L2$), the innovator makes a simultaneous offer to both firms, represented by the game $G2$, where it specifies two different fees, T_0 and T_1 from the public and private firm respectively. The two firms then simultaneously choose whether to accept or reject the offer.[7] We assume that the innovator chooses between these three alternative licensing games depending on its payoff and in case it is indifferent between licensing to both and

[7] Note that in order to keep the analysis simple, we allow for discriminatory licensing fees for the two firms. An alternative approach would be to charge a uniform fee to the firms such that both of them find it acceptable. In that case, the innovator can charge the minimum of the two firms' willingness to pay as the licensing fee. This alternative licensing scheme would reduce the attractiveness of offering two licenses for the innovator. But, such a pricing structure will complicate the algebra without adding much value to the paper.

licensing to any other single firm, it chooses to license to both firms. Any firm will accept the licensing offer when its payoff from acceptance is weakly greater than rejection. This is true for the public firm as well, as it has to pay from the profit it generates at the third stage of the game described below.

Thus, the games $G0$, $G1$ and $G2$ are designed to implement the licensing outcomes of $L0$, $L1$ and $L2$ respectively.

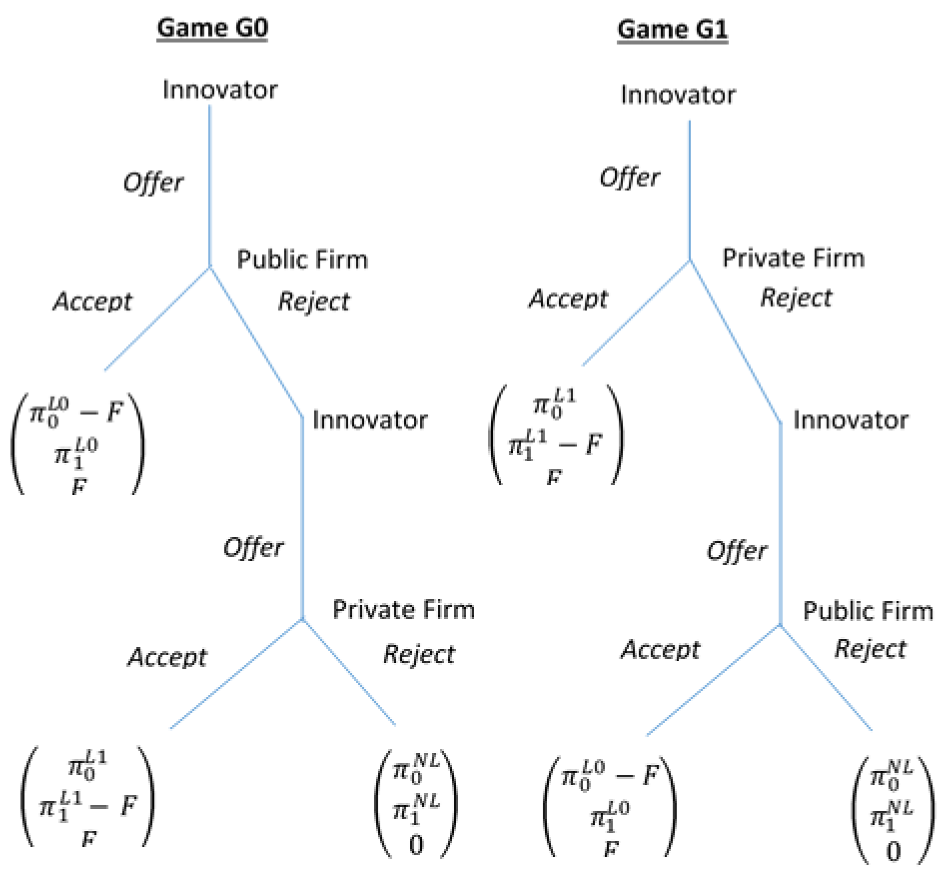

Figure 1. Licensing Games $G0$ and $G1$.

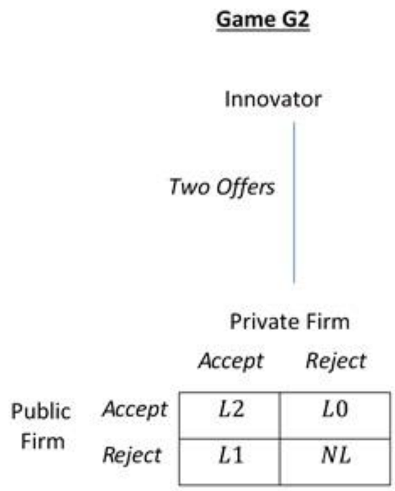

Figure 2. Licensing Game $G2$.

The Stage 1 licensing game for a single license ($G0$ or $G1$) is depicted in Figure 1. The Stage 1 game for licensing to both firms ($G2$) is depicted in Figure 2, where both firms simultaneously decide whether to accept or reject their respective licensing offers and fees.

Stage 2: The privatization stage

After the licensing regime is fixed in Stage 1, the public firm chooses a degree of privatization (α^{Li}) to maximize social welfare depending on the licensing regime, Li where $i = 0{,}1$ or 2.

Stage 3: The competition stage

Finally, in Stage 3 both firms engage in Cournot competition. The public firm maximizes a linear combination of social welfare and its own profit, weighted by its degree of privatization. The private firm maximizes its own profit. The profits are realized and the licensing fee(s) will be settled from the profit of the firms as per their Stage 1 agreement.[8]

We solve the above game through backward induction. Note that in the second stage, the optimal privatization policy is dependent on the specific regime of licensing chosen by the innovator in the first stage. So, in the first stage, the innovator chooses the game optimally given its knowledge about the implication of its choice on the optimal degree of privatization and subsequent competition, leading to payoffs for all concerned.

2.1.Benchmark no licensing game

We first consider the case when there is no licensing (depicted by a superscript NL). Suppose the degree of privatization is α in the second stage of the game. Then in the third stage, the public firm's profit is $\pi_0^{NL} = (a - q_0 - q_1)q_0 - (c + t)q_0$, which can be written as $\pi_0^{NL} = \big((a - c)(1 - x) - q_0 - q_1\big)q_0$. The private firm's profit is $\pi_1^{NL} = (a - q_0 - q_1)q_1 - (c)q_1$, and the welfare, W^{NL} is the overall welfare, consisting of the two firms' profits and consumer surplus. That is, $W^{NL} = \pi_0^{NL} + \pi_1^{NL} + \frac{(q_0^{NL}+q_1^{NL})^2}{2}$. Thus, the public firm maximizes its objective function, which is a weighted average of its own profits and social welfare, given by

$$\Omega_0^{NL} = \alpha\pi_0^{NL} + (1 - \alpha)W^{NL}$$

The public firm chooses q_0 to maximize its objective function Ω_0^{NL}, and the private firm chooses q_1 to maximize its profit π_1^{NL}. The first order conditions for the competition stage are $\frac{\partial\Omega_0^{NL}}{\partial q_0} = (a - c)(1 - x) - q_0(1 + \alpha) - q_1 = 0$ and $\frac{\partial\pi_1}{\partial q_1} = a - c - q_0 - 2q_1 = 0$. Thus, the reaction

