ISSN No. 2454 - 1427

CDE
May 2024

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Working Paper No. 348

Centre for Development Economics
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Capital Inflow, Strategic Subcontracting, and Formal Employment*

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This paper shows that capital inflow can increase formal employment with labor market rigidity even when cheaper labor is accessible outside the formal sector. In a simple duopoly setting, if subcontracting to the informal sector is strategic to deal with the bargaining power of formal workers, the model finds that better foreign technology and access to the informal sector of the foreign firm reduces the prohibitive tariff for attracting foreign capital and can raise formal employment, by limiting the wage rise due to the combined effect of competition, wage, and technology. While the increased capital flow raises the wage through the competition effect, the level of labor-saving technology and the strategic subcontracting to the informal sector moderate it, showing an inverted U-shaped wage curve against the level of technology. After a critical level of technological superiority, the union wage can be limited to a level such that the foreign firm does not need to subcontract. The firm-level analysis across countries reveals that the results are consistent with our theoretical predictions.

Keywords: Foreign Direct Investment, Foreign Technology, Subcontracting, Formal Labour, Unionised Wage
JEL Codes: F13, F14, F16, F21

*We thank Arijit Mukherjee and Uday Bhanu Sinha for their useful comments and feedback.

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1 Introduction

Whether foreign direct investments(FDI) would expand the formal sector in a typical developing economy is ambiguous in the existing literature. While a growing volume of studies (Jenkins, 2006; Jayaraman and Singh, 2007; Dao, 2020; Lipsey et al., 2010; Karlsson et al., 2009; Nunnenkamp, Peter and Bremont, José Eduardo Alatorre and Others, 2007) showed a positive impact of FDI on manufacturing employment, some other studies (Nguyen et al., 2020; Akcoraoglu and Acikgoz, 2011) found that FDI can have a negative on employment. Moreover, Massoud (2008) and Nordin (2017) did not find any significant or clear impact of FDI on employment. The recent thrust of the policy-makers of the developing world, who lack capital, encouraging the capital inflow by discouraging import competition to some extent for the benefit of gainful employment, sets such an ambiguity. A foreign firm undertaking horizontal FDI generates additional demand for labor, raising employment possibilities in the formal sector. However, the question would be whether employment in the formal sector would rise depending on the level of foreign technology, formal labor rigidity, and accessibility of informal labor. If the foreign firm uses better technology with a limited absorbable capacity of formal labor with unions and can access informal labor, one cannot firmly conclude this favorably. In a typical developing economy, workers who find employment in a formal sector complying with rules and regulations must enjoy a degree of bargaining power to negotiate for a better wage. Those who do not find employment in the formal sector move to the informal sector for survival. Firms also look for an outside option to produce at a cheaper cost by subcontracting to the informal sector. Many firms in South Asian and Latin American countries often undertake in-house production and subcontract to informal producers (Ulyssea, 2010; Mehrotra and Biggeri, 2007). Major pharmaceutical manufacturers in India produce in-house and subcontract to the unregistered sector. Similarly, Bata, a well-known shoe manufacturer in India, produces in-house and subcontracts to outside producers (Bacchetta and Bustamante, 2009). So, while negotiating with the union working in the formal sector, the firm strategically chooses informal employment. Maiti and Mukherjee (2013) modelled this possibility of strategic production allocation between in-house production and outsourcing to the informal sector in the context of both one-way and two-way trade liberalization and showed that the increased trade competition (by lowering tariff) in the domestic market raises union wage of the formal sector workers. Labor market consequences in the formal sector due to FDI liberalization in the unionized framework would have similar implications, and this issue has been ignored in the

literature. If the foreign firm competes with the domestic firm, the competition will raise the demand for formal labor and the demand would increase union wage, discouraging employment in the formal sector. These two forces, competition and wage effects, would essentially determine the size of formal employment. Of course, the superiority of foreign technology and strategic choice of informal labor add to these forces denoted as the technology effect, which influences the union wage and, thereby, the resultant formal employment. The additional demand generated by the capital flow with labor-saving technology may not necessarily increase the wage and, therefore, must have implications on the choice between imports and FDI. This paper investigated the impact of FDI accompanied by superior foreign technology on formal employment and wage inequality under the strategic choice behavior for informal labor.

In the current scenario, FDI is the largest source of external financing for emerging and developing countries, totalling US$ 700 billion in 2018 (Giroud and Ivarsson, 2020). Between 2006 and 2016, the stock of worldwide FDI as a share of world GDP has risen by about 60% (Carril-Caccia et al., 2018). Studies covering developing countries suggest that FDI is essential for transferring technology and significantly contributes to income and economic growth (Borensztein et al., 1998). However, whether it has raised formal employment does not have clear evidence. In anticipation of more formal sector jobs, policymakers in developing countries design strategies to attract, retain, and increase FDI stocks. To facilitate inward FDI, governments undertake reforms to liberalize trade and investment policy regimes, improve the business environment and offer investment incentives since a country's investment climate plays a significant role in reaping the benefits of FDI, (Farole and Winkler, 2014). Javorcik (2015) suggested that FDI inflows generate good jobs through higher wages at the firm level than domestic firms and enhance productivity in developing countries. Feenstra and Hanson (Feenstra and Hanson) found an increase in the relative demand for skilled labor and an increase in the wage share of skilled workers due to an increase in FDI in the creation of in-bond foreign assembly plants in Mexico. Multinational enterprises (MNEs) are attracted to the large market size and abundant low-cost labor availability in labor-abundant developing economies. Therefore, MNEs are keen to make both market- and efficiency-seeking outward FDI to developing countries, thereby setting up manufacturing units in such nations to cater to large markets and save on tariffs and other iceberg trade costs. Empirical evidence suggests that wage differences between countries remain substantial on a global scale and incline firms to relocate production processes from high-wage to low-wage countries[1].

[1]Sinn et al. (2006) find that the average wage cost in the manufacturing industry

Secondly, if there is an excellent technological difference between foreign and host country firms, FDI increases. Cao and Mukherjee (2018) find that multinational firms may prefer FDI in the presence of labor unions if it is sufficiently technologically superior to its domestic counterpart. Thirdly, the informal sector is a prominent feature of many developing countries[2], which is likely to have profound economic implications. Substantial wage differential exists across formal and informal sector laborers, and less productive firms compete with more productive firms, leading to the misallocation of resources and significant total factor productivity losses (Hsieh and Klenow, 2009). The informal sector provides an alternate production option to manufacturers[3]. Therefore, manufacturing enterprises, both domestic and foreign, subcontract a part of their production to the informal sector to cut down production costs, take advantage of tax evasions, relax bureaucratic requirements and regulations and contain the bargaining power of labor unions and formal wages. The significant presence of informal production activities in developing countries and the advanced foreign technology that comes with inward FDI motivates us to raise the following research questions: Can FDI increase formal employment when the formal sector is highly unionized? Can FDI increase formal employment when FDI brings better technology and has access to the informal sector? How does it affect the bargaining power of the labor union, which may either increase or decrease unionized wages?

Gaston and Trefler (1995) showed that unilateral trade liberalization may either increase or decrease unionized wage under the efficient firm-union bargaining for both employment and wage, where union and firm bargain over both employment and wage. Cao and Mukherjee (2018) show that labor market imperfections may create incentives for FDI. Most of the existing work looked at the aspects of the FDI on contemporary issues by looking at the role of skilled and unskilled workers and the union structure of the world but ignored the strategic choice of the informal sector in the production process. Maiti

ranged from 1,10 Euro in China to above 27 Euro in countries like Denmark, West Germany and Norway.

[2]According to recent estimates provided by WTO, informality in South Asia, Sub-Saharan Africa, and Latin America and the Caribbean is 70%, 74%, and 33% of the total workforce in 2018.

[3]An important way formal enterprises interact with informal enterprises is by subcontracting a part of their production process to the informal enterprises. Subcontracting has gained relevance in times of increased competition by trade liberalization and globalization (ILO and WTO, 2009)

and Mukherjee (2013) showed the negative effects of trade cost reduction on formal employment, unionized wage, and formal-informal wage gap in the presence of labor unions when the firm strategically allocates production between formal and informal sectors. They used Cournot competition in the final goods market and the right-to-manage model of labor union to show the effects of formal-informal output composition effect of a trade cost reduction. The impact of FDI on formal employment in a dual labor market setting has not been dealt with, especially when informal labor is strategically chosen. In our current work, we fill the gaps in the literature on FDI by using the international Cournot-duopoly framework of Maiti and Mukherjee (2013) of a foreign firm and a domestic firm competing in the final goods domestic market. We modify it in the context of inward FDI and extend the model to incorporate both foreign technology[4] and subcontracting jointly to analyze their impact on the labor market structure for both formal and informal workers when the formal sector is highly unionized.

We develop a Cournot duopoly model of strategic competition of inward greenfield FDI, where a foreign firm with advanced technology sets up a manufacturing unit in the host country and has access to the informal sector. Both firms strategically allocate production between formal in-house and subcontract partially to the informal producers[5]. The formal wage is assumed to be fixed by the centralized labor union. It is believed that homogeneous workers are better off being organized in a single union (centralized union) rather than in separate unions (decentralized union), which bargain independently with the employer[6]. The informal sector workers are paid minimum reservation wage. We show that through their strategic allocation to the informal sector at the first stage, firms restrict the bargaining power of labor unions and, therefore, control unionized wages. In this scenario, an increase in inward FDI increases demand for formal labor and increases formal wage but widens formal and informal wage inequality[7], and reduces informal pro-

[4]See, e.g., Acemoglu and Autor (2011) for arguments related to technology, skills, tasks, and its implications related to employment and earnings.

[5]We assume that formal and informal sector workers are homogeneous regarding their skills. Individuals who do not find work in the formal sector start working in the informal sector, where they are not given any social security benefits.

[6]A single union eliminates competition between the individual unions and results in better bargaining power, which will benefit the workforce, (Horn and Wolinsky, 1988; Davidson, 1988). Ulph (1989) showed this may not be true when workers and the firm can not commit to a long-term contract.

[7]Refer Acemoglu (1999) for discussions related to changes in the proportion of skilled

duction. Maiti and Mukherjee (2013) considering unilateral trade showed that low trade cost increases unionized wage, in-house employment, union utility, the formal-informal wage gap, and consumer surplus in the presence of strategic output allocation between formal in-house production and subcontracting to the informal sector. Naylor (1998, 1999) and Borensztein et al. (1998) showed that a two-way trade liberalization increases unionized wage. Konings and Vandenbussche (1995) used UK firm-level panel data between 1982 and 1989 to show that intensified foreign competition positively affects wages but does not affect employment. Braun and Scheffel (2007) found empirical evidence for their hypothesis that outsourcing deteriorates unions' bargaining position and reduces the bargained wage for low-skilled workers. An explanation for this could be that outsourcing puts the jobs of the low-skilled at risk. While the wages of medium-skilled workers are largely unaffected by outsourcing, high-skilled workers see their wages rise in industries with high levels of outsourcing.

Our analysis explains the effect of FDI on formal employment and unionized wages using competition, technology, and wage effects[8]. Given the amount of subcontracting, an inflow of FDI by a foreign firm leads to an increase in in-house production in the host country and creates a competition effect. This increases the demand for formal labor and union wages. As a foreign firm uses advanced technology and subcontracts to the domestic informal sector, this reduces per unit labor requirement and demand for formal labor and union wage, creating a technological effect. This technological effect dampens the competition effect, which creates a wage effect, which tends to increase firms' incentives for in-house production and decrease their incentive for subcontracting. This increases the union's demand for formal workers and the unionized wages. The strength of the competition, technology, and wage effects determine the net impact on formal employment and unionized wages.

The mechanism behind our results can be related to the firms' investment incentives decreasing with a rise in union bargaining power. The union demands higher wages once the investment is sunk to the appropriate part of the rent. The in-house union extracts rent from the firms, and this induces firms to reduce their domestic production[9]. The possibility of subcontracting to the informal sector reduces labor unions' bargaining

workers or skill-biased technical change can create a qualitative change in the composition of jobs, increasing the demand for skills, wage inequality, and unemployment.

[8]Maiti and Mukherjee (2013) use the competition and wage effect framework in the context of trade tariffs.

[9]Refer Mukherjee and Pennings (2011) and Haucap and Wey (2004) for rigidity prob-

strength and rent extraction capacity, therefore incentivizing higher domestic and foreign investment in production.

We empirically analyze the model's predictions using world enterprise survey data. This database is uniquely suitable for investigating firm-level determinants of formal employment and unionized wages. We take pooled cross-sectional data for 97,624 manufacturing firms at the four-digit level across 23 industries and 155 countries post-2006. Using fixed effects, we control for observed and unobserved heterogeneity across countries and industries. We find a significant positive relationship between formal employment and FDI accompanied by foreign technology and subcontracting. Similarly, the unionized wage is also positively associated with advanced technology and subcontracting. Lee and Park (2020) also find a positive impact of greenfield FDI on the employment of Korean firms using panel data for 1328 firms across 20 industries from 2004–2015.

The remainder of the paper is organized as follows: Section 2 presents the theory model. Section 3 describes the empirical approach and the data and reports the estimation results. Section 5 concludes.

2 Theoretical Framework

In this section, we design a simple international duopoly model to capture the strategic competition between the domestic and foreign firms to show how inward green-field FDI[10] accompanied by better technology (labor-saving) and access to the informal sector, generates and reallocates employment opportunities across formal and informal sectors in the presence of a unionized formal labor market in the host country. The wage is fixed by a centralized labor union [11] and the informal laborers are strategically chosen on the subcontracting form to control the union's bargaining strength. Maiti and Mukherjee (2013) use the framework of strategic output allocation between formal in-house production and subcontracting to the informal sector to show the impact of trade cost reduction (tariffs) on unionized wage, formal employment, formal-informal wage gap, and the union utility using the competition and wage effects. In their analysis, given the amount of

lems created by labor unions on the R&D investment and Tirole (1988) for a general theory of investment.

[10]We consider here unilateral FDI and trade liberalization.

[11]Refer Haucap and Wey (2004) for unionization structures.

subcontracting to the informal sector, if there is a fall in tariffs, it increases the exports of a foreign firm and reduces the domestic firm's in-house production. This competition effect reduces the union's labor demand and the unionized wage. This wage effect tends to increase the domestic firm's incentive for in-house production, raise the union's labor demand, reduce the incentive for subcontracting, and increase the formal wage. The strengths of the competition and wage effects determine the net effect of low tariffs on formal production, formal employment, and formal wage. They find that the wage effect, which increases the unionized wage, dominates the competition effect in the one-way trade model.

We modify (Maiti and Mukherjee, 2013) framework by considering the inward green-field FDI instead of exports under high prohibitive tariffs and technology effect, which has been ignored in their work. Where subcontracting to the informal sector links formal and informal production parts, and foreign technology may change the formal labor requirement. The inward green-field FDI increases production and creates a higher demand for formal workers. This competition effect increases the union's labor demand and the union wage. This wage effect may decrease incentives for formal in-house production and increase incentives for subcontracting. The direction of competition, technology, and wage effects changes in the FDI framework compared to the trade scenario. However, if FDI is associated with better foreign technology, this may reduce formal labor requirements and demand, creating a technological effect. Further, if the foreign firm is given access to the informal sector and strategically subcontracts to it, this may have different implications on the competition and wage effects. To identify the effect of these forces and their joint impact analytically, we design a model in a sequential framework through different cases where the domestic firm uses indigenous technology and continues to have the same set-up in all the cases. The foreign firm exports in case 1 with no FDI, undertakes green-field FDI, uses domestic technology in case 2, brings its superior foreign technology along with FDI in case 3, and, in case 4, the foreign firm is also given additional access to the informal sector of the host country for production.

Assume an open economy with two countries, called domestic and foreign, and each country has one firm. The firm in the domestic country is called firm 1, and the firm in the foreign country is called firm 2. These firms produce a homogeneous product and compete in the domestic country product market as Cournot duopolists. The domestic country's demand is served by firms 1 and 2, operating under constant returns to scale, with labor being homogeneous and the only factor of production. Firm 2 can serve the

domestic country either through exports or through FDI. Both firms face a linear inverse demand function in the domestic country: $P(Q) = a - Q$, where P is the product price, $Q = q_1 + q_2 + k_1$ is the total output, q_1 and q_2 are the individual outputs of domestic and foreign firms, and k_1 represents the output outsourced to the informal sector by firm 1, respectively. Here, a is the intercept and represents the strength of the market demand function. We assume that firm 1's output can be produced in-house and/or outsourced to the domestic informal sector. For simplicity, we further assume that one unit of labor is required to produce one unit of output, irrespective of the in-house or informal production. A centralized[12] labor union determines the formal wage w for the in-house workers. However, if firm 2 undertakes FDI, both firms hire formal workers from a single (or an industry-wide) labor union in the domestic country and face uniform wage. In contrast, the minimum reservation wage of the informal worker is z, which is assumed to be zero. However, constant transaction and/or administrative costs c exist to manage the informal sector; therefore, the effective cost of informal production is c. This minimum reservation wage prevailing in the informal sector creates different labor market institutions in the formal and informal sectors. We assume here the "right-to-manage" model[13], where firms have autonomy over employment as per their needs, and the union chooses a uniform wage rate to maximize their joint rent (López and Naylor, 2004; Vannini and Bughin, 2000; Bughin and Vannini, 1995). The presence of the informal sector would act as an alternative source of production and allow firms to make their subcontracting decisions strategically to limit the bargaining power of labor unions and their rent extraction capacity, therefore, unionized wages to a greater extent.

The model has four cases; the benchmark case 1 sets up the model here; the domestic firm 1 takes a strategic decision on formal in-house production and subcontracting to the informal producers[14]. Firm 2 is a foreign firm serving the domestic country market through exports with prohibitive tariffs imposed in the benchmark case. In case 2, firm 2 chooses to set up a production plant in the host country instead of exporting under high prohibitive tariffs, i.e., undertakes greenfield horizontal FDI with no foreign technology and does not have access to the informal sector. Both firms use the same technology

[12]There is one industry-wide union which sets up a uniform industry wage for both firms to maximize their utility, defined in terms of the industry wage bills.

[13]See Layard et al. (2005) for discussions in favor of right-to-manage models.

[14]Subcontracting acts as an alternative production option. Firms may restrict the labor union's bargaining power and the unionized wage by choosing a higher level of subcontracting.

in production and face uniform marginal production costs in the formal sector. In case 3, firm 2 brings its superior foreign technology in production and is, therefore, technologically superior to firm 1. Firm 1 uses its domestic technology in production and has access to the informal sector, as in case 1. We assume that production requires only workers, and the firms differ in technologies and, therefore, face asymmetric marginal costs. Finally, in case 4, firm 1 continues to have the same set-up and firm 2 has now given access to the informal sector. Both firms strategically decide on formal in-house production and subcontracting to the informal sector. We determine the formal and informal employment generation mechanism across sectors through foreign competition in producing final goods under the unionized labor markets. How does this impact the bargaining power of labor union and, therefore, endogenous union wage and employment in the formal sector?

We consider the following game. At stage 1, firm 1 decides between whether to export or to undertake green-field FDI. At stage 2, the firms strategically decide on in-house formal production and subcontracting to the informal sector. At stage 3, the centralized labor union determines the formal wage. At stage 4, the firms hire workers according to their requirements and compete like Cournot duopolists in the final goods market, and the profits are realized. We solve the game through backward induction.

Case 1: Baseline Model

The benchmark case explains the basic trade model between two firms producing homogeneous goods across countries, domestic and foreign. Firm 1, strategically allocates output to the informal sector before it allocates to the in-house formal production. The foreign firm 2 exports to the domestic country under prohibitive tariffs. Through the international Cournot duopoly model under strategic competition, we try to identify the impact of domestic production and exports of final goods by a foreign firm on the host country's formal wage, formal employment and the formal-informal wage gap under the unionized labor market.

We consider a three-stage game with the following timings: At stage 1, the domestic firm determines its subcontracting decision k_1 to the informal sector to internalize its effect on formal sector production, i.e., q_1, q_2 and the unionized wage, w. At stage 2, the centralized labor union determines the formal wage, w, to internalize its effects on q_1 and q_2. At stage 3, given the k_1 and w levels, firms 1 and 2 simultaneously determine q_1 and q_2 and the profits are realized. The game is solved through backward induction.

Firms 1 and 2 maximize the following profit expressions to determine q_1 and q_2, respectively:

$$\pi_1 = (a - Q - w)q_1 + (a - Q - c)k_1 \tag{1}$$

$$\pi_2 = (a - Q - t)q_2 \tag{2}$$

The equilibrium values can be found as

$$q_1 = \frac{a - 3k_1 + t - 2w}{3} \tag{3}$$

$$q_2 = \frac{a - 2t + w}{3} \tag{4}$$

Equation (3) gives the labor union's demand for labor based on the assumption that one unit of labor produces one unit of output irrespective of formal or informal sector. Firm 1's formal sector production q_1 is inversely related to the amount of subcontracting k_1 and unionized wage w but it is directly related to the trade cost (tariffs) t. This implies that the lower the unionized wage, the higher the domestic firm's in-house production, called the wage effect. The higher the trade cost, the lower the exports by firm 2, and the higher the domestic production by firm 1, i.e., q_1. Further, if firm 1 allocates a large portion of output to the informal sector, its in-house production would be lower. This implies that firm 1 can limit the union's bargaining power and formal wage by strategically choosing k_1. Equation (4) shows that firm 2's output is directly related to unionized wage and inversely related to the trade cost. The subcontracting by firm 1 does not directly impact the export decision of firm 2 because firm 2 considers that the total output of firm 1, i.e., $q_1 + k_1$, is given while taking its output decision. Now, we determine the unionized wage[15], w by maximizing the following union's utility expression

$$\max_w \frac{w(a - 3k_1 + t - 2w)}{3} \tag{5}$$

The equilibrium wage is

$$w = \frac{(a - 3k_1 + t)}{4} \tag{6}$$

It follows from equation (6) that unionized wage is inversely related to subcontracting k_1. Firm 1 can reduce unionized wage by subcontracting more to the informal sector at stage 1. This availability of alternative production options reduces the labor union's bargaining power. Further, low trade cost also reduces unionized wage because of higher

[15]The utility of the labor union is $U = wL$, where w is wage and $L = (q_1 + q_2)$ is the number of workers employed.

exporting by firm 2 and, thereby, lower in-house production of firm 1 and low demand for formal labor. Substituting equation (6) into equations (3) and (4), we get

$$q_1 = \frac{a - 3k_1 + t}{6} \tag{7}$$

$$q_2 = \frac{5a - 7t - 3k_1}{12} \tag{8}$$

If firm 1 produces in-house and subcontracts to the informal sector, it maximizes the following expression to determine k:

$$\max_{k_1} \big((a + 3k_1 + t)(a - 3k_1 + t) + 3(5a - 3k_1 + 5t - 12c)k_1\big)/36 \tag{9}$$

The equilibrium amount of subcontracting of firm 1 can be found as

$$k_1^{1*} = \frac{5a + 5t - 12c}{12} \tag{10}$$

Equation (10) shows that the lower trade cost reduces the amount of subcontracting by firm 1 as exports meet domestic demand. A trade-off exists, given k_1, a lower t reduces w; this is the competition effect, where at low tariffs, firm 2 exports more and reduces the domestic production by firm 1, implying low union labor demand and low formal wage. However, a lower t reduces k_1, which increases w, creating a wage effect, where at low tariffs, firm 1 subcontracts less and increases in-house production, increasing the union's labor demand and raising the formal wage. The strengths of competition and wage effects determine the net effect of a trade cost reduction on formal wage and employment. Substituting equation (10) in equations (6), (7), and (8), we get

$$w^{1*} = \frac{1}{16}(12c - a - t) \tag{11}$$

Note that $w^{1*} > 0$ if $\frac{a+t}{12} < c$. Then, the in-house production will be positive as follows:

$$q_1^{1*} = \frac{1}{24}(12c - a - t) \tag{12}$$

$$q_2^{1*} = \frac{1}{16}(5a + 4c - 11t) \tag{13}$$

The above equilibrium expressions show that firm 1's decision to produce in-house and/or to subcontract depends on a, c, and t. We aim to determine the impact of FDI on formal wages, employment, and the formal-informal wage gap. We restrict our analysis to where firm 1 allocates production between formal in-house and subcontract and firm 2 undertakes green-field FDI. Substitute equations (10), (11), (12), and (13) in the profits functions of firms to get the equilibrium profits of both firms, such as

$$\pi_1^{1*} = \frac{1}{96}\left(11a^2 - 40ac + 22at + 48c^2 - 40ct + 11t^2\right) \tag{14}$$

$$\pi_2^{1*} = \frac{1}{256}(5a + 4c - 11t)^2 \tag{15}$$

By using equation (13), we get the critical prohibitive tariff level for the foreign firm is $t^* = \frac{5a+4c}{11}$.

Lemma 1 *If the actual tariff t exceeds the prohibitive tariff (t^*), i.e., $t > t^*$, the foreign firm will stop exporting and may undertake FDI.*

Case 2: Foreign Direct Investment

In this case, firm 2 undertakes green-field FDI and sets up a production plant in the domestic country instead of exporting for tariffs set higher than the prohibitive critical tariffs level. Firm 1 continues to have the same set-up, produces in-house and subcontracts to the domestic informal sector. Firm 2, on the other hand, does not have access to the informal sector and undertakes only formal in-house production. We assume that firms 1 and 2 face symmetric wages[16] and use homogeneous technology. We see how the inward FDI would impact equilibrium unionized wage and formal employment and the formal-informal wage gap. The model is again solved using the stages mentioned above.

Firms 1 and 2 maximize their profit functions to determine q_1 and q_2

$$\pi_1 = (a - Q - w)q_1 + (a - Q - c)k_1 \tag{16}$$

$$\pi_2 = (a - Q - w)q_2 \tag{17}$$

The equilibrium values can be found as

$$q_1 = \frac{a - 3k_1 - w}{3} \tag{18}$$

$$q_2 = \frac{a - w}{3} \tag{19}$$

Equation (18) shows that firm 1's formal sector production is inversely related to k_1 and w. Equation (19) shows that firm 2's output is inversely related to w. The higher the unionized wage, the lower the formal output will be. Firm 2 considers that the total output of firm 1, i.e., $q_1 + k$, is given while taking its output decision. Now, we determine the unionized wage w by maximizing the following expression:

$$\max_w w\Big(\frac{a - 3k_1 - w}{3} + \frac{a - w}{3}\Big) \tag{20}$$

[16]Centralized union offers uniform wage to all firms across the industry as labor supply is perfectly monopolized.

The equilibrium wage can be found as

$$w = \frac{(2a - 3k_1)}{4} \tag{21}$$

Equation (21) shows that unionized wage is inversely related to subcontracting, k_1. With the entry of horizontal FDI, demand for formal labor increases, giving rise to formal wage (Competition effect). This high wage reduces the incentive for firm 1 to produce in-house and encourages more subcontracting (wage effect). Substituting equation (21) into equations (18) and (19), we get

$$q_1 = \frac{2a - 9k_1}{12} \tag{22}$$

$$q_2 = \frac{2a + 3k_1}{12} \tag{23}$$

Equation (23) shows that firm 2's in-house production is directly related to subcontracting by firm 1. The higher the k_1, the lower the w. The higher the q_2, the higher the q_1, and therefore the lower the q_1; this is the competition effect. Firm 1 produces in-house and subcontracts to the informal sector, substituting equations (21), (22), and (23) back into the profit expression (16) and maximizing it to determine k_1.

$$\max_{k_1} \frac{4a^2 + 84ak_1 - 9k_1(16c + 11k_1)}{144} \tag{24}$$

The equilibrium amount of subcontracting is

$$k_1^{2*} = \frac{2(7a - 12c)}{33} \tag{25}$$

Equation (25) shows an inverse relationship between administrative costs and subcontracting. The informal sector's high administrative cost,[17] i.e., c, lowers k and increasing w. Substituting equation (25) in equations (21), (22), and (23), we get

$$w^{2*} = \frac{2(a + 3c)}{11} \tag{26}$$

The wage is directly related to the administrative costs of the informal sector. The higher the c, the lower the k_1 from equation (25). Therefore, the demand for formal labor increases, increasing the unionized wage.

$$q_1^{2*} = \frac{8c - 5a}{33} \tag{27}$$

$$q_2^{2*} = \frac{3a - 2c}{11} \tag{28}$$

[17]Which could also be the minimum wage of informal labor.

Lemma 2 *When* $\frac{3a}{2} > c > \frac{5a}{8}$*, both firm produce in-house production in equilibrium, i.e.,* $q_1^{2*}, q_2^{2*} > 0$.

Proof: Refer to equations (27) and (28) for an immediate result.

By substituting equations (25), (26), (27), and (28) in the profits functions of firms 1 and 2, we get the equilibrium profits of firms 1 and 2, such as

$$\pi_1^{2*} = \frac{5a^2 - 14ac + 12c^2}{33} \tag{29}$$

$$\pi_2^{2*} = \frac{(3a - 2c)^2}{121} \tag{30}$$

By comparing the profit firm 2 earns in cases 1 and 2, i.e., $\pi_2^{2*} - \pi_2^{1*} = 0$, we determine the new prohibitive level of tariff, $t^{2*} = \frac{(7a+76c)}{121}$, since $t^{2*} < t^*$ implies firm 2 earns higher profits under FDI and the restrictive prohibitive level of tariff falls.

Lemma 3 *Unionized formal wage,* $w^{2*} > w^{1*}$ *under FDI.*

Proof: Refer to equations (11) and (26) for the wage expressions. For $t > t^{2*}$, firm 2 sets up a manufacturing plant in the domestic country instead of exporting and creates demand for formal labor. This creates competition in the labor market and increases the union's bargaining power, thus increasing unionized wage. This competition effect creates a wage effect; a high wage dampens the incentive for firm 1 to produce in-house and encourages it to subcontract more, lowering the demand for formal labor and unionized wage. The net result would depend on which effect is more substantial. Here, the competition effect dominates the wage effect.

Case 3: FDI with Foreign Technology

Union wage increases under homogeneous technology, which may discourage formal employment to some extent. However, if foreign firm brings better technology (more labor saving), wage can be depressed. In this case, we discuss how better foreign technology changes union wages and formal employment. High wages can be tackled if a foreign firm has access to better technology. Firm 1 continues to produce in-house and subcontracts to the domestic informal sector. Firm 2, however, uses its superior foreign technology and has no access to the informal sector. Both firms use different technologies and face asymmetric marginal production costs. The marginal cost of production of firm 2 falls

by a fraction α, where α is the improvement in productivity due to better technology or efficiency gap between domestic and foreign firms. So, the higher the α, implying superior labor-saving technology, and $(1-\alpha)$ is the new low per-unit labor requirement, which may reduce the formal labor demand. The model has three stages and is solved using the same approach as in case 1. Firms 1 and 2 maximize the following profit functions to obtain q_1 and q_2, respectively:

$$\pi_1 = (a - q_1 - q_2 - k_1 - w)q_1 + (a - q_1 - q_2 - k_1 - c)k_1 \tag{31}$$

$$\pi_2 = \big(a - q_1 - q_2 - k_1 - w(1-\alpha)\big)q_2 \tag{32}$$

The equilibrium values can be found as

$$q_1 = \frac{a - 3k_1 - 2w + w(1-\alpha)}{3} \tag{33}$$

$$q_2 = \frac{a - w(1-2\alpha)}{3} \tag{34}$$

Equation (33) shows that the output of firm 1 is inversely related to subcontracting, union wage, and superior foreign technology through wage. When a foreign firm uses superior technology, it becomes more competitive and captures a larger market share, so q_1 falls. Equation (34) shows that the output of firm 2 is directly related to technology through wage. The better the technology, the lower the per unit labor requirement, the lower the formal labor demand, the lower the formal wage, and therefore higher q_2.

Now we determine the unionized wage by maximizing the union's total utility, the following expression[18] to determine w

$$\max_{w} w\big(q_1 + q_2(1-\alpha)\big) \tag{35}$$

The equilibrium wage can be found as

$$w = \frac{a\big(1 + (1-\alpha)\big) - 3k_1}{4\big(1 - \alpha(1-\alpha)\big)} \tag{36}$$

The unionized wage is inversely related to superior foreign technology and subcontracting. The more the labor-saving technology firm 2 uses, the lower the per unit labor requirement and fall in formal labor demand and formal wage. Given α, a low k_1 increases w and given k_1, a high α reduces w.

[18]The formal labor demand union facing is L, where, $L = q_1 + q_2(1-\alpha)$. Formally, the union utility is $U \equiv wL = w\big(q_1 + q_2(1-\alpha)\big)$.

By substituting equation (36) back into equations (33) and (34), we get

$$q_1 = \frac{a(2 - 5\alpha(1-\alpha)) - 3k_1(3 - \alpha(5 - 4\alpha))}{12\,(1 - \alpha(1-\alpha))} \tag{37}$$

$$q_2 = \frac{a(2 + \alpha(1 + 2\alpha)) + 3k_1(1 - 2\alpha)}{12\,(1 - \alpha(1-\alpha))} \tag{38}$$

By substituting equations (36), (37), and (38) back into the profit equation (31) and maximizing the expression to determine k_1 in equilibrium. beginequation

$$k^{3*} = \frac{a\big(14 - \alpha(21 - (18 - \alpha)\alpha)\big) - 24c(1 - (1-\alpha)\alpha)^2}{33 - \alpha(42 - 33\alpha)} \tag{39}$$

The expression $k^{3*} > 0$ for $a > c$ and $0 < \alpha < 1$ and $\frac{\partial k^{3*}}{\partial \alpha} < 0$ and $\frac{\partial^2 k^{3*}}{\partial \alpha^2} < 0$. Subcontracting is inversely related to technology and administrative cost of subcontracting, i.e., the more superior technology used by firm 2, it captures a larger market share, and the market share of firm 1 falls due to the competition effect. To remain competitive, firm 1 increases subcontracting, but as firm 2 uses more labor-saving technology, the lower per unit labor requirement reduces the formal labor demand and the lower formal wage, creating a technology effect. This technology effect creates a wage effect that increases incentives for firm 1 to use more formal labor and less subcontracting[19]. This must have implications for the union wage.

Using the equilibrium value of k_1^{3*} we get the equilibrium $w^{3*}, q_1^{3*}, q_2^{3*}, \pi_1^{3*}$, and π_2^{3*} as

$$w^{3*} = \frac{a(4 - 5\alpha) + 12c\big(1 - \alpha(1-\alpha)\big)}{2\,(11 - \alpha(14 - 11\alpha))} > 0 \tag{40}$$

for $a > c$, $0 < \alpha < 1$.

Union wage increases with inward green-field FDI as firm 2, using superior technology, captures a larger market share and creates additional demand for formal labor and higher wages (competition effect). This higher wage lowers the demand for formal labor of firm 1 and lower wages (wage effect). The net impact depends on the strength of competition and wage effects. But with higher superior technology (labor-saving), the per-unit labor requirement falls, so firm 2's demand for formal labor and union wage also falls (technology effect). This increases incentives for firm 1 to hire more formal workers, which may increase formal wage (wage effect). The net impact on formal wage would depend on the strength of the technology and wage effects. Since α has a non-linear impact on k_1, the

[19]Refer to Appendix A for the expression of $\frac{\partial k^{3*}}{\partial \alpha} < 0$ and Figure 8 for the graph.

wage would also behave non-linearly with α.

$$\frac{\partial w^{3*}}{\partial \alpha} = \frac{a\big(1 - 11\alpha(8 - 5\alpha)\big) + 36\,(1 - \alpha^2)\,c}{2(11 - \alpha(14 - \alpha))^2} > 0$$

In the above expression, $\frac{\partial w^{3*}}{\partial \alpha} > 0$[20] for $36\big(1 - \alpha^2\big)c > a\big(1 - 11\alpha(8 - 5\alpha)\big)$, $a > c$ and $0 < \alpha < 1$ and $\frac{\partial^2 w^{3*}}{\partial \alpha^2} < 0$.