[8] We assume no breach of contract of payments or renegotiation of the licensing contract.

functions are $q_0^*(q_1) = \frac{(a-c)(1-x)-\mathrm{q}_1}{1+\alpha}$ and $q_1^*(q_0) = \frac{a-c-q_0}{2}$. Solving the two reaction functions simultaneously, we get the following two Cournot quantities and profits.[9]

$$q_0^{NL} = \frac{(a-c)(1-2x)}{1+2\alpha} \text{ and } q_1^{NL} = \frac{(a-c)(x+\alpha)}{1+2\alpha} \tag{1}$$

$$\pi_0^{NL} = \frac{(a-c)^2(1-2x)^2\alpha}{(1+2\alpha)^2} \text{ and } \pi_1^{NL} = \frac{(a-c)^2(x+\alpha)^2}{(1+2\alpha)^2} \tag{2}$$

Now in the privatization stage, the public firm maximizes the overall social welfare, given from (1) and (2) as $W^{\mathrm{NL}} = \frac{(a-c)^2(1-2x)^2\alpha}{(1+2\alpha)^2} + \frac{(a-c)^2(x+\alpha)^2}{(1+2\alpha)^2} + \frac{\left(\frac{(a-c)(1-2x)}{1+2\alpha}+\frac{(a-c)(x+\alpha)}{1+2\alpha}\right)^2}{2}$, with respect to its degree of privatization:

$$\frac{\partial W^{NL}}{\partial \alpha} = -\frac{(a-c)^2(1-2x)(\alpha - x(4\alpha+1))}{(1+2\alpha)^3}$$

From Assumption 1 and due to the natural restriction on α as $0 \leq \alpha \leq 1$, the optimal degree of privatization in case of no licensing would be

$$\alpha^{NL} = \begin{cases} \dfrac{x}{1-4x} & \text{if } 5x < 1 \\ 1 & \text{if } 1 \leq 5x \end{cases}$$

Thus, even if there is no licensing, there exists a rationale for privatization due to the public firm's relative cost inefficiency, as shown by other studies (Matsumura, 2003b; Mukherjee and Suetrong

[9] Note that $q_0^{\mathrm{NL}} > 0$ by Assumption 1.

2009; and Wang and Zeng, 2019). The cost asymmetry (represented by x) determines whether partial or full privatization is optimal under no licensing.

2.2.Licensing

We now move on to analyse the three stage game with technology licensing where an outsider innovator chooses to license its cost-reducing technology to one or both firms. Here, we introduce the public firm's budget constraint, wherein it has to pay for a license from its own profits in the absence of any government budgetary support. Note that due to this constraint, the public firm will not be able to pay for any license unless it privatizes to some extent and generates a profit. We solve the game by standard backward induction and start off with the last stage.

2.2.1. Stage 3: Competition stage

Depending on the licensing game chosen in Stage 1, the licensee's cost reduces by an amount $\varepsilon = y(a-c)$, and since it pays a fixed fee (F) for this new technology, its third stage optimal quantities are not affected by the payment of licensing fee. With this, we calculate the Stage 3 outcomes under different licensing possibilities.

Case 1: Licensing to public firm alone (L0)

Suppose the public firm is the sole licensee $(L0)$, and the degree of privatization is α. The public firm's cost now becomes $c+t-\varepsilon = c+(a-c)(x-y)$, whereas it stays c for the private firm. Then in Stage 3, the public firm maximizes $\Omega_0^{L0} = \alpha\big((a-c)(1-x+y)-q_0-q_1\big)q_0 + (1-\alpha)\left(\big((a-c)(1-x+y)-q_0-q_1\big)q_0 + (a-c-q_0-q_1)q_1 + \frac{(q_0+q_1)^2}{2}\right) - F$, and the private firm maximizes $\pi_1^{L0} = (a - c - q_0 - q_1)q_1$. The equilibrium outputs and profits are given below.

$$q_0^{L0} = \frac{(a-c)(1-2x+2y)}{1+2\alpha} \text{ and } q_1^{L0} = \frac{(a-c)(x-y+\alpha)}{1+2\alpha} \tag{3}$$

$$\pi_0^{L0} = \frac{(a-c)^2(1-2x+2y)^2\alpha}{(1+2\alpha)^2} \text{ and } \pi_1^{L0} = \frac{(a-c)^2(x-y+\alpha)^2}{(1+2\alpha)^2} \tag{4}$$

Case 2: Licensing to the private firm alone (L1)

If the private firm is the sole licensee, its cost becomes $c - \varepsilon = c - (a-c)y$, whereas the public firm's cost remains $c - t = c + (a-c)x$. Based on the choice of α in the second stage, the public and private firms maximize $\Omega_0^{L1} = \alpha\big((a-c)(1-x) - q_0 - q_1\big)q_0 + (1-\alpha)\left(\big((a-c)(1-x) - q_0 - q_1\big)q_0 + \big((a-c)(1+y) - q_0 - q_1\big)q_1 + \frac{(q_0+q_1)^2}{2}\right)$ and $\pi_1^{L1} = \big((a-c)(1+y) - q_0 - q_1\big)q_1 - F$ respectively, to give the following equilibrium outcomes. [10]

$$q_0^{L1} = \frac{(a-c)(1-2x-y)}{1+2\alpha} \text{ and } q_1^{L1} = \frac{(a-c)\big(x+y+\alpha(1+y)\big)}{1+2\alpha} \tag{5}$$

$$\pi_0^{L1} = \frac{(a-c)^2(1-2x-y)^2\alpha}{(1+2\alpha)^2} \text{ and } \pi_1^{L1} = \frac{(a-c)^2\big(x+y+\alpha(1+y)\big)^2}{(1+2\alpha)^2} \tag{6}$$

Case 3: Licensing to both firms (L2)

[10] Note that by Assumption 1, q_0^{L0} and q_0^{L1} are positive and by Assumption 2, $q_1^{L0} > 0$.

Finally, if in Stage 1 the innovator chooses to license to both firms ($L2$), the two firms maximize their respective objective functions with the public firm's cost being $c + t - \varepsilon = c + (a - c)(x - y)$ and the private firm's cost given by $c - \varepsilon = c - (a - c)y$. The equilibrium outputs are:

$$q_0^{L2} = \frac{(a-c)(1-2x+y)}{1+2\alpha} \text{ and } q_1^{L2} = \frac{(a-c)\big(x+\alpha(1+y)\big)}{1+2\alpha} \tag{7}$$

$$\pi_0^{L2} = \frac{(a-c)^2(1-2x+y)^2\alpha}{(1+2\alpha)^2} \text{ and } \pi_1^{L2} = \frac{(a-c)^2\big(x+\alpha(1+y)\big)^2}{(1+2\alpha)^2} \tag{8}$$

With these Stage 3 equilibrium outcomes, we can now move to Stage 2, where the public firm chooses the optimal degree of privatization based on the licensing regime chosen in Stage 1.

2.2.2. Stage 2: Optimal privatization

In Stage 2, the public firm chooses an optimal degree of privatization to maximize social welfare, which is the sum of the producer and consumer surplus. As the licensing fee is simply a lump-sum transfer from the licensee/s to the innovator (who, we consider to be a part of the domestic market), it does not affect the overall welfare calculations.[11]

If in Stage 1, the innovator chooses to license to the public firm alone ($L0$), the optimal degree of privatization is derived by maximizing the following welfare expression, which is a sum of the producer surplus of both firms given in (5), and the consumer surplus given by $\frac{\left(q_0^{L0}+q_1^{L0}\right)^2}{2}$, using quantities presented in equation (4).

$$W^{L0} = \frac{(a-c)^2(1-2x+2y)^2\alpha}{(1+2\alpha)^2} + \frac{(a-c)^2(x-y+\alpha)^2}{(1+2\alpha)^2} + \frac{(a-c)^2(1-x+y+\alpha)^2}{2(1+2\alpha)^2} \tag{9}$$

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[11] However, if the innovator is a foreign entity, then the licensing fees do matter for domestic welfare, as shown by Kim *et al* (2018), Niu (2015), and Wang *et al* (2020), among others.

Differentiating welfare with respect to α, we get $\frac{\partial W^{\mathrm{L0}}}{\partial \alpha} = \frac{(a-c)^2(1-2x+2y)\left(x-y-\alpha(1-4x+4y)\right)}{(1+2\alpha)^3}$. By solving for α we get the optimal α^{L0} as follows:

$$\alpha^{L0} = \begin{cases} \dfrac{x-y}{1-4x+4y} & \text{if } 5x-5y<1 \\ 1 & \text{if } 5x-5y \geq 1 \end{cases} \tag{10}$$

Here we invoke our two important assumptions to ensure that the degrees of privatization for different licensing regimes are always positive. To elaborate on α^{L0}, we note that the slope of W^{L0} evaluated at $\alpha = 0$ is positive, that is, $\left.\frac{\partial W^{L0}}{\partial \alpha}\right|_{\alpha=0} = (a-c)^2(1-2x+2y)(x-y) > 0$ (by Assumption 2). Further, the slope of W^{L0} evaluated at $\alpha = 1$, $\left.\frac{\partial W^{L0}}{\partial \alpha}\right|_{\alpha=1} = -\frac{1}{27}(a-c)^2(1-5x+5y)(1-2x+2y)$, is negative (positive) when $(5x-5y) < (\geq)\ 1$, implying an interior solution (alternatively, full privatization as a solution) to the maximization problem.[12]

If in Stage 1, the private firm is the sole licensee, then in Stage 2, the optimal degree of privatization is determined by maximizing the appropriate welfare function.

$$W^{L1} = \frac{(a-c)^2(1-2x-y)^2\alpha}{(1+2\alpha)^2} + \frac{(a-c)^2\left(x+y+\alpha(1+y)\right)^2}{(1+2\alpha)^2} + \frac{(a-c)^2(1-x+\alpha+y\alpha)^2}{2(1+2\alpha)^2} \tag{11}$$

Differentiating, we get $\frac{\partial W^{\mathrm{L1}}}{\partial \alpha} = \frac{(a-c)^2(1-2x-y)(x+y-(1-4x-3y)\alpha)}{(1+2\alpha)^3}$. Equating it to zero, we get the optimal degree of privatization as below. Note that from Assumption 1, $\alpha^{L1} > 0$.

$$\alpha^{L1} = \begin{cases} \dfrac{x+y}{1-4x-3y} & \text{if } 5x+4y<1 \\ 1 & \text{if } 5x+4y \geq 1 \end{cases} \tag{12}$$

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[12] The second order conditions for partial privatization in all regimes are satisfied under Assumptions 1 and 2.

Finally, if both firms receive the license in Stage 1, the welfare is given by

$$W^{L2} = \frac{(a-c)^2(1-2x+y)^2\alpha}{(1+2\alpha)^2} + \frac{(a-c)^2(x+\alpha+y\alpha)^2}{(1+2\alpha)^2} + \frac{(a-c)^2(1-x+y+\alpha+y\alpha)^2}{2(1+2\alpha)^2} \qquad (13)$$

The differentiation of W^{L2} with respect to α is $\frac{\partial W^{L2}}{\partial \alpha} = \frac{(a-c)^2(1-2x+y)(x+4x\alpha-(1+y)\alpha)}{(1+2\alpha)^3}$, giving the optimal degree of privatization as below.

$$\alpha^{L2} = \begin{cases} \dfrac{x}{1-4x+y} & \text{if } 5x-y<1 \\ 1 & \text{if } 5x-y \geq 1 \end{cases} \qquad (14)$$

Equations (10), (12), and (14) together give us the optimal degree of privatization under different regimes, along with the conditions for when partial or full privatization is optimal, as presented in detail in Table 1, and illustrated in Figure 3.

Region	**Parameter Space**	$\boldsymbol{\alpha^{L0}}$	$\boldsymbol{\alpha^{L1}}$	$\boldsymbol{\alpha^{L2}}$
A	$5x+4y<1$	$\frac{x-y}{1-4x+4y}$	$\frac{x+y}{1-4x-3y}$	$\frac{x}{1-4x+y}$
B	$5x+4y \geq 1$ $5x-y<1$	$\frac{x-y}{1-4x+4y}$	1	$\frac{x}{1-4x+y}$
C	$5x-y \geq 1$ $5x-5y<1$	$\frac{x-y}{1-4x+4y}$	1	1
D	$5x-5y \geq 1$	1	1	1

Table 1. Privatization under three licensing regimes for different parameter restrictions.

(N.B. The common parameter restrictions of Assumptions 1 and 2 apply to all regions).