As firm 2 uses superior technology, it becomes more competitive and captures a larger market share and the share of firm 1 falls. Since firm 2 does not have access to the informal sector, so there is a higher increase in demand for formal labor and a rise in formal wage due to the competition effect. However, with more superior technology, the per-unit formal labor requirement falls, and the demand for formal workers and the formal wage starts falling. This critical level of technology[21], at which wage starts falling is $\alpha^{'} = \frac{3\sqrt{209a^2 - 216ac + 144c^2} + 44a}{55a - 36c}$. Still, the wage under case 3 is higher than in case 2. Equating equilibrium wages under FDI (case 2) and FDI with technology (case 3), i.e., $w^{2*} = w^{3*}$, gives the critical level of technology[22] i.e., $\alpha^* = (a + 36c)/(44a)$ at which wages in cases 2 and 3 are equal. For technology higher than the critical level α^*, w^{3*} is lower than w^{2*}.

Lemma 4 *For $\alpha < \alpha^{'}$, $\frac{\partial w^{3*}}{\partial \alpha} > 0$, competition effect dominates the technology and wage effects and results in $\frac{\partial k_1^{3*}}{\partial \alpha} > 0$ and for $\alpha > \alpha^{'}$, wage starts falling, i.e., $\frac{\partial w^{3*}}{\partial \alpha} < 0$, technology effect dominates the competition and wage effects and results in $\frac{\partial k_1^{3*}}{\partial \alpha} < 0$.*

Lemma 5 *$w^{3*} > w^{2*}$ for $\alpha < \alpha^*$ and for $\alpha > \alpha^*$, $w^{3*} < w^{2*}$.*

The equilibrium output of firms 1 and 2 are

$$q_1^{3*} = \frac{a(\alpha - 2)(5 - \alpha(5 - 2\alpha)) + 12c(1 - (1 - \alpha)\alpha)(3 - \alpha(5 - 4\alpha))}{2\big(33 - \alpha(42 - 33\alpha)\big)} \tag{41}$$

$$q_2^{3*} = \frac{a(6 - \alpha(5 - 4\alpha)) - 4c(1 - 2\alpha)(1 - (1 - \alpha)\alpha)}{2\big(11 - \alpha(14 - 11\alpha)\big)} \tag{42}$$

In equation (41) $q_1^{3*} > 0$ for $a(\alpha - 2)(5 - \alpha(5 - 2\alpha)) < 12c(1 - (1 - \alpha)\alpha)(3 - \alpha(5 - 4\alpha))$ and $\frac{\partial q_1^{3*}}{\partial \alpha} < 0$ and $\frac{\partial^2 q_1^{3*}}{\partial \alpha^2} > 0$[23]. In equation (42) $q_2^{3*} > 0$ for $a(6 - \alpha(5 - 4\alpha)) > 4c(1 -$

[20] Refer to Figure 12 for the graph.

[21] Equate $\frac{\partial w^{3*}}{\partial \alpha} = 0$ and solve for $\alpha^{'}$.

[22] Refer to Figure 4 in the simulations section for the graph.

[23] Refer to Appendix A for expression of $\frac{\partial q_1^{3*}}{\partial \alpha} < 0$ and Figure 9 for the graph.

$2\alpha)(1-(1-\alpha)\alpha)$ and $\frac{\partial q_2^{3*}}{\partial \alpha} > 0$ and $\frac{\partial^2 q_2^{3*}}{\partial \alpha^2} < 0$[24]. Firm 1's demand for formal labor q_1 falls initially as firm 2 captures a larger market share using superior technology. The higher superior technology requires low per-unit labor. Therefore, firm 2's formal labor demand $\big(q_2(1-\alpha)\big)$ and formal wage fall even with higher market share. This wage effect increases firm 1's formal labor demand with the fall in formal wage and reduces subcontracting. Similarly, Firm 2's demand for formal labor rises initially with the increase in market share, but with higher superior technology, the formal labor demand for formal labor falls.

Refer to Appendix A for equilibrium quantities of profits. These vary with a, α, and c and are positive for $a > 0, c > 0, a > c$, and $0 < \alpha < 1$. By equating the profit of firm 2 in cases 3 and 1 that is $\pi_2^{3*} - \pi_2^{1*} = 0$, we determine the prohibitive level of tariff, $t^{3*} = \frac{(1-\alpha)(a(7-23\alpha)-4c(19-\alpha(19-16\alpha)))}{11(\alpha(11\alpha-14)+11)}$.

Case 4: FDI with Foreign Technology and Subcontracting

Union wage increases with non-homogeneous superior technology. This superior technology and high formal wage may discourage formal employment for a given level of aggregate demand. However, if foreign firms using superior technology are given access to the informal sector, the formal employment and wage can be depressed. In this case, we discuss how better technology and access to the informal sector can change union wage and formal employment. Firm 1 remains the same and firm 2 has access to the informal sector. All three stages of the game remain the same as in case 1, except that now firm 2 also subcontracts strategically at stage 1, and the game is solved similarly.

Both firms will maximize the following expressions to solve for q_1 and q_2.

$$\pi_1 = (a - Q - k_2 - w)q_1 + (a - Q - k_2 - c)k_1 \tag{43}$$

$$\pi_2 = \big(a - Q - k_2 - w(1-\alpha)\big)q_2 + (a - Q - k_2 - c)k_2 \tag{44}$$

The equilibrium values can be found as

$$q_1 = \frac{a - 3k_1 + w(1-\alpha) - 2w}{3} \tag{45}$$

$$q_2 = \frac{a - 3k_2 - w(1-2\alpha)}{3} \tag{46}$$

The formal in-house outputs of firms 1 and 2 are inversely related to subcontracting to the informal sector, i.e., k_1 and k_2. Further, the in-house outputs of firms 1 and 2 are

[24]Refer to Appendix A for expression of $\frac{\partial q_2^{3*}}{\partial \alpha} > 0$ and Figure 9 for the graph.

inversely and directly related to technology, i.e., α. The higher the technology used by firm 2, the higher the market share it captures. We determine the unionized wage by maximizing the following expression:

$$\max_{w} w\big(q_1 + q_2(1-\alpha)\big) \tag{47}$$

We get wage as

$$w = \frac{a(2-\alpha) - 3k_1 - 3k_2(1-\alpha)}{4\big(1-\alpha(1-\alpha)\big)} \tag{48}$$

The unionized wage is inversely related to superior foreign technology and firm 1's subcontracting. Further, for a given level of α, the wage is inversely related to k_2, and for a given level of k_2, it is directly related to α. By substituting equation (48) back into equations (45) and (46), we get

$$q_1 = \frac{2\big(2-5\alpha(1-\alpha)\big) - 3k_1\big(3-\alpha(5-4\alpha)\big) - 3k_2(1-\alpha^2)}{12\big(1-\alpha(1-\alpha)\big)} \tag{49}$$

$$q_2 = \frac{2\big(2+\alpha(1+2\alpha)\big) + 3k_1(1-2\alpha) - 3k_2\big(3-\alpha(1-2\alpha)\big)}{12\big(1-\alpha(1-\alpha)\big)} \tag{50}$$

Firm 1's formal sector production is inversely related to technology and subcontracting by both firms. On the other hand, firm 2's formal sector production is directly related to technology, subcontracting by firm 1 (higher subcontracting by firm 1, lower formal wage, and, therefore, higher in-house production by firm 2), and inversely related to its subcontracting. Firms 1 and 2 produce in-house and subcontract, they maximize their profits functions after substituting equations (48), (49), and (50) back into equations (43) and (44) to determine k_1 and k_2 in equilibrium.

$$k_1^{4*} = \frac{a(1-\alpha)\big(14-\alpha\big(7-\alpha(4+7\alpha)\big)\big) + 4c\big(1+(1-\alpha)\alpha\big)\big(6-\alpha\big(11-(11-8\alpha)\alpha\big)\big)}{3(1-\alpha)\big(16-\alpha(16-13\alpha)\big)} \tag{51}$$

$$k_2^{4*} = \frac{a(\alpha-1)\big(14-\alpha(35-2\alpha(16-9\alpha))\big) + 4c\big(1-(1-\alpha)\alpha\big)\big(6-\alpha\big(7-\alpha(7+2\alpha)\big)\big)}{3(1-\alpha)^2\big(16-\alpha(16-13\alpha)\big)} \tag{52}$$

$k_1^{4*} > 0$ for $a > c$ and $0 < \alpha < 1$ and $\frac{\partial k_1^{4*}}{\partial \alpha} > 0$ and $\frac{\partial^2 k_1^{4*}}{\partial \alpha^2} > 0$ and $k_2^{4*} > 0$ for $(\alpha-1)\big(14-\alpha(35-2\alpha(16-9\alpha))\big) > 4c\big(1-(1-\alpha)\alpha\big)\big(6-\alpha\big(7-\alpha(7+2\alpha)\big)\big)$ and $\frac{\partial k_2^{4*}}{\partial \alpha} < 0$ and $\frac{\partial^2 k_2^{4*}}{\partial \alpha^2} < 0$. Equations (51) and (52) show that the subcontracting of firm 1 is directly related to technology and firm 2's subcontracting is inversely related to technology[25]. With better technology (labor-saving), firm 2 needs less labor, and the need

[25]Refer to Appendix A for expressions and Figure 11 for graphs of $\frac{\partial k_1^{4*}}{\partial \alpha} > 0$ and $\frac{\partial k_2^{4*}}{\partial \alpha} < 0$.

for subcontracting may fall, so after a certain level of technology, the foreign firm stops subcontracting. On the other hand, firm 1 subcontracts more to remain competitive.

By using the equilibrium values of k_1^{4*} and k_2^{4*} we get equilibrium values of $w^{4*}, q_1^{4*}, q_2^{4*}, \pi_1^{4*}$, and π_2^{4*}.

$$w^{4*} = \frac{(2-\alpha)\big(a(1-\alpha)+12c(1-\alpha(1-\alpha)\big)}{2(1-\alpha)(16-\alpha(16-13\alpha)} \tag{53}$$

$$q_1^{4*} = \frac{a\left(2+14\alpha^4-33\alpha^3+38\alpha^2-21\alpha\right)+4c\left(6-16\alpha^5+41\alpha^4-66\alpha^3+56\alpha^2-31\alpha\right)}{6(1-\alpha)\left(16-\alpha(16-13\alpha)\right)} \tag{54}$$

$$q_2^{4*} = \frac{a\left(2+25\alpha^3-12\alpha^4-26\alpha^2+11\alpha\right)+4c\left(6+\alpha+10\alpha^5-17\alpha^4+24\alpha^3-8\alpha^2\right)}{6(1-\alpha)^2\left(16-\alpha(16-13\alpha)\right)} \tag{55}$$

In the above equation (53) $w^{4*} > 0$, $\frac{\partial w^{4*}}{\partial \alpha} > 0$, and $\frac{\partial^2 w^{4*}}{\partial \alpha^2} > 0$, in equation (54) $q_1^{4*} > 0$, $\frac{\partial q_1^{4*}}{\partial \alpha} < 0$, and $\frac{\partial^2 q_1^{4*}}{\partial \alpha_2} < 0$ and in equation (55) $q_2^{4*} > 0$, and $\frac{\partial q_2^{4*}}{\partial \alpha} > 0$, and $\frac{\partial^2 q_2^{4*}}{\partial \alpha_2} > 0$, for $a > c$ and $0 < \alpha < 1$[26]. Foreign firms with better technology and access to the informal sector may become very competitive, capture a large market share, and dominate formal production. On the other hand, domestic firms may quit formal in-house production at some critical level of technology and remain in the informal sector.

There may exist two critical level[27] of technologies such as α^{**} at which firm 2 stops subcontracting, i.e., $k_2^{4*} = 0$ and α'' at which firm 1 exits the formal in-house production. There are three possible cases of how firms undertake production such as:

- **Case 4:** If the level of technology used is $0 < \alpha < \alpha^{**} < \alpha''$.
- **Case 3:** If the level of technology used is $\alpha^{**} < \alpha < \alpha''$ and $\alpha'' < \alpha < \alpha^{**}$.
- **Special case:** If the level of technology used is $\alpha^{**} < \alpha'' < \alpha$ or $\alpha'' < \alpha^{**} < \alpha$, at this level firm 1 quits the formal in-house production and firm 2 stops subcontracting.

Special Case: $\alpha^{**} < \alpha'' < \alpha$ or $\alpha'' < \alpha^{**} < \alpha$

We solve this third special option, in which a domestic firm continues in the informal sector while a foreign firm undertakes only in-house formal production.

[26]Refer to appendix A (i) for expressions of $\frac{\partial w^{4*}}{\partial \alpha} > 0$, $\frac{\partial q_1^{4*}}{\partial \alpha} < 0$, and $\frac{\partial q_2^{4*}}{\partial \alpha} > 0$ and Figures 10 and 12 for the graphs, and (ii) for equilibrium profit expressions.

[27]By equating $k_2^{4*} = 0$ and $q_1^{4*} = 0$ and solve for α^{**} and α''.

Firm 2 maximizes its profit function to solve for q_2

$$\pi_2 = \big(a - q_2 - k_1 - w(1-\alpha)\big)q_2 \tag{56}$$

The equilibrium values can be found as

$$q_2 = \frac{1}{2}(a - k_1 - (1-\alpha)w) \tag{57}$$

The formal in-house production is inversely related to subcontracting by firm 1 and directly related to technology. The union maximizes its utility to solve for wage:

$$\max_w wq_2(1-\alpha)$$

$$w = \frac{a - k1}{2(1-\alpha)} \tag{58}$$

The union wage directly relates to superior foreign technology and inversely to firm 1's subcontracting. By substituting equation (58) back into equation (57), and we get

$$q_2 = \frac{1}{4}(a - k_1) \tag{59}$$

Given the market demand, higher subcontracting by firm 1 reduces formal production by firm 2. By substituting equation (59) back into the profit function of firm 1 to determine k_1 in equilibrium

$$\max_{k_1} \pi_1 = \big(a - q_2 - k_1 - c\big)k_1$$

$$k_1^{**} = \frac{3a - 4c}{6} \tag{60}$$

Subcontracting is inversely related to the administrative cost of subcontracting. By substituting equation (60) into equations (58) and (59) to get equilibrium wage and formal output:

$$w^{**} = \frac{3a + 4c}{12(1-\alpha)} \tag{61}$$

$$q_2^{**} = \frac{3a + 4c}{24} \tag{62}$$

$$\frac{\partial w^{**}}{\partial \alpha} = \frac{10}{12(1-\alpha)^2} > 0 \tag{63}$$

In the above equations $w^{**} > 0$, $\frac{\partial w^{**}}{\partial \alpha} > 0$, and $\frac{\partial^2 w^{**}}{\partial \alpha^2} > 0$[28], and $q_2^{**} > 0$ for $a > c$ and $0 < \alpha < 1$. When a foreign firm sets up a manufacturing plant in the host country (greenfield FDI) with its superior production technology and access to the informal sector, it becomes very competitive and captures a huge market share. With better technology,

[28]Refer to Appendix A for the expression of $\frac{\partial w^{**}}{\partial \alpha} > 0$ and the Figure 13 for the graph.

per-unit labor requirement falls, and firm 2 stops subcontracting and only undertakes in-house formal production, resulting in an increase in union wage with better technology. Firm 1 can not withstand this competition, so it quits formal in-house production and continues informal subcontracting to remain competitive. So, a foreign firm undertakes complete formal in-house production, and a domestic firm remains in complete informal sector production. In this kind of setup, the equilibrium formal production undertaken by a foreign firm is a function of the market strength and the subcontracting cost. The higher the subcontracting cost, the smaller the domestic market catered through informal production by firm 1, and the higher the formal production undertaken by firm 2. Our results vary from the results of Cao and Mukherjee (2018), where they find that a foreign firm acts as a monopolist in the host country in the presence of industry-wide and firm-specific labor unions if the host country's labor union sets wages higher than the foreign firm. In our model, because of the domestic firm's subcontracting to the informal sector, the foreign firm does not turn into a monopolist despite rising union wage.

In the next section, we compare cases, draw Propositions, and carry out mathematical simulations since it is difficult to determine the signs of the equilibrium quantities analytically and compare the results.

3 Simulations and Propositions

In this section, we compare all the above cases and draw Propositions. Analytically, it is very difficult to compare the equilibrium expressions of wage, formal output, and informal output across cases, so we carry out software simulations to prove the Propositions.

Proposition 1 *Formal wage increases with inward green-field FDI, i.e., (i)* $w^{2*} > w^{1*}$*, under homogeneous technology, (ii) increases further when FDI is accompanied by superior foreign technology* $w^{3*} > w^{2*} > w^{1*}$ *for* $\alpha < \alpha^*$*, and (iii)* $w^{1*} < w^{4*} < w^{2*} < w^{3*}$ *for* $\alpha < \alpha^{**}$ *when FDI is accompanied by foreign technology and access to the informal sector.*

Proof: The proof directly follows from the Lemmas 3, 4, and 5. The expression $(w^{2*} - w^{1*})$, i.e., $\frac{1}{176}(43a - 36c + 11t) > 0$ for $a > c$ and $0 < \alpha < 1$ and $(w^{3*} - w^{2*})$, i.e., $\frac{\alpha(a+36c-44\alpha a)}{22(11\alpha^2-14\alpha+11)} > 0$ for $a > c$, $0 < \alpha < \alpha^*$. The expression $(w^{4*} - w^{3*}) < 0$ for

$\frac{(2-\alpha)\left(a(1-\alpha)+12c\left(1-\alpha(1-\alpha)\right)\right.}{(1-\alpha)\left(16-\alpha(16-13\alpha^2)\right)} < \frac{-a(4-5\alpha)-12c\left(1-\alpha(1-\alpha)\right)}{\left(11-\alpha(14-11\alpha)\right)}$ and $\alpha < \alpha^{**}$.