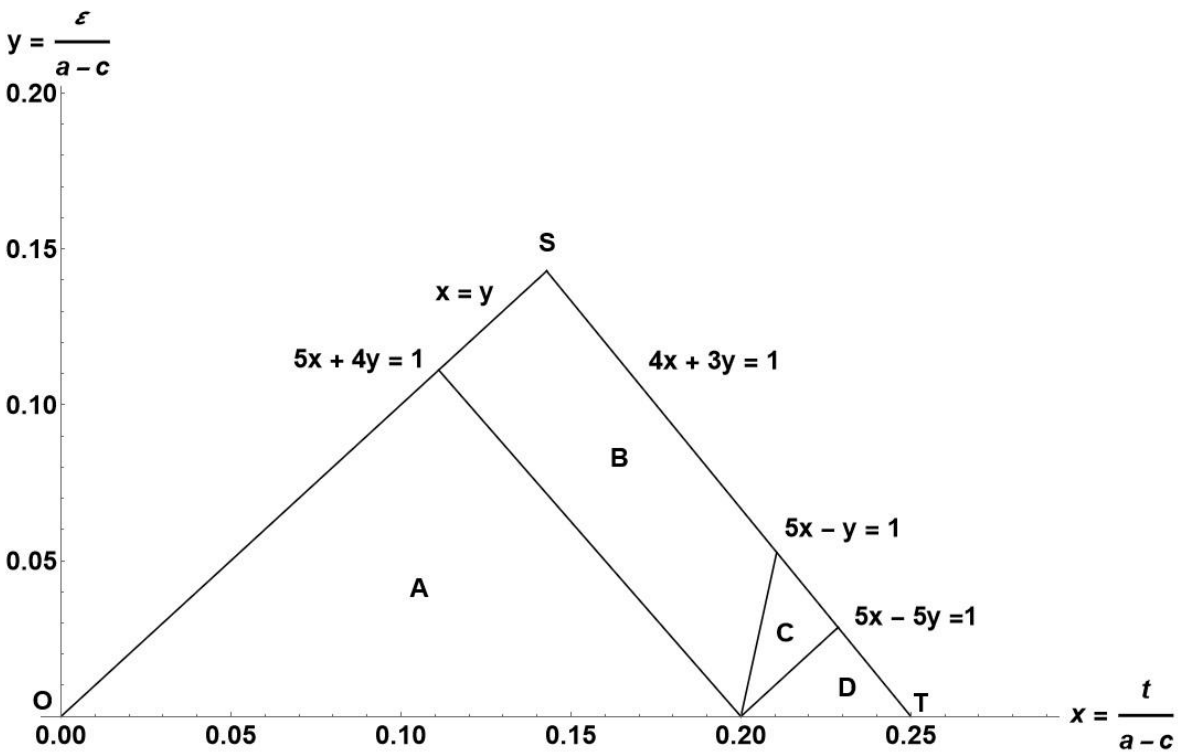

Figure 3. Feasible set of parameters for partial or full privatization.

With the help of the conditions derived above, the feasible set of parameters for positive privatization are represented by the triangle OST in Figure 3. Within OST, we have Region A, where the optimal degree of privatization is a fraction for all three licensing regimes. Region B is where partial privatization is optimal in case of $L0$ and $L2$, but full privatization is optimal in case of $L1$. Further, Region C is where partial (full) privatization is optimal in case of $L0$ ($L1$, $L2$); and finally, D represents the region where full privatization is optimal under all licensing regimes.

Comparing the optimal degree of privatization for various licensing outcomes given in (10), (12), and (14), we can see that $\alpha^{L0} \leq \alpha^{L2} \leq \alpha^{L1}$, that is $\frac{x-y}{1-4x+4y} \leq \frac{x}{1-4x+y} \leq \frac{x+y}{1-4x-3y}$. There is a strict ordering when all three degrees of privatization are a fraction, and even if some of the values reach the corner solution of full privatization, the above ordering is not violated. We can thus, state the following lemma.

Lemma 1. *In a mixed duopoly with an outsider innovator, licensing to the public firm leads to the lowest degree of privatization under partial privatization, while licensing to the private firm leads to the highest degree of optimal privatization and the degree of privatization is intermediate for the case of licensing to both firms.*

The result is similar to that of Wang and Zeng (2019) in their insider innovator model. The degree of cost asymmetry under different licensing regimes dictates this ordering of privatization degrees. When the private firm is the sole licensee, the cost asymmetry between the two firms is highest. Welfare increases as production shifts from the inefficient public frim to the more efficient private firm, thus prompting a higher degree of optimal privatization (or a higher likelihood of full privatization). On the other hand, if the public firm is the sole licensee, it is relatively less inefficient, and welfare increases if the public firm produces more. This reduces the motivation for privatization (consequently, the likelihood of full privatization is also lower).

The above analysis gives us each firm's exact willingness to pay for each licensing regime, given initial cost difference, size of the innovation, and the corresponding optimal degree of privatization. We can now move on to Stage 1, where the innovator chooses its most profitable licensing option.

2.2.3. Stage 1: Optimal licensing

In Stage 1, the innovator chooses between games $G0$, $G1$ and $G2$ based on which alternative gives it the highest licensing revenue. Recall that the innovator has perfect foresight and therefore, it implements its desired outcome subject to the recipient's participation constraints.

The public firm cares about social welfare, and will accept a licensing offer only if welfare when it accepts is weakly greater than welfare when it rejects. The private firm, on the other hand, is a profit maximizer, and accepts the licensing offer if its profit is weakly greater than when it rejects. With this in mind, the innovator makes an appropriate licensing offer that gets accepted. We now calculate the innovator's payoff for each possible licensing game.

Analysis of game $G0$

First consider the game where the innovator approaches the public firm first, i.e., $G0$. We proceed to analyse the game backwards. In the last node of the extensive form game the private firm will accept a licensing offer if $\pi_1^{L1}(\alpha^{L1}) - F \geq \pi_1^{NL}(\alpha^{NL})$. Therefore in this case, it can pay $F = \pi_1^{L1}(\alpha^{L1}) - \pi_1^{NL}(\alpha^{NL})$, at best. The innovator would charge that maximum fee, and the outcome would be $L1$. On the other hand, in the previous stage, the public firm will accept the licensing offer if $W^{L0}(\alpha^{L0}) \geq W^{L1}(\alpha^{L1})$. If this condition holds, then it is willing to pay its entire profit, $\pi_0^{L0}(\alpha^{L0})$, and thus the innovator gets the maximum payoff of $RG0 = \pi_0^{L0}(\alpha^{L0})$ by choosing the game G0.

Analysis of Game $G1$

Next, we solve the game $G1$ by backward induction. Here, the public firm accepts the license if $W^{L0}(\alpha^{L0}) \geq W^{NL}(\alpha^{NL})$. In $G1$ as well, it is willing to pay its entire profit $\pi_0^{L0}(\alpha^{L0})$ for the license. Since licensing improves welfare over no licensing, the public firm will accept an offer in this circumstance. In the previous stage of the game, the private firm is aware that upon rejection, the resulting outcome would be $L0$, and thus it is now willing to accept an offer if $\pi_1^{L1}(\alpha^{L1}) - F \geq \pi_1^{L0}(\alpha^{L0})$. The innovator would thus make a licensing offer by charging $F = \pi_1^{L1}(\alpha^{L1}) - \pi_1^{L0}(\alpha^{L0})$. Given the game structure, the maximum revenue the innovator can earn from the choice of $G1$ is $RG1 = \pi_1^{L1}(\alpha^{L1}) - \pi_1^{L0}(\alpha^{L0})$.

Analysis of Game $G2$

Finally, in $G2$, the innovator makes a simultaneous offer to both firms by charging two different fees T_0 and T_1. Both firms simultaneously decide whether to accept or reject the offer under the assumption that the other firm accepts the offer. Thus, the public firm will accept an offer if $W^{L2}(\alpha^{L2}) \geq W^{L1}(\alpha^{L1})$. It can pay its entire profit $\pi_0^{L2}(\alpha^{L2})$ for the license, so the innovator charges $T_0 = \pi_0^{L2}(\alpha^{L2})$ to the public firm. The private firm will accept an offer if $\pi_1^{L2}(\alpha^{L2}) - T_1 \geq \pi_1^{L0}(\alpha^{L0})$. So, the innovator charges $T_1 = \pi_1^{L2}(\alpha^{L2}) - \pi_1^{L0}(\alpha^{L0})$. The innovator's total revenue from the choice of game $G2$ is the sum of the two firms' willingness to pay. Thus, $RG2 = \pi_0^{L2}(\alpha^{L2}) + \left(\pi_1^{L2}(\alpha^{L2}) - \pi_1^{L0}(\alpha^{L0})\right)$.

The innovator chooses one among the three games depending on the payoffs $RG0$, $RG1$, and $RG2$. Note that the licensing revenue for each regime is calculated based on the acceptance of the offer by the intended licensee.[13] We now examine the innovator's optimal licensing choice based on the various parametric zones described in Table 1.

Analysis of Region A:

In Region A, the optimal degree of privatization is a fraction for all possible outcomes: NL, $L0$, $L1$, and $L2$. Irrespective of whether it is approached first (as in $G0$) or not (as in $G1$), the public firm is willing to pay its entire profits (from (4)).

$$\pi_0^{L0}(\alpha^{L0}) = (a-c)^2(1-4x+4y)(x-y)$$

[13] Suppose under some parameter values, the innovator foresees a rejection by the public firm in game $G0$ due to its welfare consideration (i.e., if $W^{L0} < W^{L1}$) then the innovator will not choose $G0$, but will instead choose one of the other two games depending on its payoff.

On the other hand, if the private firm is approached first ($G1$), it is willing to pay the following (from (6) and (4)).

$$\pi_1^{L1}(\alpha^{L1}) - \pi_1^{L0}(\alpha^{L0}) = 4(a-c)^2(x+y)^2 - 4(a-c)^2(x-y)^2 = 16(a-c)^2xy$$

If it is approached after the public firm has rejected a licensing offer (in $G0$), then it is willing to pay (from (6) and (2)),

$$\pi_1^{L1}(\alpha^{L1}) - \pi_1^{NL}(\alpha^{NL}) = 4(a-c)^2(x+y)^2 - 4(a-c)^2x^2 = 4(a-c)^2y(2x+y)$$

From the above calculations we note that the private firm's willingness to pay when it is approached first (in $G1$) is always higher than if it is approached second ($G0$); that is, $\pi_1^{L1} - \pi_1^{L0} > \pi_1^{L1} - \pi_1^{NL}$. From (4) and (2) we can see that this is because $\pi_1^{L0} < \pi_1^{NL}$, as $\frac{(a-c)^2(x-y+\alpha)^2}{(1+2\alpha)^2} < \frac{(a-c)^2(x+\alpha)^2}{(1+2\alpha)^2}$ due to $y > 0$, from Assumption 1. Therefore, the innovator has no incentive to implement the $L1$ licensing through $G0$ and vice versa. So, the innovator's payoff from $G0$ is

$$RG0^A = \pi_0^{L0}(\alpha^{L0}) = (a-c)^2(1-4x+4y)(x-y) \tag{15}$$

and from $G1$ is

$$RG1^A = \pi_1^{L1}(\alpha^{L1}) - \pi_1^{L0}(\alpha^{L0}) = 16(a-c)^2xy \tag{16}$$

If the two firms are approached simultaneously ($G2$), the public firm is willing to pay a fee of $\pi_0^{L2}(\alpha^{L2}) = \frac{(a-c)^2(1-2x+y)^2\alpha}{(1+2\alpha)^2} = (a-c)^2x(1-4x+y)$ provided $W^{L2}(\alpha^{L2}) \geq W^{L1}(\alpha^{L1})$; and the private firm is willing to pay $\pi_1^{L2}(\alpha^{L2}) - \pi_1^{L0}(\alpha^{L0}) = 4(a-c)^2x^2 - 4(a-c)^2(x-y)^2 = 4(a-c)^2(2x-y)y$. If both firms accept, the innovator gets a total revenue of $(a-c)^2x(1-4x+y) + 4(a-c)^2(2x-y)y$ which can be written as

$$RG2^A \ = (a-c)^2(x+9xy-4x^2-4y^2) \tag{17}$$

Comparing (15) and (17) we have $RG2^A - RG0^A = (a-c)^2(1+x)y > 0$, therefore, $RG0$ is never chosen by the innovator. Further, from (16) and (17) we have $RG1^A - RG2^A = (a-c)^2(4x^2+4y^2-x+7xy)$, which can be positive or negative. So, the innovator chooses $G1$ if $4x^2+4y^2-x+7xy > 0$, else it chooses $G2$. In case of $G2$, the public firm's acceptance is conditional on welfare improvement. $W^{L2}(\alpha^{L2}) - W^{L1}(\alpha^{L1}) = (a-c)^2 y(1-4x-y)$, which is positive from Assumption 1. Hence, the public firm accepts the licensing offer in case Game G2 is chosen.

Thus, the innovator chooses the game $G1$ (alternatively, $G2$) leading to the outcome of licensing to the private firm alone, $L1$ (or to both firms, $L2$). These corresponding regions of licensing to one or both firms are depicted in Figure 4 as regions $A1$ and $A2$ respectively.

Analysis of Region B

In Region B, privatization is partial under $L0$ and $L2$, but full under $L1$. We see that in this region, $RG0^B = \ (a-c)^2(1-4x+4y)(x-y)$, which is the same as in Region A because the public firm can pay the same π_0^{L0} by partially privatizing up to α^{L0} under $L0$. However, the innovator's payoff under $G1$ is $RG1^B = \frac{(a-c)^2\left((1+x+2y)^2-36(x-y)^2\right)}{9}$, which is different from the payoff from game $G1$ in Region A. This is because the degree of privatization under $L1$ is different for Regions A and B, consequently bringing about a change in the private firm's willingness to pay for a license. Finally, if the innovator offers the license to both firms, and they accept, its payoff is $RG2^B = (a-c)^2(x+9xy-4x^2-4y^2)$, the same as Region A. Comparing between the three possible payoffs, it is easy to see that $RG2^B$ is always greater than $RG0^B$, as $RG2^B - RG0^B =$

$(a-c)^2(1+x)y > 0$. On the other hand, $RG2^B - RG1^B = \frac{(a-c)^2(7x+5xy-x^2-(1+2y)^2)}{9}$, which is negative (positive) if $y \geq (<) \frac{(3\sqrt{8x+x^2}+5x-4)}{8}$.[14] Therefore, the innovator chooses $L1$ or $L2$ accordingly, which is respectively represented as $B1$ and $B2$ in Figure 4.

Analysis of Region C

In Region C, privatization is partial under $L0$, but full under $L1$ and $L2$. Here, $RG0^C = (a-c)^2(1-4x+4y)(x-y)$, as in Regions A and B; and $RG1^C = \frac{(a-c)^2((1+x+2y)^2-36(x-y)^2)}{9}$ as in Region B. But, $RG2^C = \frac{(a-c)^2(1+x+y)^2}{9} - 4(a-c)^2(x-y)^2 + \frac{(a-c)^2(1-2x+y)^2}{9}$.