Based on the assumption that one unit of labor input produces one unit of output, irrespective of the formal or informal sector. So total formal employment under various cases is measured by $Q^{1*} = q^{1*}$, $Q^{2}* = q_1^{2*} + q_2^{2*}$, $Q^{3}* = q_1^{3*} + q_2^{3*}(1-\alpha)$, and $Q^{4}* = q_1^{4*} + q_2^{4*}(1-\alpha)$.

Proposition 2 *Formal employment increases under inward green-field FDI, i.e., (i) $Q^{2*} > Q^{1*}$ under homogeneous technology, (ii) Q^{3*} is monotonically falling and lower than the case of homogeneous technology but greater than the level under imports, i.e., $Q^{1*} < Q^{3*} < Q^{2*}$, and (iii) for $\alpha < \alpha^{**}$, $Q^{1*} < Q^{4*} < Q^{3*} < Q^{2*}$, when FDI is accompanied by technology and access to the informal sector.*

Proof: The expressions $(Q^{3*} - Q^{1*}) > 0$, for $\frac{\left(1-(1-\alpha)\alpha\right)\left(a(4-5\alpha)+12c(1-(1-\alpha)\alpha)\right)}{33\alpha^2-42\alpha+33} > 1/24(12c - a - t)$ and $(Q^{2*} - Q^{3*}) > 0$ for $\frac{\alpha(a(43-55(1-\alpha)\alpha)+12(8-11\alpha(2-(2-\alpha)\alpha))c)}{33(11-\alpha(14-11\alpha))} > 0$ for $a > c$ and $0 < \alpha < 1$, implying $Q^{2*} > Q^{1*}$. The expression $(Q^{3*} - Q^{4*}) > 0$ for $\frac{a(4-5\alpha)+12c(1-(1-\alpha)\alpha)}{11-\alpha(14-11\alpha)} > \frac{(2-\alpha)(a(1-\alpha)+12c(1-(1-\alpha)\alpha))}{(1-\alpha)(16-\alpha(16-13\alpha))}$ for $\alpha < \alpha^{**}$.

Proposition 3 *Subcontracting to the informal sector falls with the inflow of green-field FDI, i.e., $k^{2*} < k^{1*}$ under homogeneous technology, (ii) it increases under non-homogeneous technology, i.e., $k^{2*} < k^{3*} < k^{1*}$, (iii) increases further with more labor-saving technology and access to the informal sector, i.e., $k^{2*} < k^{3*} < k^{4*} < k^{1*}$ for $\alpha < \alpha^{**}$, and (iv) $k_2^{4*} = 0$ and $k_1^{4*} > 0$ at $\alpha = \alpha^{**}$.*

Proof: The expression $(k^{1*} - k^{4*}) > 0$ for $\left(\frac{16((\alpha-1)\alpha+1)(\alpha(\alpha(\alpha(8\alpha-17)+29)-24)+12)c-a(\alpha-1)(\alpha(\alpha(\alpha(28\alpha-5)-27)+64)-32)}{(\alpha-1)^2(\alpha(13\alpha-16)+16)} + 5t\right) > 12c$ and $\alpha < \alpha^{**}$. The expressions $(k^{3*} - k^{2*}) > 0$ for $24(8 - 11\alpha^3 + 22\alpha^2 - 22\alpha)c > a(11\alpha^2 - 44\alpha + 35)$ and $(k^{4*} - k^{3*}) > 0$ for $a(14+18\alpha^4-50\alpha^3+67\alpha^2-49\alpha) > 4(6+2\alpha^5+5\alpha^4-12\alpha^3+20\alpha^2-13\alpha)c$ and $\alpha < \alpha^{**}$.

Proposition 4 *The prohibitive level of tariff to attract FDI declines consistently if FDI is associated with superior foreign technology and access to the informal sector for subcontracting, i.e., $t^{3*} < t^{2*} < t^{*}$.*

Proof: The expression for $(t^* - t^{2*})$, i.e., $\frac{16}{121}(3a - 2c) > 0$ for $a > c$ and the expression for $(t^{3*} - t^{2*})$, i.e., $\frac{8\alpha(a(29-22\alpha)-88(1-\alpha)\alpha c+76c)}{121(11-\alpha(14-11\alpha))} > 0$ for $a > c$ and $0 < \alpha < 1$.

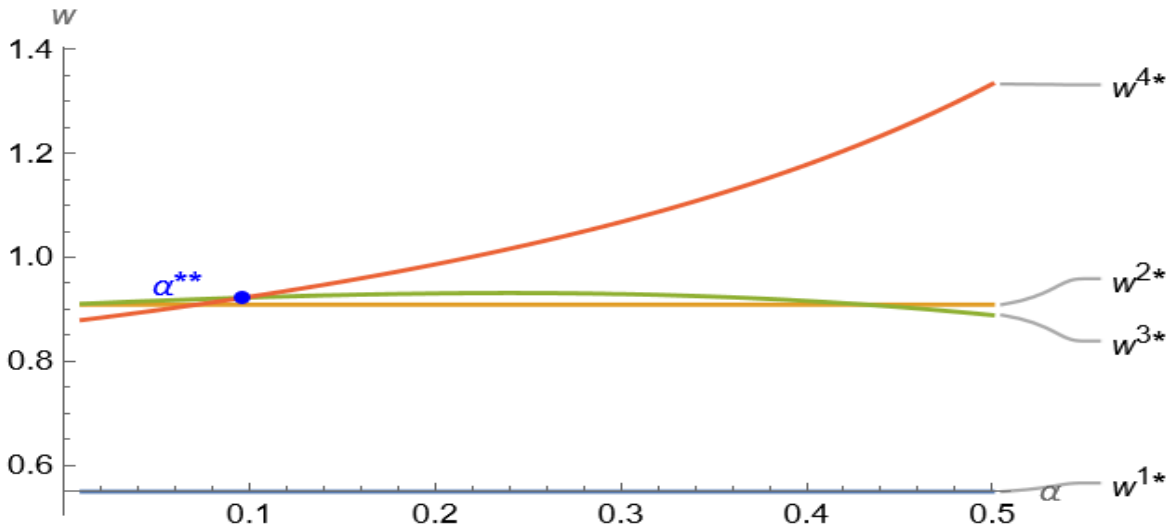
We conduct mathematical simulations in the next section to prove the above Propositions.

3.1 Simulations

In this section, we carry out mathematical numerical simulations to prove the Propositions in the above section because the signs and size of the equilibrium quantities in the theory section are not tractable and difficult to compare across sections. Following the linear demand condition $a > w > c$, we normalize $c = 1$ and assume $a = 2$ to conduct all simulations.

Proof Proposition 1: We find the rising wage under homogeneous technology using mathematical numerical simulation (refer to Table 1), i.e., $w^{2*} > w^{1*}$ for $0 \leq \alpha < 1$ and $w^{3*} > w^{2*}$ for $0.00 < \alpha \leq 0.431$ under different technologies. Access to the informal sector results in a fall in wage, i.e., $w^{4*} < w^{3*}$ for $0 < \alpha \leq 0.096$. When foreign firms enter with homogeneous technology, wage is higher than the case of importing. If the level of foreign technology improves, wage rises and falls after the level of technology α^* ($\alpha^* = 0.431$). If the foreign firm with improved technology is given access to the informal sector, wage drops and gets equal to w^{3*} at $\alpha^{**} = .096$, and the foreign firm does not use the informal labor after this level of technology.

Formal wage increases with inward FDI inflows as demand for formal labor increases, refer to Figure 1, graph w^{2*}, competition effect dominates the wage effect. When FDI is accompanied by labor-saving technology, per-unit labor requirement declines as technology upgrades. Still, formal wage increases due to rising labor demand as firm 2 becomes more competitive and captures a large market share, $w^{3*} > w^{2*}$ for $\alpha < \alpha^* = 0.431$, and competition effect dominates both wage and technology effects. For $\alpha > \alpha^*$, a more labor saving technology, $w^{3*} < w^{2*}$. As shown in Figure 1, w^{3*} cuts w^{2*} from above at α^*, resulting in a inverted U-shaped curve. When the foreign firm uses foreign technology and has access to the informal sector, for $\alpha < \alpha^{**} = .096$, $w^{1*} < w^{4*} < w^{2*} < w^{3*}$ since firm 2 also strategically subcontracts. Therefore, the demand for formal labor is low. For $\alpha > \alpha^{**} = .096$, per-unit labor requirement falls, and firm 2 quits subcontracting. Formal wage monotonically increases as shown by graph w^{4*}. Table 1 gives simulated



Source: Author's simulation

Figure 1: Equilibrium Unionized Wage under FDI, FDI with Technology, and FDI with Technology and Subcontracting
Note: α^{**} is the level of technology at which $w^{3*} = w^{4*}$

wages under all four cases at different levels of technology.

Proof Proposition 2: The mathematical numerical simulation (refer to Table 4) gives higher formal employment under FDI with homogeneous technology than the decision to import, i.e., $Q^{2*} > Q^{1*}$, however formal employment starts falling when FDI is associated with foreign technology i.e., $Q^{3*} < Q^{2*}$ and monotonically falls as technology improves and tends to converge to the formal employment under the decision to import as technology becomes quite superior (more labor-saving). When the foreign firm with improved technology is given access to the informal sector, formal employment falls, i.e., $Q^{1*} < Q^{4*} < Q^{3*} < Q^{2*}$ for $0 < \alpha < .096$ as firm 2 also subcontracts to the informal sector and $Q^{4*} = Q^{3*}$ at $\alpha^{**} = 0.096$.

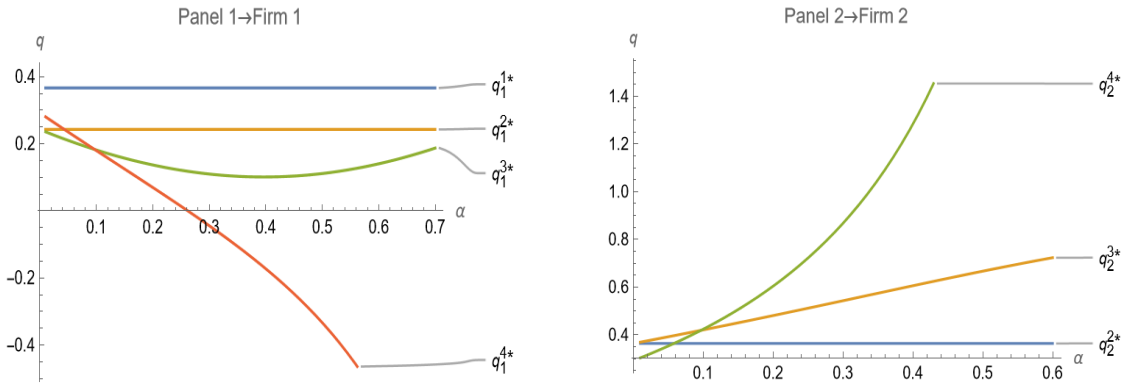
The market share of firm 1 is higher under importing than FDI as shown in Figure 2, panel 1, by graphs of q_1^{1*} and q_1^{2*}. A foreign firm captures a larger market share when FDI is associated with technology and the market share of firm 1 falls. Figure 2, panels 1 and 2, graphs q_1^{3*} and q_2^{3*} show the falling and rising market shares of firms 1 and 2. Firm 1 increases subcontracting to remain competitive; refer to Figure 5, the rising part of the graph k^{3*}. Figure 3, panel 2, graph $q_2^{3*}(1-\alpha)$ shows that firm 2 demands more formal labor as α rises initially. But with higher α, the per-unit labor requirement and demand for formal labor fall, leading to a fall in formal wage and a rise in formal labor

α	W^{1*}	W^{2*}	W^{3*}	W^{4*}
.04	0.549375	0.909091	0.915086	0.893296
.08	0.549375	0.909091	0.920405	0.913325
.12	0.549375	0.909091	0.924882	0.935333
.16	0.549375	0.909091	0.928331	0.959606
.20	0.549375	0.909091	0.930556	0.986486
.24	0.549375	0.909091	0.931348	1.01639
.28	0.549375	0.909091	0.9305	1.04981
.32	0.549375	0.909091	0.927809	1.08736
.36	0.549375	0.909091	0.923094	1.1298
.40	0.549375	0.909091	0.916201	1.17808
.44	0.549375	0.909091	0.907025	1.23343
.48	0.549375	0.909091	0.895515	1.29745

Table 1: Formal Equilibrium Wage under Exporting, FDI, FDI with Technology, FDI with Technology and Subcontracting

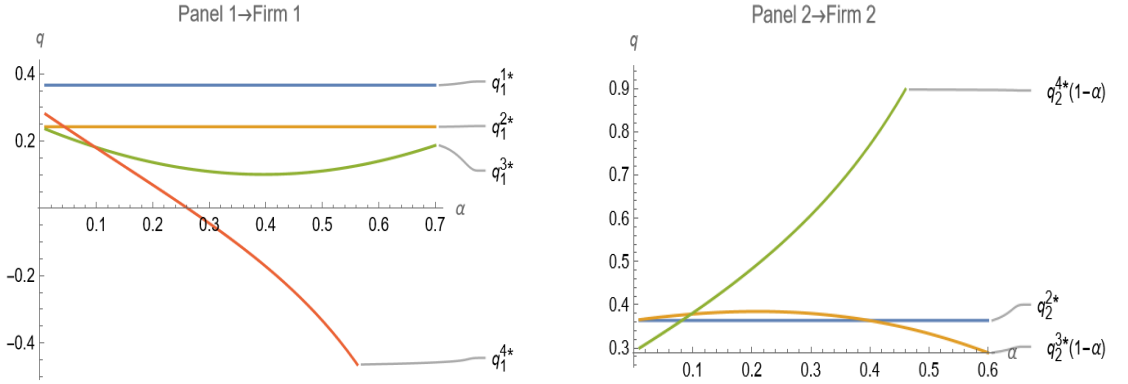
demand of firm 1 as shown in the graph of q_2^{3*} in panel 1, and fall in subcontracting, refer to Figure 5, the graph of k^{3*}. Figure 3, panel 1, graph q_1^{4*} shows that when a foreign firm is also given access to the informal sector, demand for formal labor continuously falls for firm 1. Panel 2, Figure 3, graph $q_2^{4*}(1-\alpha)$ shows that formal labor demand rises for firm 2 as α rises but $q_2^{4*}(1-\alpha) < q_2^{3*}(1-\alpha)$ for $\alpha < \alpha^{**}$. Domestic firm 1 has a higher market share, i.e., $q_1^{4*} > q_1^{3*}$ for $\alpha < \alpha^{**}$ due to its domestic advantage, but the formal in-house production share falls as firm 2's technology upgrades and its subcontracting increases (Figure 6, graph k_1^{4*}). It quits formal production at $\alpha^{''} = 0.261$.

Figure 4 shows the total demand for formal labor under different scenarios. Assuming one unit of output is produced by one unit of labor, the total formal labor demand, Q, is the total number of units produced in equilibrium. The formal labor demand increases with the inflow of FDI, i.e., $Q^{2*} > Q^{1*}$ and this increase is dampened when FDI is associated with better technology due to the technology effect and rising formal wage, as shown by the curve Q^{3*}, which is lower than $Q^{1*} < Q^{3*} < Q^{2*}$. When the foreign firm is given access to the informal sector, it starts strategic subcontracting and has a low demand for formal labor. But as technology upgrades, the technology effect reduces the per-unit labor requirement, and its demand for formal labor falls, i.e., $Q^{1*} < Q^{4*} < Q^{3*} < Q^{2*}$ for $\alpha < \alpha^{**}$ and it quits subcontracting at $\alpha < \alpha^{**} = .096$. This implies that for a



Source: Author's simulation

Figure 2: Formal Output of Firms 1 and 2 under FDI, FDI with Technology, and FDI with Technology and Subcontracting

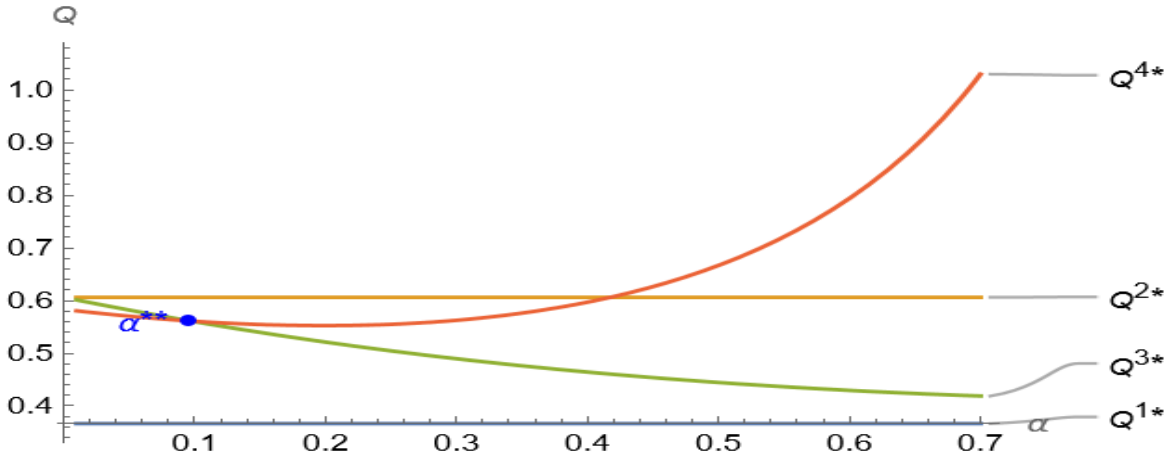


Source: Author's simulation

Figure 3: Formal Labour Demand of Firms 1 and 2 under FDI, FDI with Technology, and FDI with Technology and Subcontracting

technology-superior foreign firm, the relevant model is case 3. Tables 2, 3, and 4 below give the simulated values of the firms' 1, 2, and total formal employment under all cases at different levels of technology, i.e., $0.04 \leq \alpha \leq 0.2$.

Proof Proposition 3: Subcontracting is lower under FDI than the decision to import under homogeneous technology, i.e., $k^{2*} < k^{1*}$ for $0 < \alpha < 1$, refer to table 5. It starts increasing when FDI is accompanied by labor-saving technology, i.e., $k^{3*} > k^{2*}$ for $0 < \alpha < 0.467$ and increases further when the foreign firm using labor-saving technology is also given access to the informal sector, i.e., $k^{4*} > k^{3*} > k^{2*}$ for $0 < \alpha < 0.096$. Foreign firm exits subcontracting at $\alpha^{**} = .096$, and the domestic firm continues to use the informal sector.



Source: Author's simulation

Figure 4: Formal Employment under FDI and FDI and Subcontracting
Note: α^{**} is the level of technology at which $Q^{3*} = Q^{4*}$

α	q_1^{1*}	k^{1*}	q_1^{2*}	k^{2*}	q_1^{3*}	k^{3*}	q_1^{4*}	k_1^{4*}
.04	0.36625	0.3375	0.242424	0.121212	0.216032	0.133405	0.246287	0.110704
.06	0.36625	0.3375	0.242424	0.121212	0.203795	0.138568	0.223895	0.123683
.08	0.36625	0.3375	0.242424	0.121212	0.192205	0.143115	0.201664	0.136206
.10	0.36625	0.3375	0.242424	0.121212	0.18127	0.147051	0.179559	0.148284
.12	0.36625	0.3375	0.242424	0.121212	0.170997	0.150381	0.157545	0.159931
.14	0.36625	0.3375	0.242424	0.121212	0.161392	0.153111	0.135582	0.171162
.16	0.36625	0.3375	0.242424	0.121212	0.152463	0.155249	0.113627	0.181992
.18	0.36625	0.3375	0.242424	0.121212	0.144219	0.156801	0.0916339	0.192439
.20	0.36625	0.3375	0.242424	0.121212	0.136667	0.157778	0.0695495	0.202523

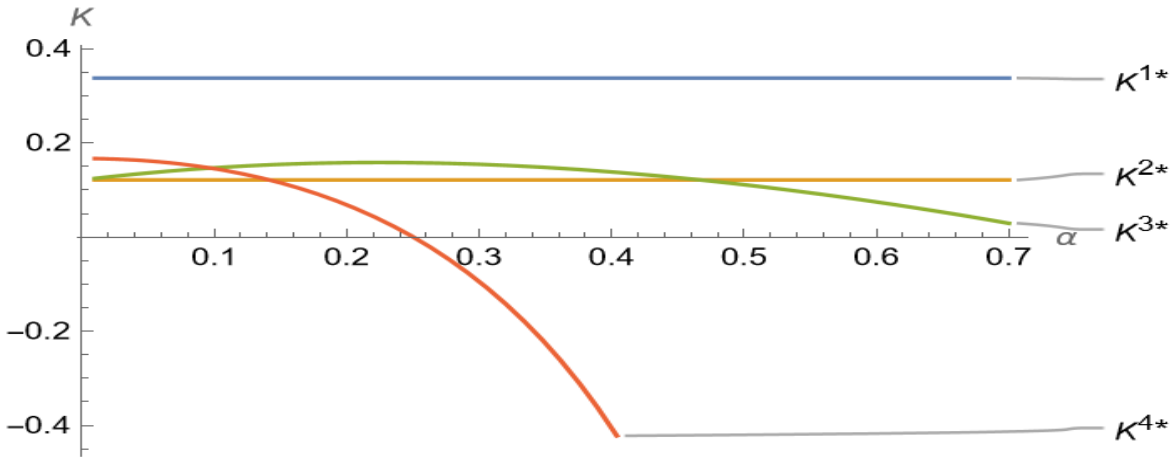
Table 2: Formal Employment and Subcontracting of Firm 1 under Exporting, FDI, FDI with Technology, FDI with Technology and Subcontracting

α	q_2^{2*}	q_2^{3*}	q_2^{4*}	k_2^{4*}
.04	0.363636	0.370599	0.326375	0.0527483
.06	0.363636	0.373588	0.344202	0.0355913
.08	0.363636	0.376237	0.362406	0.0170161
.10	0.363636	0.378538	0.381041	00
.12	0.363636	0.38048	0.400163	00
.14	0.363636	0.382053	0.419836	00
.16	0.363636	0.383246	0.440129	00
.18	0.363636	0.384047	0.461117	00
.20	0.363636	0.384444	0.482883	00

Table 3: Formal Employment and Subcontracting of Firm 2 under Exporting, FDI, FDI with Technology, FDI with Technology and Subcontracting

α	Q^{1*}	Q^{2*}	Q^{3*}	Q^{4*}
.04	0.36625	0.606061	0.586631	0.572662
.06	0.36625	0.606061	0.577382	0.568097
.08	0.36625	0.606061	0.568442	0.56407
.10	0.36625	0.606061	0.559808	0.5606
.12	0.36625	0.606061	0.551476	0.557708
.14	0.36625	0.606061	0.543444	0.555418
.16	0.36625	0.606061	0.535709	0.553756
.18	0.36625	0.606061	0.528266	0.552751
.20	0.36625	0.606061	0.521111	0.552432

Table 4: Total Formal Employment under Exporting, FDI, FDI with Technology, FDI with Technology and Subcontracting

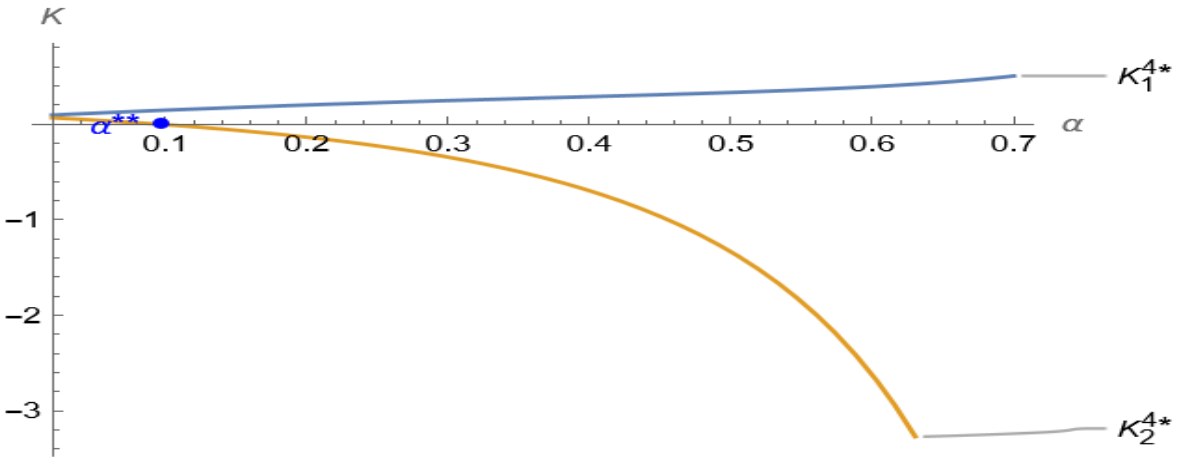


Source: Author's simulation

Figure 5: Total Informal Employment under FDI, FDI with Technology, FDI with Technology and Subcontracting

Figure 5, $k^{1*} > k^{2*}$, shows subcontracting is lower under FDI as foreign firms capture a large market share. Subcontracting by firm 1 increases when FDI is accompanied by labor-saving technology. The foreign firm becomes more competitive, captures a larger market share, and creates a high demand for formal labor and a higher formal wage. To remain competitive, firm 1 subcontracts more, and with a fall in union wage, formal labor demand increases, and subcontracting falls. The curve k^{3*} shows that total informal employment is higher under FDI with technology for $\alpha < \alpha^* = 0.471$, i.e., $k^{1*} > k^{3*} > k^{2*}$ and falls for $\alpha > \alpha^*$. When FDI is accompanied by technology and access to the informal sector, total informal employment increases as both firms subcontract, i.e., $k^{1*} > k^{4*} > k^{3*} > k^{2*}$ but firm 2 stops subcontracting as technology upgrades. So $k_2^{4*} = 0$ and $k_1^{4*} > 0$ at $\alpha^{**} = .096$, Figure 6. Table 5 below gives the total informal employment at different levels of α, and Tables 2 and 3 give firm-level informal employment.

Proof Proposition 4: Figure 7, panel 1 shows that for $\alpha = 0.5$, the prohibitive tariff level falls (shown by the intersection of profit curves) as technology improves and access to the informal sector is granted. Similarly, panel 2 shows that with more advanced technology and subcontracting, i.e., $0.5 \leq \alpha$, firm 2's profit is much higher at any tariff rate under cases 3 and 4. Therefore, firm 2 prefers FDI over trade.

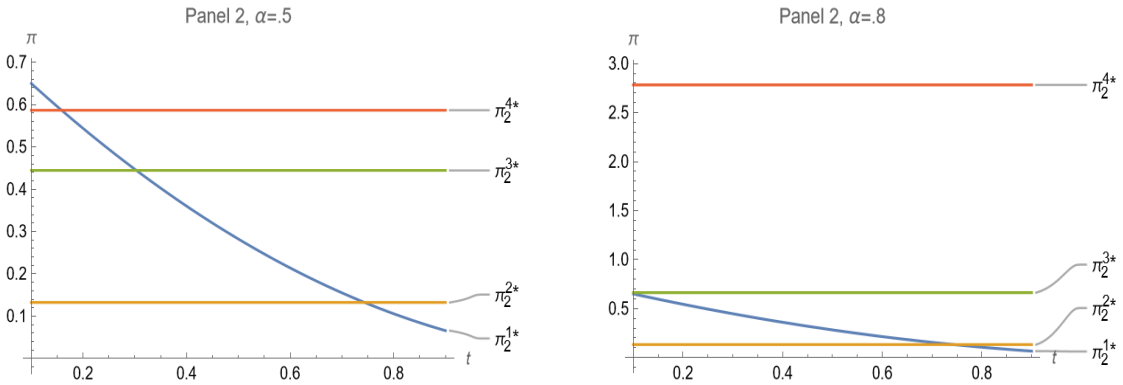


Source: Author's simulation

Figure 6: Total Informal Employment under FDI and FDI and Subcontracting
Note: α^{**} is the level of technology at which a foreign firm stops subcontracting.

α	k^{1*}	k^{2*}	k^{3*}	k^{4*}
.05	0.3375	0.121212	0.136064	0.16159
.10	0.3375	0.121212	0.147051	0.145155
.15	0.3375	0.121212	0.154253	0.115088
.20	0.3375	0.121212	0.157778	0.0682883
.25	0.3375	0.121212	0.157761	0.000451671
.30	0.3375	0.121212	0.154369	00
.35	0.3375	0.121212	0.147795	00
.40	0.3375	0.121212	0.13825	00
.45	0.3375	0.121212		00

Table 5: Total Informal Employment FDI and FDI and Subcontracting



Source: Author's simulation

Figure 7: Profit of Foreign Firm under Exports, FDI, FDI with Technology, FDI with Technology and Subcontracting

4 The Empirical Analysis

4.0.1 Econometric Specification

The international duopoly model of the previous section suggests that green-field FDI accompanied by labor-saving foreign technology and strategic subcontracting to the informal sector raises formal wage, which is limited by the level of technology. This rise in wage and labor-saving technology limits the increase in formal employment. We exploit WBES cross-sectional firm-level data for 97,624 manufacturing firms for 23 industries at two-digit groups across 155 countries to investigate whether these predictions are borne out in the data. We conduct our analysis in two consecutive steps, employing an extensive list of control variables and country- and industry-specific fixed effects at each step. By applying the ordinary least squares (OLS) regression technique to the alternative formal employment and formal wage measures, we estimate the impact of FDI and subcontracting on formal employment in the first step. In the second step, we estimate the impact of FDI on formal wage. To investigate the impact of FDI inflows on formal employment and formal wages, we consider the following econometric model equations:

$$lnFE_{ijct} = \alpha_0 + \alpha_1 FO_{ijct} + \alpha_2 FO_{ijct} \star FT_{ijct} + \alpha_3 FO_{ijct} \star FT_{ijct} \star S_{ijct} + \alpha Z_{ijct} + \gamma_j + \delta_c + \epsilon_{ijct} \tag{64}$$

$$lnW_{ijct} = \beta_0 + \beta_1 FO_{ijct} + \beta_2 FO_{ijct} \star FT_{ijct} + \beta_3 FO_{ijct} \star FT_{ijct} \star S_{ijct} + \beta X_{ijct} + \gamma_j + \delta_c + \epsilon_{ijct} \tag{65}$$

We estimate equations (64) and (65) to establish the competition, technology, and wage effects of FDI inflows. In the above equations, i represents the firm, j captures the firm's industry affiliation, c represents the country where the firm operates, and t denotes time. We use two proxy measures for the dependent variable $lnFE_{ijct}$ in equation (64). First is the log of total formal workers[29] and the second is the share of formal workers, which is the ratio of permanent workers to the total workers (permanent plus temporary [30]). The dependent variable lnW_{ijct} in equation (65) has two measures, the log of average wage[31] and the log of the total labor cost of firm i of industry j of country c for the year t. FO represents foreign ownership and is a continuous variable representing the actual percentage of the equity stake in the host country firm. FT stands for foreign technology and is a binary variable taking value one if a firm uses foreign technology and takes value zero otherwise. S represents the subcontracting and the log of total informal payment is taken as a proxy. The interaction terms $FO_{ijct} \star FT_{ijct}$ and $FO_{ijct} \star FT_{ijct} \star lnS_{ijct}$ capture the impact of a foreign firm using foreign technology and a foreign firm using foreign technology and subcontracting on formal employment and unionized wage in the host country. We also account for various firm-specific control variables represented by Z_{ijct} and X_{ijct} in equations (64) and (65). The Z_{ijct} control variables are total sales, capacity utilization, tax administration, capital intensity, quality certificate, foreign technology, and total informal payment. The X_{ijct} control variables are education level, labor regulations, corruption, training of production workers, union membership, foreign technology, and total informal payment. We also control for country-specific and industry-specific fixed effects. We can not capture firm-specific effects due to missing information on the firm identifiers in the data. Coefficient δ_c controls for heterogeneity across countries for time-invariant characteristics such as openness policies towards FDI and attractiveness factors such as infrastructure, market size, history or geography, and relatively stable factors, e.g., quality of institutions. Industry fixed effects, γ_j, control for a broad spectrum of industry-specific factors such as technological factors, union structure, size, contractability, relationship specificity of the inputs, capital intensity, etc.

[29]We take total permanent workers to represent formal workers. Permanent workers are all full-time paid employees contracted for one year or more years and have a guarantee of renewal of their employment contract and work for eight or more hours per day.

[30]Temporary workers are defined as all paid short-term employees with no guarantee of renewal of employment contract and that work for eight or more hours per day.

[31]Wage is calculated by taking the log of total labor cost divided by total permanent workers.

4.0.2 Data and Variables

We take pooled cross-sectional data from the World Bank survey data at the firm level. The WBES[32] conducts a face-to-face interview of the enterprise's owner or the managers drawn from the stratified random sample. The survey aims to gather information about a country's investment climate and help develop policies and programs that enhance economic growth and employment. The survey provides detailed firm-level information on various dimensions such as infrastructure, sales and supplies, degree of competition and innovation, business and government relations, finance, employment, productivity, land, and other related firm-level aspects. The standardization of the survey questionnaire was done in 2006, which allows for a cross-country comparison. Therefore, we restrict our analysis to the period post-2006.

The variables corruption, tax administration, license, finance, and labor regulations are measured on an ascending scale of obstacle severity from 0 to 4. Refer to Table 8 for variable definitions (discussed in Appendix A in detail). To remove the scale effect and facilitate interpretation, we divide each variable's value by 4. Similarly, the education level, measured on an ascending scale of 1 to 5, is divided by 5 to remove the scale effect. The measure of capital intensity is constructed by taking a ratio of the amount required to hypothetically purchase the machinery and equipment in use now as they are in current condition to the number of total production workers. We take a log of this measure as a proxy for the capital intensity of firms in the host country. The expected signs of coefficients of these control variables in both equations are positive for education level, labor regulations, training of production workers, union membership, firm size, total sales, capacity utilization, and quality certificate. The expected signs are negative for tax administration, licenses and permits, capital intensity, corruption, and finance.

We estimate the econometric model using the OLS technique with heteroskedasticity-corrected robust standard errors, refer to Appendix A, Table 9, for the summary statistics. The key variables of interest in our analysis are foreign ownership, the interaction term of foreign ownership and foreign technology, the interaction term of foreign ownership, and foreign technology and subcontracting. The respective coefficients are expected to be positive for α_1, β_1, and β_2 and negative for α_2, α_3, and β_3 to verify Propositions 1 and 2.