Comparing the payoff from $G2$ and $G0$, we have $RG2^C - RG0^C = \frac{(2+5x^2+13y+2y^2-x(11+2y))}{9}$. To show that this value is positive, we evaluate the expression at all three vertices of the triangular region C, and show that it is positive everywhere. The vertex formed by the intersection of $4x+3y=1$ and $5x-5y=1$ is $(x,y) = \left(\frac{8}{35}, \frac{1}{35}\right)$. At this point, $RG2^C - RG0^C = \frac{(a-c)^2 131}{9(1225)}$, which is positive. Next, the intersection of $4x+3y=1$ and $5x-y=1$ gives us the point $(x,y) = \left(\frac{4}{19}, \frac{1}{19}\right)$. Here, $RG2^C - RG0^C = \frac{(a-c)^2 23}{361}$, which is also positive. Finally, the third vertex is $(x,y) = \left(\frac{1}{5}, 0\right)$. Here, $RG2^C - RG0^C = 0$. Since Region C is a convex combination of the three vertices, the result, $RG2^C \geq RG0^C$ holds for the entire region. So, the innovator does not choose $G0$.

[14] Given the complicated expressions, we use Mathematica software for our analysis.

Next, the difference between the payoff from $G2$ and $G1$ is $RG2^C - RG1^C = \frac{(a-c)^2(1-4x-6y+4x^2-2y^2)}{9}$. At the point $(x,y) = \left(\frac{8}{35}, \frac{1}{35}\right)$, $RG2^C - RG1^C = \frac{(a-c)^2 311}{9(1225)}$, which is greater than zero. At the second vertex, $(x,y) = \left(\frac{4}{19}, \frac{1}{19}\right)$, $RG2^C - RG1^C = \frac{(a-c)^2 5}{9(19)}$, which is also greater than zero. Finally, at $(x,y) = \left(\frac{1}{5}, 0\right)$, $RG2^C - RG1^C = \frac{(a-c)^2}{25}$ which is positive. Since Region C is a convex combination of the three vertices, $RG2^C > RG1^C$ for the entire region. So, the innovator does not choose $G1$ either. The innovator thus, prefers to offer two licenses and chooses $G2$ in Region C, represented by $C2$ in Figure 4.

Analysis of Region D

Next, we examine Region D where the optimal degree of privatization is 1 (i.e., full privatization) under all possible outcomes. With similar analysis as above, the innovator's payoff under the different alternatives are: $RG0^D = \frac{(a-c)^2(1-2x+2y)^2}{9}$, $RG1^D = \frac{(a-c)^2 y(2+2x+y)}{3}$, and $RG2^D = \frac{(a-c)^2(1-4x+4x^2+6y+y^2)}{9}$. In this region, $RG2^D - RG0^D = \frac{(a-c)^2 y(2+8x-3y)}{9}$, which is positive from Assumption 1. Next, in Region D, $RG2^D - RG1^D = \frac{(a-c)^2(1-4x-6y+4x^2-2y^2)}{9}$ as in Region C. We employ the same analysis as before where we evaluate the difference for each vertex of the triangular region D and show that the difference is positive. It shares two vertices with Region C, namely $(x,y) = \left(\frac{8}{35}, \frac{1}{35}\right)$, and $(x,y) = \left(\frac{1}{5}, 0\right)$, where the difference is already shown to be positive. The third vertex is $(x,y) = \left(\frac{1}{4}, 0\right)$, where the difference is $\frac{(a-c)^2}{36}$, which is clearly positive. Since Region D is the convex combination of the three vertices, it holds that $RG2^D > RG1^D$ for the entire region. Thus, the innovator always prefers to offer two licenses simultaneously, depicted as $D2$ in Figure 4.

Thus, we summarise the licensing outcome of our model in the following proposition.

Proposition 1. *The innovator's optimal licensing strategy is to choose*

(i) *simultaneous licensing to both firms in regions $A2$, $B2$, $C2$, and $D2$.*

(ii) *licensing to the private firm alone in regions $A1$ and $B1$.*

The optimal licensing outcome can be visualized in the parameter space with the help of Figure 4.

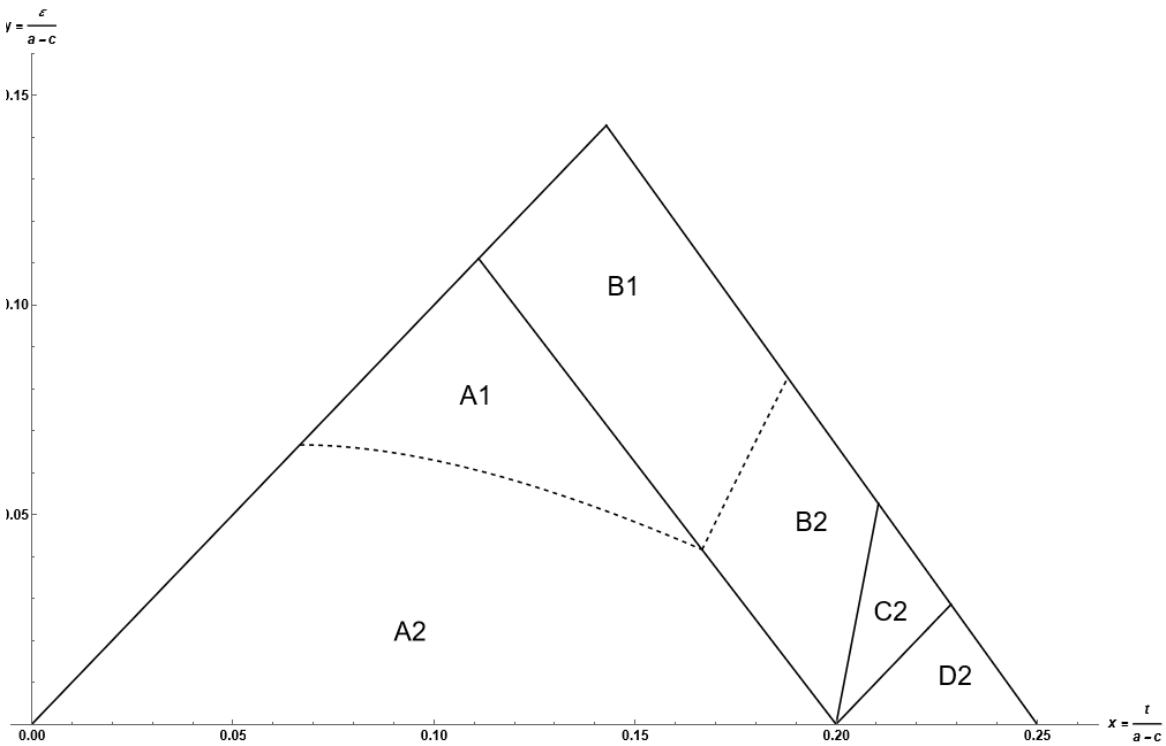

Figure 4. Innovator's optimal choice of outcome.

From Figure 4, we can clearly see that the higher levels of innovation (y) are offered to the private firm alone (in regions $A1$ and $B1$). Licensing is offered to both firms for relatively lower levels of innovation (in regions $A2$, $B2$, $C2$ and $D2$). Another important observation is that for an intermediate range of innovation (y), it is possible to see that the innovator offers two licenses both for higher and lower degree of cost asymmetry (x values) but for intermediate levels of cost asymmetry the innovator's choice is to offer licensing to the private firm only. This can be seen by drawing a horizontal line at a level cutting across the parameter space such that $A2$ and $B2$ are the outcomes for lower as well as higher degrees of cost asymmetry (x values) respectively, but for the intermediate range of x values, the innovator chooses licensing to private firm alone (in regions $A1$ and $B1$). This non-monotonicity of innovator's preference with respect to the level of cost asymmetry is a result of the interaction of the degree of privatization for some given intermediate sizes of innovation, which is an interesting feature of our model.

Moreover, licensing to the public firm alone is never an outcome. Proposition 1, thus, has important policy implications. In a market setting without government intervention, the public firm can never catch up or bridge its technology gap with the efficient private firm by licensing a new technology from a private outsider innovator. In addition, relatively more efficient technologies are licensed exclusively to the private firm. So, the clear implication is that the technology gap between the public and private firms gets further aggravated for large innovation. Thus, an inefficient public firm that wants to catch up with the private firm through licensing, *cannot* do so in a market setting when it has to pay for the license through its own profits.

Based on the licensing outcome, we can summarise the degree of privatization undertaken by the public firm at the second stage of the game as below.

Proposition 2. *In a market setting, given the innovator's licensing choice,*

- *(i)* *partial privatization is optimal in Regions $A1$, $A2$, and $B2$.*
- *(ii)* *full privatization is optimal in Regions $B1$, $C2$ and $D2$.*

This demonstrates the significance of the interaction between initial cost asymmetry and the size of innovation in determining the degree of privatization.

We now focus on an economic environment where a social planner would choose the optimal licensing outcome and the corresponding optimal privatization in order to maximize social welfare.

3. Social planner's problem of allocation of innovation

What is the best possible allocation of technology to maximize social welfare in the present context? To address this question, we can think of a situation where the social planner owns the innovation and then allocates the license so as to maximize social welfare, provided that both firms subsequently compete in the market. Alternatively, the social planner can buy the innovation from the innovator at a fee and then license it to the producers. This fee payment to the innovator can be financed by imposing a lump-sum tax on the citizens. Note that the fee or lump-sum tax is a transfer between parties within the economy, and does not affect social welfare in our analysis.

In Stage 1, the social planner would allocate the innovation to maximize social welfare without being concerned about licensing fees/revenue (as opposed to the case of the private innovator considered in Section 2.2.3). Then in Stage 2, it would optimally privatize the public firm based on the first stage allocation of licensing. In Stage 3, the public firm will choose quantities to maximize the weighted average of profit and welfare depending on the degree of privatization chosen in the second stage. The private firm will choose q_1 to maximize its profits. We solve this game by backward induction.

To avoid duplicating the analysis, we note that the third and second stage outcomes of the social planner's game coincide with the market outcomes. That is, in the competition stage, the profits and quantities are given by equations (1) to (8), and the optimal degree of privatization, depending on the Stage 1 licensing outcome is given by equations (10), (12), and (14).

In Stage 1, the social planner allocates the technology (or licenses the technology) based on which regime yields the highest social welfare for the given parameter combinations. While allocating the innovation, the social planner takes into consideration the Stage 2 optimal degree of privatization. Thus, for each specific regime of allocation we can arrive at the welfare expressions depending on whether the Stage 2 privatization is partial or full. So, the welfare in Stage 1 can be written as (from equations (9), (11), and (13)):

$$W^{L0} = \begin{cases} (a-c)^2(1-4x+4y)(x-y) + 4(a-c)^2(x-y)^2 + \dfrac{(a-c)^2(1-2x+2y)^2}{2} & \text{if } 5x-5y<1 \\ \dfrac{(a-c)^2(1-2x+2y)^2}{9} + \dfrac{(a-c)^2(x-y+1)^2}{9} + \dfrac{(a-c)^2(2-x+y)^2}{18} & \text{if } 5x-5y \geq 1 \end{cases} \tag{18}$$

$$W^{L1} = \begin{cases} (a-c)^2(x+y)(1-4x-3y) + 4(a-c)^2(x+y)^2 + \dfrac{(a-c)^2(1-2x-y)^2}{2} & \text{if } 5x+4y<1 \\ \dfrac{(a-c)^2(1-2x-y)^2}{9} + \dfrac{(a-c)^2(x+2y+1)^2}{9} + \dfrac{(a-c)^2(2-x+y)^2}{18} & \text{if } 5x+4y \geq 1 \end{cases} \tag{19}$$

$$W^{L2} = \begin{cases} (a-c)^2x(1-4x+y) + 4(a-c)^2x^2 + \dfrac{(a-c)^2(1-2x+y)^2}{2} & \text{if } 5x-y<1 \\ \dfrac{(a-c)^2(1-2x+y)^2}{9} + \dfrac{(a-c)^2(x+1+y)^2}{9} + \dfrac{(a-c)^2(2-x+2y)^2}{18} & \text{if } 5x-y \geq 1 \end{cases} \tag{20}$$

In each of the above welfare values, the top (bottom) expression reports welfare under partial (full) privatization. By comparing social welfare, the social planner chooses the best outcome, which we state in the following proposition.

Proposition 3. *It is a dominant strategy for the social planner to allocate the innovation to both firms under all feasible parametric configurations.*

Proof: We need to show that welfare when both firms have the technology (corresponding to the $L2$ regime) is higher than welfare under the other two regimes for all feasible parametric configurations. So, we prove this proposition in two parts, by pairwise comparison between welfare under licensing to both firms and the other two regimes.

(i) Proof of $W^{L2} > W^{L0}$:

$$W^{L2}(\alpha) - W^{L0}(\alpha) = \frac{(a-c)^2 y(4x - 2y + (2 + 10x - 4y)\alpha + (6 + 3y)\alpha^2)}{2(1+2\alpha)^2}$$

The above expression is positive for all $y < x$ by Assumption 2. Hence,

$$W^{L2}(\alpha) > W^{L0}(\alpha)$$

Thus, welfare for any degree of privatization is higher under licensing to both firms ($L2$) than under licensing to the public firm ($L0$) alone. This implies that the following must also hold:

$$W^{L2}(\alpha^{L0}) > W^{L0}(\alpha^{L0}) \tag{21}$$

Further, since by the definition of optimal degree of privatization, it must also be true that

$$W^{L2}(\alpha^{L2}) > W^{L2}(\alpha') \ \forall \alpha' \neq \alpha^{L2} \tag{22}$$

Combining equations (21) and (22), we can thus, write

$$W^{L2}(\alpha^{L2}) \geq W^{L2}(\alpha^{L0}) > W^{L0}(\alpha^{L0})$$

(ii) Proof of $W^{L2} > W^{L1}$:

$$W^{L2}(\alpha) - W^{L1}(\alpha) = \frac{(a-c)^2 y\big((2 - 6x - y) + 2\alpha(3 - 8x - y)\big)}{2(1+2\alpha)^2}$$

From Assumption 1, we have $0 < y < \frac{1-4x}{3}$. Therefore, the first term within parentheses, $(2 - 6x - y)$ is positive. Also, $(3 - 8x - y) > 0$ for $1 - 4x > 0$. With a similar logic as in part (i) of the proof, we can state that

$$W^{L2}(\alpha^{L2}) \geq W^{L2}(\alpha^{L1}) > W^{L1}(\alpha^{L1})$$ ■

Now that we have the social planner's optimal choices, we can compare them with the market outcomes, which brings us to the following two propositions.