[32]Kindly refer to http://www.enterprisesurveys.org

4.0.3 Results

We start our empirical investigation of the econometric equations (64) and (65) by cleaning[33] the repeated cross-sectional data.[34]

Table 6 presents the results of the impact of inward green-field FDI on formal employment accompanied by foreign technology and subcontracting. We empirically investigate the econometric model from equation (64), taking the formal share as an outcome variable in columns 1 and 2 and the log of formal employment in columns 3 to 5. The formal share and log of formal employment are regressed against foreign ownership, the interaction of foreign technology and foreign ownership, the interaction of foreign technology, foreign ownership and subcontracting, and the control variables. We also control for country- and industry-specific fixed effects in columns 1 and 5. The variable of interest, foreign ownership shows an insignificant positive association for formal share and a positive significant association for a log of formal employment. The coefficient of the interaction term foreign ownership and foreign technology is negatively significant in columns 1 to 3 and insignificant in columns 4 and 5, implying that foreign firms using labor-saving foreign technology dampen the positive effect of FDI on demand for formal employment. Further, this impact is amplified by access to the informal sector, indicated by the coefficient of the interaction term foreign ownership, foreign technology, and subcontracting is positive and negative insignificant in columns 2 and 5. The results are consistent with our theoretical prediction that inward market-seeking greenfield FDI significantly impacts formal employment. The inflow of FDI creates formal jobs in the host country through backward and forward linkages and multiplier effects. This may suggest that rather than crowding out competing firms in the same industry, foreign firms bring new business opportunities and may also create more jobs in other industries that supply goods and services to foreign firms. The direct and interaction effects retain the predicted signs in columns 1 to 5 and column 5 controls for the country- and industry-specific fixed effects.

[33]Data is cleaned by dropping those observations for which either no data is available or where the surveyed individual answered that either "don't know" or "does not apply" or "application rejected" or "refused to respond" or "application still in process".

[34]WBES surveys are not carried out regularly. Moreover, WBES collects sensitive information on aspects of bribery and crime. As a result, the identity of the survey respondent is kept confidential. Given the respondent's anonymity and the survey year's specificity, it is unfeasible to construct a true firm-level panel structure (Gopalan et al., 2022).

Furthermore, regarding controls, all the control variables retain predicted signs and show statistically significant impact on formal employment in all the columns. The positive significant coefficient of foreign technology in columns 1 to 5 indicates better technology creates more demand for formal labor. The negative significant coefficients imply that poor access to finance and a high fixed capital-to-labor ratio reduce a firm's formal employment. The positive significant coefficients of a firm's total sales, higher capacity utilization, possession of an international quality certificate, and stringent tax administration increase formal employment.

Table 7 presents the regression results of the wage equation (65). We investigate the impact of FDI on union wage in the presence of foreign technology and subcontracting. We take two proxies for union wage, a log of the average wage and a log of total labor cost, which are regressed against foreign ownership, the interaction term of foreign technology with foreign ownership, further its interaction with subcontracting, a set of control variables and country- and industry-specific fixed effects. The regression results in columns 1 to 4 reinforce our theoretical predictions of Proposition 1, which states the positive effect of green-field FDI on union wage. The coefficients of interest are foreign ownership and its interaction terms with foreign technology; the results in columns 1 to 4 imply that green-field FDI has a positive and significant impact on formal wages, and the FDI accompanied by technology has a positive but insignificant impact on the formal wage. The impact of interaction terms of FDI, technology, and informal payment is negative and significant, implying subcontracting to the informal sector dampens the positive impact of FDI on formal wage. Advanced foreign technology and high informal costs create more demand for formal labor and increase formal wage. The positive coefficients of control variables, such as union membership, labor regulation, training of production workers, and education levels, imply that higher education, regulation, advanced training, and union membership help workers earn higher wages. The negative coefficient of corruption implies that a high level of corruption may encourage more subcontracting and low demand for formal labor and formal wage.

VARIABLES	(1) Formal Share	(2) Formal Share	(3) Log Formal Workers	(4) Log Formal Workers	(5) Log Formal Workers
Foreign Own.	9.98e-05	0.000100	0.00308***	0.00328***	0.00328***
	(9.55e-05)	(9.55e-05)	(0.000243)	(0.000609)	(0.000609)
Foreign Own.⋆ Foreign Tech.	-0.000312**	-0.000370**	-0.00106**	-0.00117	-0.00103
	(0.000147)	(0.000147)	(0.000414)	(0.00111)	(0.00110)
Foreign Own.⋆ Foreign Tech.					
⋆ Informal Cost		2.49e-05			-6.08e-05
		(2.27e-05)			(0.000240)
Foreign Tech.	0.0101*	0.0100*	0.180***	0.171***	0.171***
	(0.00577)	(0.00577)	(0.0165)	(0.0425)	(0.0425)
log Informal Cost	-0.00167**	-0.00158**		0.00698*	0.00721*
	(0.000756)	(0.000747)		(0.00387)	(0.00388)
Log Fixed Capital/Labor	-0.00986***	-0.00990***	-0.134***	-0.175***	-0.175***
	(0.00151)	(0.00151)	(0.00404)	(0.00969)	(0.00975)
Access to Finance	-0.0100	-0.0100	-0.0673***	-0.0888**	-0.0890**
	(0.00717)	(0.00718)	(0.0177)	(0.0424)	(0.0424)
Log Annual Sales	0.00463***	0.00470***	0.451***	0.464***	0.464***
	(0.00117)	(0.00117)	(0.00560)	(0.0111)	(0.0110)
Tax Administration			0.0200	0.00254	0.00292
			(0.0183)	(0.0472)	(0.0471)
Quality Certificate	0.00257	0.00244	0.337***	0.396***	0.396***
	(0.00555)	(0.00555)	(0.0169)	(0.0360)	(0.0361)
Capacity Utilization	0.000378***	0.000375***	0.00214***	0.00267***	0.00268***
	(0.000119)	(0.000119)	(0.000267)	(0.000673)	(0.000671)
Constant	1.012***	1.012***	-1.835***	-2.281***	-2.280***
	(0.144)	(0.144)	(0.252)	(0.500)	(0.499)
Observations	4,733	4,733	26,551	4,733	4,733
R-squared	0.203	0.203	0.676	0.683	0.683
Industry FE	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 6: OLS Results of Formal Employment.

	(1)	(2)	(3)	(4)
VARIABLES	Log Wage	Log Wage	Log Labor Cost	Log Labor Cost
Foreign Own.	0.00537	0.00592	0.0223***	0.0221***
	(0.00504)	(0.00569)	(0.00607)	(0.00614)
Foreign Own.⋆ Foreign Tech.	0.00514	0.00650	0.00107	0.00224
	(0.00674)	(0.00774)	(0.00955)	(0.0100)
Foreign Own.⋆ Foreign Tech.				
⋆ Informal Cost		-0.00112***		-0.000945
		(0.000433)		(0.000721)
Foreign Technology	0.0567	0.0413	0.888*	0.892*
	(0.336)	(0.350)	(0.458)	(0.459)
Log Informal Cost	0.0289*	0.0349**	0.0689**	0.0703**
	(0.0172)	(0.0177)	(0.0300)	(0.0302)
Labor Regulation	0.282	0.292	0.881**	0.891**
	(0.258)	(0.268)	(0.442)	(0.444)
Corruption	-0.180	-0.144	-0.179	-0.193
	(0.196)	(0.205)	(0.377)	(0.380)
Education Level	0.631	0.577	0.521	0.529
	(0.537)	(0.558)	(0.803)	(0.803)
Training	0.000227	0.000412	0.000958	0.00105
	(0.00198)	(0.00222)	(0.00314)	(0.00315)
Union Membership	0.00281	0.00162	0.0118**	0.0118**
	(0.00394)	(0.00394)	(0.00524)	(0.00525)
Constant	12.93***	13.58***	13.78***	13.92***
	(0.235)	(0.590)	(0.779)	(0.801)
Observations	290	290	290	290
R-squared	0.880	0.885	0.737	0.737
Country FE	YES	YES	YES	YES
Industry FE	NO	YES	YES	YES

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 7: OLS Results of Union Wage

5 Conclusion

This paper considers green-field FDI and shows the impact of labour-saving foreign technology and subcontracting to the informal sector by multinational enterprises in the host country on formal employment generation and wages in a highly regulated and unionized labour market. The general perception is that labour-saving technology may reduce labour requirements and, therefore, employment, which is unsuitable for labour-abundant countries. The government in such a country needs help to create a sizable number of formal sector jobs at reasonable wages in the absence of much-needed capital. The existing literature discussed the factors that create the rationale for FDI, such as exchange rate uncertainty and volatility, production cost advantages, large market size, supply chain logistics, environmental regulations, tax incentives, etc., and formalisation or informalization of labour. However, whether multinationals can create formal employment in the presence of an option to subcontract to the informal sector still needs to be explored. While the empirical evidence on the relationship between labour unions and FDI is mixed, the theoretical literature mainly explains that the presence of labour unions reduces the incentive for FDI. We explore and provide a positive relationship between formal employment, formal wage, foreign technology, and the informal sector in our work, thus providing a new rationale for seeking FDI.

We employ a strategic competition model to analyze the behavior of domestic and foreign firms, demonstrating that multinationals prefer FDI over exports when faced with high tariffs. Our model incorporates the use of non-homogeneous technologies, operation in a unionized formal sector, and the informal sector as an alternative production option. We show that firms strategically choose a mix of formal in-house and informal production to counter the bargaining power of centrally unionized formal workers. Our model predicts that foreign firms using labor-saving technology create demand for formal labor, which is limited by the level of technology and the wage rise. Furthermore, when a foreign firm with improved technology gains access to the informal sector, the demand for formal labor and the wage rise are further limited by the level of technology and strategic subcontracting.

We conduct mathematical simulations to support our findings, as the complexity of our research makes it challenging to establish the results analytically. Our simulations demonstrate that FDI can increase formal employment, depending on how wages are influenced by the level of technology and access to the informal sector. As technology improves, the

demand for formal labor decreases, eventually converging to the level of import decision at a critical level of technology. The speed at which employment converges to import decisions is determined by the size of aggregate demand. The level of technology and strategic subcontracting influence the wage rise, following an inverted U-shaped curve. If the level of technology is high enough and the foreign firm also strategically subcontracts, the foreign firm ceases to use the informal sector at a critical level of technology.

Further, we empirically establish the Propositions of an increase in formal employment and unionized wage using firm-level pooled cross-sectional data for 97,624 manufacturing firms across 23 industries at two-digit groups across 155 countries. The OLS results indicate that foreign technology and strategic output allocation between formal in-house production and subcontracting to the informal sector under inward green-field FDI increases formal labor demand and the unionized wage. The results are robust to alternative wage measures, control variables, and country- and industry-specific fixed effects. The rise in formal employment and wage implies few technological differences between domestic and foreign firms. Furthermore, this implies that foreign firms may not bring their highly superior technologies to the developing countries. So far, the literature has yet to pay much attention to the joint role of foreign technology and the informal sector in formal employment and wages. Our paper fills this gap in the literature.

This study suggests that developing countries' governments must invest sufficiently in research and development expenditure, technological innovation and up-gradation, and human capital to advance technological innovations and ensure better and easier availability of skills and technology to the domestic manufacturing sector to compete with foreign competition to retain and expand their market share. Furthermore, governments should strategize in such a way as to motivate and attract larger green-field FDI, which improves the formalization of workers and formal wages, both of which are conducive to economic development. The present work can be extended further by considering the framework of decentralized labor unions and liberal bilateral trade and analyzing the consumer and social welfare implications of green-field FDI.

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Appendix A

Variable	Definition	Expected Sign
Wage	Log(labor cost/total permanent workers)	+
Wage Share	Log(labor cost)	+
Formal Employment	Log(permanent workers)	+
Formal Share	Permanent workers/total workers	+
F-Ownership	Binary variable taking value 1 or 0 OR Percent of firm owned by private foreign individuals, companies and organizations	+ +
Subcontracting	Log(total informal payment)	+
Foreign Technology	Binary variable taking value 1 or 0	
Total Sales	Total annual sales in domestic and foreign markets	+
Capacity Utilization	Current output % of maximum output possible	+
Tax Administration	Scale variable taking values from 0 to 4 (Values divide by 4 to descale)	 +
Quality Certificate	Binary variable taking value 1 or 0	+
Capital Intensity	Log(machinery cost/production workers)	-
Corruption	Scale variable taking values from 0 to 4 (Values divide by 4 to descale)	 -
Finance	Scale variable taking values from 0 to 4 (Values divide by 4 to descale)	 -
Education	Scale variable taking values from 0 to 5 (Values divide by 5 to descale)	 +
Labor Regulation	Scale variable taking values from 0 to 4 (Values divide by 4 to descale)	 +
Training	% of Production workers trained	-
Union	% of Permanent workers unionized	+

Table 8: Variable definition

Variables	Obs.	Mean	Std. Dev.	Min.	Max.
Formal Workers	2151	115	380	1	7000
Labor Share	2151	115	380	1	7000
F-Ownership	2151	7.9	25.62	0	100
Total Sales	2151	86.99	27.72	0	100
Foreign Technology	2151	23.44	31.30	0	100
Corruption	2151	1.846	1.514	0	4
Subcontracting	2151	1.78	1.067	0	4.75
Capacity Utilization	2151	13.49	102.05	0	4324
Tax Administration	2151	1.613	1.434	0	4
Quality Certificate	2151	1.868	1.536	0	4
Capital Intensity	2151	1.472	1.214	0	4
Finance	2151	0.893	1.134	0	4
Education	2151	1.517	1.240	0	4
Labor Regulation	2151	1.380	1.283	0	4
Training	2151	3.49e+9	3.09e+10	0	1.00e+12

Table 9: Summary Statistics

Profit Expressions of Cases 3 and 4.

$$\pi_1^{3*} = \frac{(\alpha(23\alpha-32)+20)a^2+4(\alpha((\alpha-18)\alpha+21)-14)ac+48((\alpha-1)\alpha c+c)^2}{12(\alpha(11\alpha-14)+11)}$$

$$\pi_2^{3*} = \frac{(a(\alpha(4\alpha-5)+6)+4(2\alpha-1)((\alpha-1)\alpha+1)c)^2}{4(\alpha(11\alpha-14)+11)^2}$$

$$\pi_1^{4*} = \frac{a^2(\alpha-1)^2\left(229\alpha^4-574\alpha^3+933\alpha^2-716\alpha+356\right)-4a\left(87\alpha^7-240\alpha^6+214\alpha^5+252\alpha^4-901\alpha^3+1084\alpha^2-696\alpha+200\right)c}{12(\alpha-1)^2(13\alpha^2-16\alpha+16)^2}$$
$$+\frac{16\left(59\alpha^8-233\alpha^7+543\alpha^6-818\alpha^5+899\alpha^4-717\alpha^3+419\alpha^2-164\alpha+36\right)c^2}{12(\alpha-1)^2(13\alpha^2-16\alpha+16)^2}$$

$$\pi_2^{4*} = \frac{a^2(\alpha-1)^2\left(228\alpha^4-568\alpha^3+921\alpha^2-708\alpha+356\right)-4a\left(16\alpha^7+148\alpha^6-732\alpha^5+1647\alpha^4-2187\alpha^3+1812\alpha^2-904\alpha+200\right)c}{12(\alpha-1)^2(13\alpha^2-16\alpha+16)^2}$$
$$+\frac{(16c^2(24\alpha^8-64\alpha^7+162\alpha^6-264\alpha^5+379\alpha^4-369\alpha^3+279\alpha^2-124\alpha+36))}{(12(\alpha-1)^2(13\alpha^2-16\alpha+16)^2)}$$

First Derivative Expressions and Graphs

$$\frac{\partial k^{3*}}{\partial \alpha} = \frac{a\left(-11\alpha^4+28\alpha^3-54\alpha^2+88\alpha-35\right)-48\left(11\alpha^5-32\alpha^4+50\alpha^3-43\alpha^2+22\alpha-4\right)c}{3\left(11\alpha^2-14\alpha+11\right)^2} < 0$$

if

$$a\left(-11\alpha^4+28\alpha^3-54\alpha^2+88\alpha-35\right) < 48\left(11\alpha^5-32\alpha^4+50\alpha^3-43\alpha^2+22\alpha-4\right)c$$

$$\frac{\partial k_1^{4*}}{\partial \alpha} = \frac{a(1-\alpha)^2\left(112+91\alpha^4-224\alpha^3+363\alpha^2-236\alpha\right)}{3(\alpha-1)^2\left(13\alpha^2-16\alpha+16\right)^2} + \frac{4\left(80-208\alpha^7+943\alpha^6-2126\alpha^5+2970\alpha^4-2694\alpha^3+1609\alpha^2-548\alpha\right)c}{3(\alpha-1)^2\left(13\alpha^2-16\alpha+16\right)^2} > 0$$

if

$$a(1-\alpha)^2\left(112+91\alpha^4-224\alpha^3+363\alpha^2-236\alpha\right) > 4\left(80-208\alpha^7+943\alpha^6-2126\alpha^5+2970\alpha^4-2694\alpha^3+1609\alpha^2-548\alpha\right)c$$

$$\frac{\partial k_2^{4*}}{\partial \alpha} = \frac{a\left(112-106\alpha^5+348\alpha^4-569\alpha^3+539\alpha^2-324\alpha\right)}{3(\alpha-1)^3\left(13\alpha^2-16\alpha+16\right)^2} + \frac{4\left(80-26\alpha^7+142\alpha^6-170\alpha^5+124\alpha^4+69\alpha^3+\alpha^2-12\alpha\right)c}{3(\alpha-1)^3\left(13\alpha^2-16\alpha+16\right)^2} < 0$$

if

$$a\left(112-106\alpha^5+348\alpha^4-569\alpha^3+539\alpha^2-324\alpha\right) < 4\left(80-26\alpha^7+142\alpha^6-170\alpha^5+124\alpha^4+69\alpha^3+\alpha^2-12\alpha\right)c$$

$$\frac{\partial q_1^{3*}}{\partial \alpha} = \frac{-251 + \alpha(1210 + 528\alpha^4 - 1580\alpha^3 + 2512\alpha^2 - 2235\alpha)}{3\big(11 - \alpha(14 - 11\alpha)\big)^2} < 0$$

$$\frac{\partial q_2^{3*}}{\partial \alpha} = \frac{67 - \alpha(132 - 44\alpha^3 + 112\alpha^2 - 149\alpha)}{\big(11 - \alpha(14 - 11\alpha)\big)^2} > 0$$

$$\frac{\partial q_1^{4*}}{\partial \alpha} = \frac{4\left(416\alpha^7 - 1925\alpha^6 + 4426\alpha^5 - 6402\alpha^4 + 6042\alpha^3 - 3827\alpha^2 + 1444\alpha - 304\right)c}{6(1-\alpha)^2\left(16 - \alpha(16 - 13\alpha)\right)^2} - \frac{a(\alpha-1)^2\left(182\alpha^4 - 448\alpha^3 + 729\alpha^2 - 556\alpha + 272\right)}{6(1-\alpha)^2\left(16 - \alpha(16 - 13\alpha)\right)^2} > 0$$

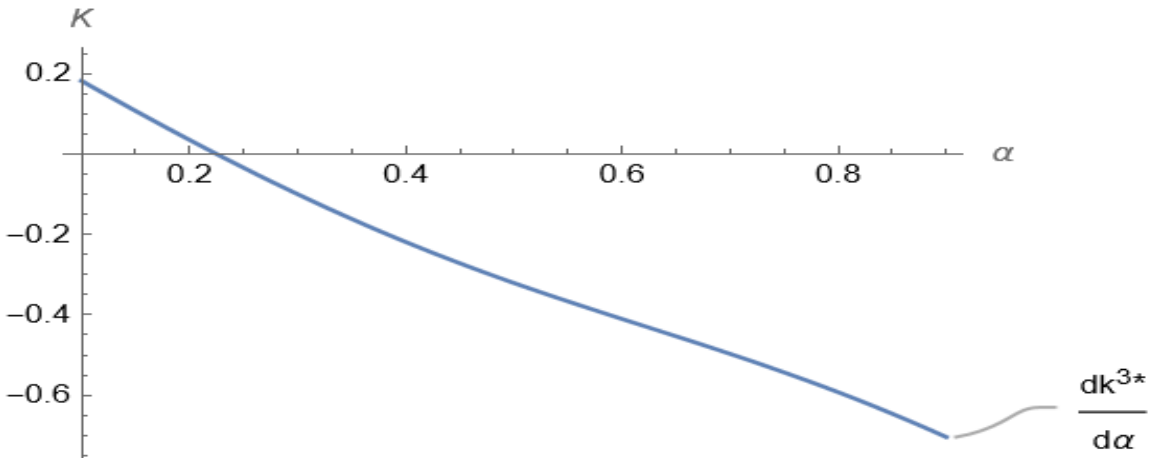
if

$$4\left(416\alpha^7 - 1925\alpha^6 + 4426\alpha^5 - 6402\alpha^4 + 6042\alpha^3 - 3827\alpha^2 + 1444\alpha - 304\right)c > a(\alpha-1)^2\left(182\alpha^4 - 448\alpha^3 + 729\alpha^2 - 556\alpha + 272\right)$$

$$\frac{\partial q_2^{4*}}{\partial \alpha} = \frac{880 - \alpha(2172 + 260\alpha^6 - 1420\alpha^5 + 3223\alpha^4 - 5137\alpha^3 + 5269\alpha^2 - 4319\alpha)}{3(1-\alpha)^3\left(16 - \alpha(16 - 13\alpha)\right)^2} > 0$$

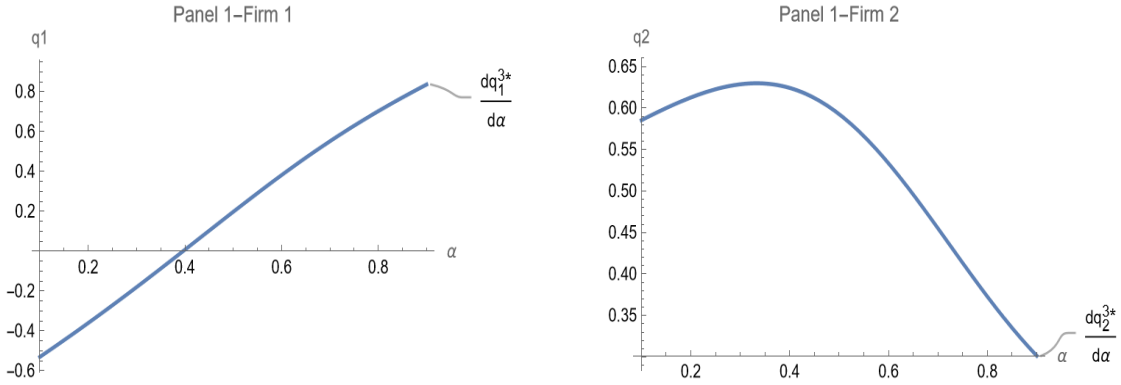
$$\frac{\partial w^{3*}}{\partial \alpha} = \frac{36\left(1-\alpha^2\right)c + a\big(1 - \alpha(88 - 55\alpha)\big)}{2\big(11 - \alpha(14 - 11\alpha)\big)^2} > 0$$

$$\frac{\partial w^{4*}}{\partial \alpha} = \frac{a(1-\alpha)^2\big(16 - \alpha(52 - 13\alpha)\big) + 12c\big(16 - \alpha(20 - 10\alpha^3 + 14\alpha^2 - 21\alpha\big)}{2(1-\alpha)^2\big(16 - \alpha(16 - 13\alpha)\big)^2} > 0$$



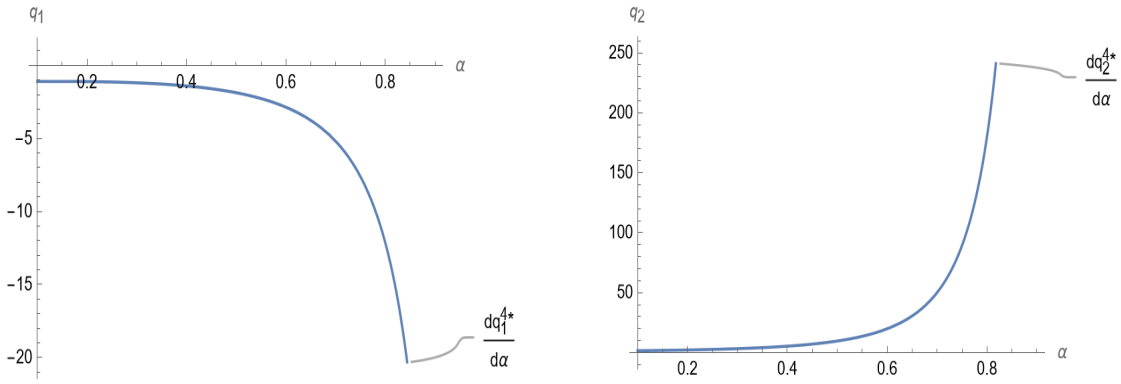
Source: Author's simulation

Figure 8: First Derivative of Subcontracting in Case 3



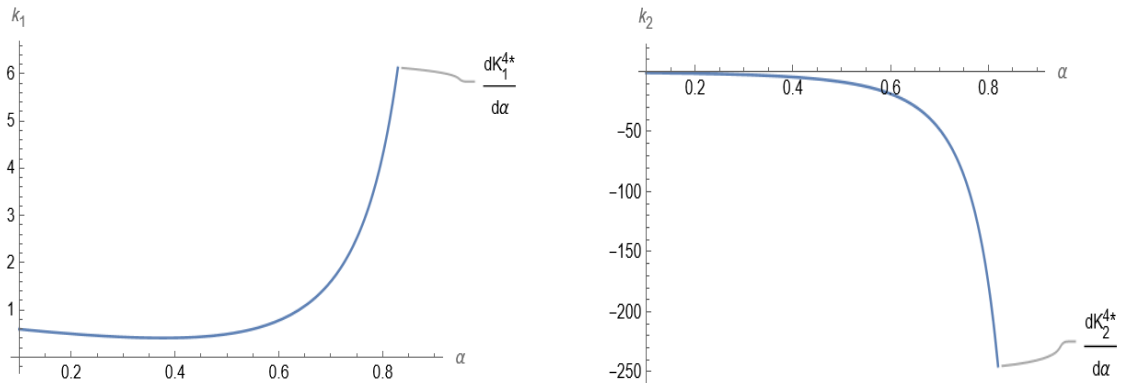
Source: Author's simulation

Figure 9: First Derivative of Formal Output of Firms 1 and 2 in Case 3



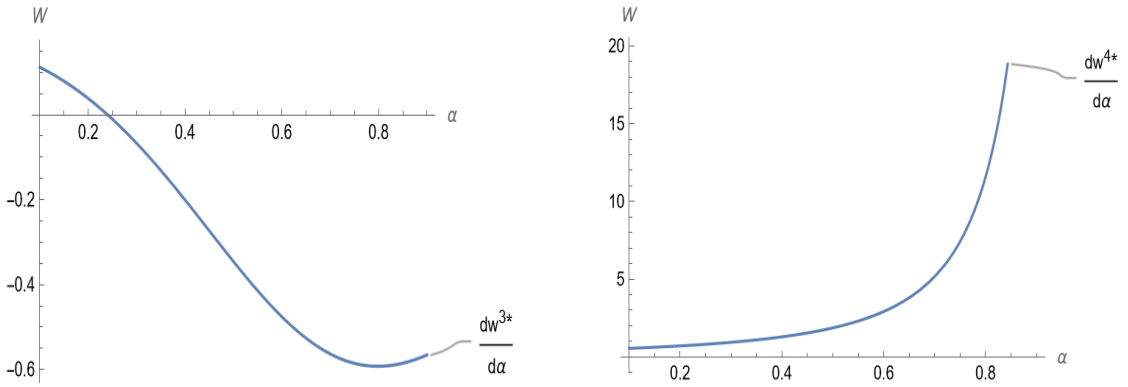
Source: Author's simulation

Figure 10: First Derivative of Formal Output of Firms 1 and 2 in Case 4



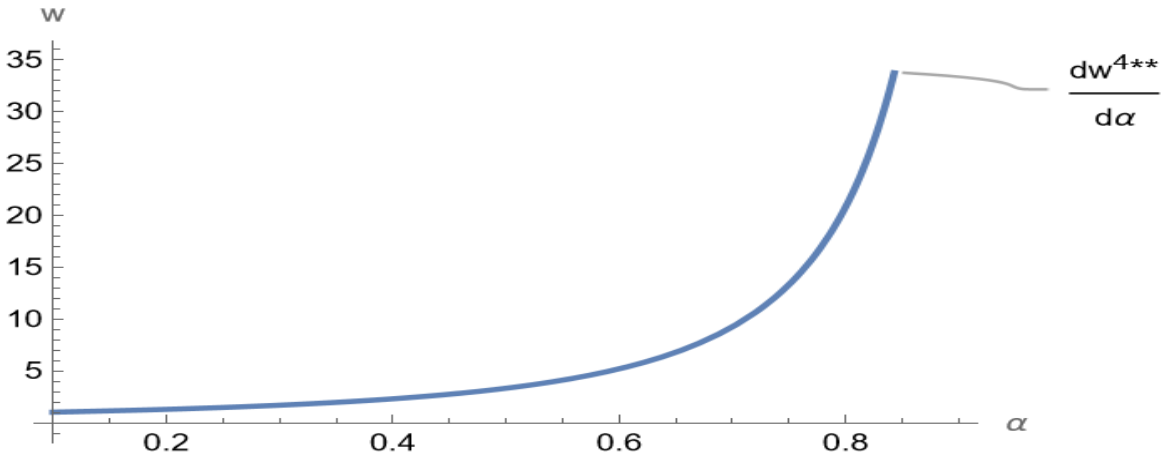
Source: Author's simulation

Figure 11: First Derivative of Subcontracting of Firms 1 and 2 in Case 4



Source: Author's simulation

Figure 12: First Derivative of Wage in Cases 3 and 4



Source: Author's simulation

Figure 13: First Derivative of Wage in Special Case of Case 4

Mathematica Codes and Output for all Four Cases of the Theory Model:

Case 1:

Stage 3: Solve for q1 and q2

$p = a - Q;$
$Q = \text{q1} + \text{q2} + \text{k1};$
$\text{P11} = (a - Q - w)\text{q1} + (a - Q - c)\text{k1}$
$\text{P21} = (a - Q - t)\text{q2}$
$\text{DP11} = D[\text{P11}, \text{q1}]$
$\text{DP21} = D[\text{P21}, \text{q2}]$
$\text{SQ1} = \text{FullSimplify}[\text{Solve}[\text{DP11}==0\&\&\text{DP21}==0, \{\text{q1}, \text{q2}\}]]$

$\text{k1}(a - c - \text{k1} - \text{q1} - \text{q2}) + \text{q1}(a - \text{k1} - \text{q1} - \text{q2} - w)$

$\text{q2}(a - \text{k1} - \text{q1} - \text{q2} - t)$

$a - 2\text{k1} - 2\text{q1} - \text{q2} - w$

$a - \text{k1} - \text{q1} - 2\text{q2} - t$

$\left\{\left\{\text{q1} \to \frac{1}{3}(a - 3\text{k1} + t - 2w), \text{q2} \to \frac{1}{3}(a - 2t + w)\right\}\right\}$

Stage 2: Solve for union wage

$\text{q1} = \frac{1}{3}(a - 3\text{k1} - 2w + t);$
$\text{q2} = \frac{1}{3}(a - 2t + w);$
$\text{U1} = w(\text{q1});$
$\text{DU1} = D[\text{U1}, w];$
$\text{SU1} = \text{FullSimplify}[\text{Solve}[\text{DU1}==0, w]]$

$\left\{\left\{w \to \frac{1}{4}(a - 3\text{k1} + t)\right\}\right\}$

Substitute wage back into q1 and q2

$w = \frac{1}{4}(a - 3\text{k1} + t);$
$\text{qw1} = \text{Simplify}\left[\frac{1}{3}(a - 3\text{k1} - 2w + t)\right]$

$\mathbf{qw2 = Simplify\left[\frac{1}{3}(a - 2t + w)\right]}$

$\frac{1}{6}(a - 3\text{k1} + t)$

$\frac{1}{12}(5a - 3\text{k1} - 7t)$

Stage 1: Solve for k1(Subcontracting by firm 1)

$\mathbf{w = \frac{1}{4}(a - 3k1 + t);}$
$\mathbf{qw1 = \frac{1}{6}(a - 3k1 + t);}$
$\mathbf{qw2 = \frac{1}{12}(5a - 3k1 - 7t);}$
$\mathbf{p = a - Q;}$
$\mathbf{Q = qw1 + qw2 + k1;}$
$\mathbf{PW11 = (a - Q - w)qw1 + (a - Q - c)k1;}$
$\mathbf{PW21 = (a - Q - t)qw2;}$
$\mathbf{DPW11 = D[PW11, k1];}$
$\mathbf{SQW1 = FullSimplify[Solve[DPW11==0, k1]]}$

$\left\{\left\{\text{k1} \to \frac{1}{12}(5a - 12c + 5t)\right\}\right\}$

Solve for q1, q2, w, and profits of firms 1 and 2

$\mathbf{p = a - Q;}$
$\mathbf{Q = qw1 + qw2 + k1;}$
$\mathbf{k1 = \frac{1}{12}(5a - 12c + 5t);}$
$\mathbf{w = FullSimplify\left[\frac{1}{4}(a - 3k1 + t)\right]}$
$\mathbf{qw1 = FullSimplify\left[\frac{1}{6}(a - 3k1 + t)\right]}$
$\mathbf{qw2 = FullSimplify\left[\frac{1}{12}(5a - 3k1 - 7t)\right]}$
$\mathbf{PWF11 = FullSimplify[(a - Q - w)qw1 + (a - Q - c)k1]}$
$\mathbf{PWF21 = FullSimplify[(a - Q - t)qw2]}$

$\frac{1}{16}(-a + 12c - t)$

$\frac{1}{24}(-a + 12c - t)$

$\frac{1}{16}(5a + 4c - 11t)$

$\frac{1}{96}\left(11a^2 - 40ac + 48c^2 + 22at - 40ct + 11t^2\right)$

$\frac{1}{256}(5a + 4c - 11t)^2$

Case 2:

Stage 3: Solve for q1 and q2

$\boldsymbol{p = a - Q;}$
$\boldsymbol{Q = \text{q12} + \text{q22} + \text{k12};}$
$\boldsymbol{\text{P12} = (a - Q - \text{w2})\text{q12} + (a - Q - c)\text{k12}}$
$\boldsymbol{\text{P22} = (a - Q - \text{w2})\text{q22}}$
$\boldsymbol{\text{DP12} = D[\text{P12}, \text{q12}]}$
$\boldsymbol{\text{DP22} = D[\text{P22}, \text{q22}]}$
$\boldsymbol{\text{SQ1} = \text{FullSimplify}[\text{Solve}[\text{DP12}==0\&\&\text{DP22}==0, \{\text{q12}, \text{q22}\}]]}$

$\text{k12}(a - c - \text{k12} - \text{q12} - \text{q22}) + \text{q12}(a - \text{k12} - \text{q12} - \text{q22} - \text{w2})$

$\text{q22}(a - \text{k12} - \text{q12} - \text{q22} - \text{w2})$

$a - 2\text{k12} - 2\text{q12} - \text{q22} - \text{w2}$

$a - \text{k12} - \text{q12} - 2\text{q22} - \text{w2}$

$\left\{\left\{\text{q12} \to \frac{1}{3}(a - 3\text{k12} - \text{w2}), \text{q22} \to \frac{a - \text{w2}}{3}\right\}\right\}$

Substitute q1 and q2 in the wage function to solve for union wage

Stage 2: Solve for union wage

$\boldsymbol{\text{q12} = \frac{1}{3}(a - 3\text{k12} - \text{w2});}$
$\boldsymbol{\text{q22} = \frac{a - \text{w2}}{3};}$
$\boldsymbol{\text{U12} = \text{w2}(\text{q12} + \text{q22});}$
$\boldsymbol{\text{DU12} = D[\text{U12}, \text{w2}];}$
$\boldsymbol{\text{SU12} = \text{FullSimplify}[\text{Solve}[\text{DU12}==0, \text{w2}]]}$