Proposition 4. *Comparing the optimal privatization in the market outcome with a social planner's optimal privatization choice, we find that privatization under the market outcome is weakly higher than the degree of privatization chosen by the social planner.*

Proposition 5. *Comparing the licensing outcomes in the market situation with a social planner's optimal licensing outcomes, we find that*

(i) *Regions $A1$ and $B1$ are areas of conflict – while the innovator prefers to offer the license to the private firm alone, the social planner would offer the license to both firms.*

(ii) *Regions $A2$, $B2$, $C2$, and $D2$ are areas of convergence – the market outcomes and social planner's optimal licensing outcomes are the same, i.e., licensing to both firms.*

One can view this conflict of choices in Figure 4 in region $A1$ and $B1$, and the details of the conditions under which various market and social outcomes occur are also presented in Table 2. The primary reason why the market outcome is different from the socially optimal outcome is that the innovator as a profit maximizer tries to maximize its own payoff. For large innovation, the private firm's willingness to pay for an exclusive license is much higher than the two firms' combined willingness to pay under $L2$. Recall from Lemma 1, that the degree of privatization is higher under $L1$ than under $L2$ and $L0$. If $L1$ is chosen by the innovator, the private firm is in competition with a highly privatized, very inefficient public firm. So, $\pi_1^{L1}(\alpha^{L1})$ is large, while the outside option of $\pi_1^{L0}(\alpha^{L0})$ is much lower, thus the private firm's willingness to pay, i.e., $\pi_1^{L1}(\alpha^{L1}) - \pi_1^{L0}(\alpha^{L0})$ is high. On the other hand, if the innovator chooses $L2$, the private firm is in competition with a relatively less privatized and less inefficient public firm. Consequently, the private firm's willingness to pay is much lower. Further, due to its budget constraint, the public firm can only pay $\pi_0^{L2}(\alpha^{L2})$, which is also small due to a lower degree of privatization, α^{L2}. Thus, the total payoff to the innovator under $L2$ cannot exceed its payoff from $L1$, and it will only offer the license to the private firm alone.

Proposition 5 has clear policy implications regarding government budgetary support to a public firm for technology upgradation in regions $A1$ and $B1$. The government can provide budgetary support equivalent to the difference between the innovator's payoffs from $G1$ and $G2$ and improve social welfare. However, in regions $A2$, $B2$, $C2$, and $D2$, the market outcome of licensing to both firms will also be welfare improving. Hence, government intervention will not be necessary.

Region	Conditions	Market Outcome		Social Planner's Optimum	
		Licensing	**Privatization**	**Licensing**	**Privatization**
A1	If $(4x^2 + 4y^2 - x + 7xy) > 0$	$L1$	$\frac{x+y}{1-4x-3y}$	$L2$	$\frac{x}{1-4x+y}$
A2	If $(4x^2 + 4y^2 - x + 7xy) \leq 0$	$L2$	$\frac{x}{1-4x+y}$	$L2$	$\frac{x}{1-4x+y}$
B1	If $(x^2 + (1+2y)^2 - 7x - 5xy) > 0$	$L1$	1	$L2$	$\frac{x}{1-4x+y}$
B2	If $(x^2 + (1+2y)^2 - 7x - 5xy) \leq 0$	$L2$	$\frac{x}{1-4x+y}$	$L2$	$\frac{x}{1-4x+y}$
C2	-	$L2$	1	$L2$	1
D2	-	$L2$	1	$L2$	1

Table 2. Detailed outcomes in each permissible region.

(N.B. Assumptions 1 and 2 describe the admissible parameter space).

4. Conclusion

We have considered a licensing game in a mixed duopoly where an outsider innovator chooses the optimal licensing strategy and then, the public firm chooses its optimal degree of privatization. To the best of our knowledge, our paper is the first to introduce the public firm's budget constraint when it wants to acquire a license for a new technology from an innovator. In our model, the public firm has to pay for a new technology with its own profits, as government financial support may be inadequate or absent altogether.

We find that the innovator chooses to offer two licenses when the size of innovation is small, and only one license to the private firm when the innovation is large. Licensing to the public firm alone is never chosen by the innovator. An important implication of our analysis is that higher levels of innovation are not shared with the inefficient public firm, and thus its cost-difference with the private rival will aggravate. For smaller levels of innovation, both firms receive the technology, and as a result, their absolute cost difference would remain the same. Therefore, in our model, it is evident that market outcomes under certain situations will not allow the public firm to catch up or even maintain the technology difference with its rival private firm.

The optimal degree of privatization in this model is similar to some earlier findings. In a market setting we find that partial privatization is optimal for smaller initial cost difference, but full privatization is optimal for relatively large innovation, or larger initial cost difference.

We also determine the socially optimal licensing outcomes from the perspective of a social planner, and find that it is always optimal to license to both firms. When compared to the social planner's choices, for larger innovation sizes, we show that the market outcomes are not in sync with socially optimal outcomes. But for smaller innovation sizes, the two outcomes converge, and licensing to both firms is optimal. Our results differ significantly from the existing literature which discusses the possibility of an outcome where the public firm is the sole licensee, as that outcome is never optimal in our model. This is because the public firm has to pay the license fee from the profit it generates. (e.g., Mukherjee and Sinha, 2014; Chen et. al, 2014; Wang and Zeng, 2019; Wang et. al, 2020).

There are some interesting policy implications of our model. If there is moderate initial cost asymmetry, and a large size of innovation, then to improve social welfare, the government can directly intervene and provide budgetary support equivalent to the difference between the innovator's payoffs from offering a single license to the private firm, and offering licenses to both firms. This can ensure an outcome where both firms have access to the new technology, and can thus improve social welfare. Another situation can arise for some intermediate innovation size, where the licensing outcome can be influenced by manipulating the cost asymmetry between the two firms. This can be brought about by imposing a per unit tax on the private firm's output and/or by subsidising the public firm's output, thereby leading to a shift in the outcome from regions $A1$ or $B1$ to $A2$ or $B2$. This move may increase the welfare by ensuring wider technology dissemination. On the other hand, if there is a relatively small size of innovation, irrespective of the initial cost asymmetry, the market outcome (of licensing to both firms) is socially optimal as well, and therefore, government intervention is not required.

One limitation of our work is that we have only considered fixed-fee licensing rather than a general two-part tariff licensing scheme. Thus, it would be interesting to analyse the possibility of a two-part tariff licensing contract in a setting similar to ours. Future research can also examine what would be the outcome in case of a foreign R&D firm, an insider innovator, or a public innovator. Further, one may also extend the analysis by considering a convex cost structure. Another interesting point to note is that our model generates an empirically testable hypothesis that when the cost asymmetry between the public and private firms is moderate, and the size of the innovation is large, then the innovator may prefer to license to the private firm only, rather than offering the license to both the public and private firms.

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