$\left\{\left\{\text{w2} \to \frac{1}{4}(2a - 3\text{k12})\right\}\right\}$

Substitute wage back into the q1 and q2

$\boldsymbol{\text{w2} = \frac{1}{4}(2a - 3\text{k12});}$
$\boldsymbol{\text{qw12} = \text{Simplify}\left[\frac{1}{3}(a - 3\text{k12} - \text{w2})\right]}$
$\boldsymbol{\text{qw22} = \text{Simplify}\left[\frac{1}{3}(a - \text{w2})\right]}$

$\frac{a}{6} - \frac{3\text{k12}}{4}$

$\frac{a}{6} + \frac{\text{k12}}{4}$

Substitute q1, q2, and w back into profit functions and solve for k1

Stage 1: Solve for k1

```
w2 = 1/4 (2a - 3k12);
qw12 = a/6 - (3k12)/4;
qw22 = a/6 + k12/4;
p2 = a - Q;
Q = qw12 + qw22 + k12;
PW12 = Simplify[(a - Q - w2)qw12 + (a - Q - c)k12];
PW22 = Simplify[(a - Q - w2)qw22];
DPW12 = D[PW12, k12]
SQW12 = FullSimplify[Solve[DPW12==0, k12]]
```

$\frac{1}{144}(84a - 99\text{k12} - 9(16c + 11\text{k12}))$

$\left\{\left\{\text{k12} \to \frac{2}{33}(7a - 12c)\right\}\right\}$

Substitute k1 back into w, q1, q2, and profit functions

```
p = a - Q;
Q = qw12 + qw22 + k12;
k12 = 2/33 (7a - 12c)
w2 = FullSimplify [1/4 (2a - 3k12)]
qw12 = FullSimplify [a/6 - (3k12)/4]
qw22 = FullSimplify [a/6 + k12/4]
PWF12 = FullSimplify[(a - Q - w2)qw12 + (a - Q - c)k12]
PWF22 = FullSimplify[(a - Q - w2)qw22]
```

$\frac{2}{33}(7a - 12c)$

$\frac{2}{11}(a + 3c)$

$\frac{1}{33}(-5a + 18c)$

$\frac{1}{11}(3a - 2c)$

$\frac{1}{33}\left(5a^2 - 14ac + 12c^2\right)$

$\frac{1}{121}(3a - 2c)^2$

Case 3:

Stage 3: Solve for q1 and q2

ClearAll[a, c, α, t]
$p = a - Q$;
Q = q13 + q23 + k13;
P13 = ($a - Q$ − w3)q13 + ($a - Q - c$)k13
P23 = ($a - Q$ − w3(1 − α))q23
DP13 = D[P13, q13]
DP23 = D[P23, q23]
SQ3 = FullSimplify[Solve[DP13==0&&DP23==0, {q13, q23}]]

k13$(a - c -$ k13 − q13 − q23) + q13$(a -$ k13 − q13 − q23 − w3)

q23$(a -$ k13 − q13 − q23 − w3$(1 - \alpha)$)

$a -$ 2k13 − 2q13 − q23 − w3

$a -$ k13 − q13 − 2q23 − w3$(1 - \alpha)$

$\left\{\left\{\text{q13} \to \frac{1}{3}(a - 3\text{k13} - \text{w3}(1 + \alpha)), \text{q23} \to \frac{1}{3}(a + \text{w3}(-1 + 2\alpha))\right\}\right\}$

Stage 2: Solve for union wage

q13 = $\frac{1}{3}$(a − 3k13 − (1 + α)w3);
q23 = $\frac{1}{3}$(a + (−1 + 2α)w3);
U13 = FullSimplify[w3(q13 + q23(1 − α))]
DU13 = D[U13, w3];
SU13 = FullSimplify[Solve[DU13==0, w3]]

$-\frac{1}{3}$w3(3k13 + $a(-2 + \alpha)$ + 2(w3 + w3$(-1 + \alpha)\alpha$))

$\left\{\left\{\text{w3} \to -\frac{3\text{k13} + a(-2+\alpha)}{4+4(-1+\alpha)\alpha}\right\}\right\}$

Substitute union wage back into q1 and q2

$$\mathbf{w3 = Factor\left[-\frac{3k13+a(-2+\alpha)}{4+4(-1+\alpha)\alpha}\right]}$$
$$\mathbf{qw13 = Factor\left[\frac{1}{3}(a - 3k13 - (1+\alpha)w3)\right]}$$
$$\mathbf{qw23 = Factor\left[\frac{1}{3}(a + (-1+2\alpha)w3)\right]}$$

$$-\frac{-2a+3k13+a\alpha}{4(1-\alpha+\alpha^2)}$$

$$\frac{2a-9k13-5a\alpha+15k13\alpha+5a\alpha^2-12k13\alpha^2}{12(1-\alpha+\alpha^2)}$$

$$\frac{2a+3k13+a\alpha-6k13\alpha+2a\alpha^2}{12(1-\alpha+\alpha^2)}$$

Stage 1: Solve for k1

$$\mathbf{ClearAll[a, c, \alpha, k13]}$$
$$\mathbf{w3 = -\frac{3k13+a(-2+\alpha)}{4+4(-1+\alpha)\alpha};}$$
$$\mathbf{qw13 = \frac{2a-9k13-5a\alpha+15k13\alpha+5a\alpha^2-12k13\alpha^2}{12(1-\alpha+\alpha^2)};}$$
$$\mathbf{qw23 = \frac{k13(3-6\alpha)+a\left(2+\alpha+2\alpha^2\right)}{12(1-\alpha+\alpha^2)};}$$
$$\mathbf{p = a - Q;}$$
$$\mathbf{Q = qw13 + qw23 + k13;}$$
$$\mathbf{Pw13 = (a - Q - w3)qw13 + (a - Q - c)k13;}$$
$$\mathbf{Pw23 = (a - Q - w3(1-\alpha))qw23;}$$
$$\mathbf{DPW13 = D[Pw13, k13];}$$
$$\mathbf{SQW13 = FullSimplify[Solve[DPW13==0, k13]]}$$

$$\left\{\left\{\text{k13} \to \frac{14a-a\alpha(21+(-18+\alpha)\alpha)-24c(1+(-1+\alpha)\alpha)^2}{33-42\alpha+33\alpha^2}\right\}\right\}$$

Substitute k1 back into w, q1, q2, and profits of firms 1 and 2

$$\mathbf{ClearAll[a, c, \alpha, k13]}$$
$$\mathbf{p = a - Q;}$$
$$\mathbf{Q = qw13 + qw23 + k13;}$$
$$\mathbf{k13 = \frac{14a-a\alpha(21+(-18+\alpha)\alpha)-24c(1+(-1+\alpha)\alpha)^2}{33-42\alpha+33\alpha^2};}$$
$$\mathbf{w3 = FullSimplify\left[-\frac{3k13+a(-2+\alpha)}{4+4(-1+\alpha)\alpha}\right]}$$
$$\mathbf{qw13 = FullSimplify\left[\frac{2a-9k13-5a\alpha+15k13\alpha+5a\alpha^2-12k13\alpha^2}{12(1-\alpha+\alpha^2)}\right]}$$
$$\mathbf{qw23 = FullSimplify\left[\frac{k13(3-6\alpha)+a\left(2+\alpha+2\alpha^2\right)}{12(1-\alpha+\alpha^2)}\right]}$$
$$\mathbf{PWF13 = FullSimplify[(a - Q - w3)qw13 + (a - Q - c)k13]}$$
$$\mathbf{PWF23 = FullSimplify[(a - Q - w3(1-\alpha))qw23]}$$

$\frac{a(4-5\alpha)+12(c+c(-1+\alpha)\alpha)}{22-28\alpha+22\alpha^2}$

$\frac{a(-2+\alpha)(5+\alpha(-5+2\alpha))+12c(1+(-1+\alpha)\alpha)(3+\alpha(-5+4\alpha))}{66-84\alpha+66\alpha^2}$

$\frac{4c(-1+2\alpha)(1+(-1+\alpha)\alpha)+a(6+\alpha(-5+4\alpha))}{22-28\alpha+22\alpha^2}$

$\frac{48(c+c(-1+\alpha)\alpha)^2+a^2(20+\alpha(-32+23\alpha))+4ac(-14+\alpha(21+(-18+\alpha)\alpha))}{12(11+\alpha(-14+11\alpha))}$

$\frac{(4c(-1+2\alpha)(1+(-1+\alpha)\alpha)+a(6+\alpha(-5+4\alpha)))^2}{4(11+\alpha(-14+11\alpha))^2}$

Case 4:

Stage 3: Solve for q1 and q2

ClearAll[a, α, c, t]
$p = a - Q$;
Q = q1 + q2 + k1 + k2;
P14 = ($a - Q - w$)q1 + ($a - Q - c$)k1
P24 = ($a - Q - w(1 - \alpha)$)q2 + ($a - Q - c$)k2
DP14 = D[P14, q1];
DP24 = D[P24, q2];
SQ4 = FullSimplify[Solve[DP14==0&&DP24==0, {q1, q2}]]

k1$(a - c -$ k1 $-$ k2 $-$ q1 $-$ q2$)$ + q1$(a -$ k1 $-$ k2 $-$ q1 $-$ q2 $- w)$

k2$(a - c -$ k1 $-$ k2 $-$ q1 $-$ q2$)$ + q2$(a -$ k1 $-$ k2 $-$ q1 $-$ q2 $- w(1 - \alpha))$

$\{\{\text{q1} \to \frac{1}{3}(a - 3\text{k1} - w(1 + \alpha)), \text{q2} \to \frac{1}{3}(a - 3\text{k2} + w(-1 + 2\alpha))\}\}$

Stage 2: Solve for union wage

ClearAll[a, α, c, t]
q1 = $\frac{1}{3}$(a − 3k1 − w(1 + α));
q2 = $\frac{1}{3}$(a − 3k2 + w(−1 + 2α));
U4 = w(q1 + q2(1 − α))
DU4 = D[U4, w];
SU4 = FullSimplify[Solve[DU4==0, w]]

$w\left(\frac{1}{3}(a-3\text{k1}-w(1+\alpha))+\frac{1}{3}(1-\alpha)(a-3\text{k2}+w(-1+2\alpha))\right)$

$\left\{\left\{w\to \frac{-3(\text{k1}+\text{k2})-a(-2+\alpha)+3\text{k2}\alpha}{4+4(-1+\alpha)\alpha}\right\}\right\}$

```
ClearAll[a, α, c, t]
w = (-3(k1+k2)-a(-2+α)+3k2α)/(4+4(-1+α)α);
qw1 = Factor [1/3 (a - 3k1 - w(1 + α))]
qw2 = Factor [1/3 (a - 3k2 + w(-1 + 2α))]
```

$\frac{2a-9\text{k1}+3\text{k2}-5a\alpha+15\text{k1}\alpha+5a\alpha^2-12\text{k1}\alpha^2-3\text{k2}\alpha^2}{12(1-\alpha+\alpha^2)}$

$\frac{2a+3\text{k1}-9\text{k2}+a\alpha-6\text{k1}\alpha+3\text{k2}\alpha+2a\alpha^2-6\text{k2}\alpha^2}{12(1-\alpha+\alpha^2)}$

Further simplifying the above expressions

```
ClearAll[a, α, c, t]
q1 = Simplify [(2a-9k1+3k2-5aα+15k1α+5aα^2-12k1α^2-3k2α^2)/(12(1-α+α^2))]
q2 = Simplify [(2a+3k1-9k2+aα-6k1α+3k2α+2aα^2-6k2α^2)/(12(1-α+α^2))]
```

$\frac{a\left(2-5\alpha+5\alpha^2\right)-3\left(\text{k2}\left(-1+\alpha^2\right)+\text{k1}\left(3-5\alpha+4\alpha^2\right)\right)}{12(1-\alpha+\alpha^2)}$

$\frac{a\left(2+\alpha+2\alpha^2\right)+3\left(\text{k1}-2\text{k1}\alpha+\text{k2}\left(-3+\alpha-2\alpha^2\right)\right)}{12(1-\alpha+\alpha^2)}$

Stage 1: Solve for k1 and k2

```
ClearAll[a, α, c, t]
w = (-3(k1+k2)-a(-2+α)+3k2α)/(4+4(-1+α)α);
qw1 = (a(2-5α+5α^2)-3(k2(-1+α^2)+k1(3-5α+4α^2)))/(12(1-α+α^2));
qw2 = (a(2+α+2α^2)+3(k1-2k1α+k2(-3+α-2α^2)))/(12(1-α+α^2));
p = a - Q;
Q = qw1 + qw2 + k1 + k2;
PW14 = (a - Q - w)qw1 + (a - Q - c)k1;
PW24 = (a - Q - w(1 - α))qw2 + (a - Q - c)k2;
DPW14 = D[PW14, k1];
DPW24 = D[PW24, k2];
SQW4 = FullSimplify[Solve[DPW14==0&&DPW24==0, {k1, k2}]]
```

$$\left\{\left\{\text{k1} \to \frac{a(-1+\alpha)(14+\alpha(-7+\alpha(4+7\alpha)))-4c(1+(-1+\alpha)\alpha)(-6+\alpha(11+\alpha(-11+8\alpha)))}{3(-1+\alpha)(16+\alpha(-16+13\alpha))},\right.\right.$$

$$\text{k2} \to \frac{-4c(1+(-1+\alpha)\alpha)(6+\alpha(-7+\alpha(7+2\alpha)))+a(-1+\alpha)(-14+\alpha(35+2\alpha(-16+9\alpha)))}{3(-1+\alpha)^2(16+\alpha(-16+13\alpha))}$$

Equilibrium wage, output, and profits:

$\textbf{ClearAll}[a, \alpha, c, t]$

$p = a - Q;$

$Q = \text{qw1} + \text{qw2} + \text{k1} + \text{k2};$

$\text{k1} = \text{Simplify}\left[\frac{a(-1+\alpha)(14+\alpha(-7+\alpha(4+7\alpha)))-4c(1+(-1+\alpha)\alpha)(-6+\alpha(11+\alpha(-11+8\alpha)))}{3(-1+\alpha)(16+\alpha(-16+13\alpha))}\right]$

$\text{k2} = \text{Simplify}\left[\frac{-4c(1+(-1+\alpha)\alpha)(6+\alpha(-7+\alpha(7+2\alpha)))+a(-1+\alpha)(-14+\alpha(35+2\alpha(-16+9\alpha)))}{3(-1+\alpha)^2(16+\alpha(-16+13\alpha))}\right]$

$w = \text{Simplify}\left[\frac{-3(\text{k1}+\text{k2})-a(-2+\alpha)+3\text{k2}\alpha}{4+4(-1+\alpha)\alpha}\right]$

$\text{qw1} = \text{Simplify}\left[\frac{a\left(2-5\alpha+5\alpha^2\right)-3\left(\text{k2}\left(-1+\alpha^2\right)+\text{k1}\left(3-5\alpha+4\alpha^2\right)\right)}{12(1-\alpha+\alpha^2)}\right]$

$\text{qw2} = \text{Simplify}\left[\frac{a\left(2+\alpha+2\alpha^2\right)+3\left(\text{k1}-2\text{k1}\alpha+\text{k2}\left(-3+\alpha-2\alpha^2\right)\right)}{12(1-\alpha+\alpha^2)}\right]$

$\text{PWF14} = \text{Simplify}[(a - Q - w)\text{qw1} + (a - Q - c)\text{k1}]$

$\text{PWF24} = \text{Simplify}[(a - Q - w(1 - \alpha))\text{qw2} + (a - Q - c)\text{k2}]$

$$\frac{a(-1+\alpha)(14+\alpha(-7+\alpha(4+7\alpha)))-4c(1+(-1+\alpha)\alpha)(-6+\alpha(11+\alpha(-11+8\alpha)))}{3(-1+\alpha)(16+\alpha(-16+13\alpha))}$$

$$\frac{-4c(1+(-1+\alpha)\alpha)(6+\alpha(-7+\alpha(7+2\alpha)))+a(-1+\alpha)(-14+\alpha(35+2\alpha(-16+9\alpha)))}{3(-1+\alpha)^2(16+\alpha(-16+13\alpha))}$$

$$\frac{(-2+\alpha)\left(a-a\alpha+12c\left(1-\alpha+\alpha^2\right)\right)}{2(-1+\alpha)(16-16\alpha+13\alpha^2)}$$

$$\frac{a\left(-2+21\alpha-38\alpha^2+33\alpha^3-14\alpha^4\right)+4c\left(-6+31\alpha-56\alpha^2+66\alpha^3-41\alpha^4+16\alpha^5\right)}{6(-1+\alpha)(16-16\alpha+13\alpha^2)}$$

$$\frac{a\left(2+11\alpha-26\alpha^2+25\alpha^3-12\alpha^4\right)+4c\left(6+\alpha-8\alpha^2+24\alpha^3-17\alpha^4+10\alpha^5\right)}{6(-1+\alpha)^2(16-16\alpha+13\alpha^2)}$$

$$(\text{a}^2(-1+\alpha)^2\left(356 - 716\alpha + 933\alpha^2 - 574\alpha^3 + 229\alpha^4 - 4ac\left(200 - 696\alpha + 1084\alpha^2 - 901\alpha^3 + 252\alpha^4 + 214\alpha^5 - 24\right.\right.$$

$$(\text{a}^2(-1+\alpha)^2\left(356 - 708\alpha + 921\alpha^2 - 568\alpha^3 + 228\alpha^4 - 4ac\left(200 - 904\alpha + 1812\alpha^2 - 2187\alpha^3 + 1647\alpha^4 - 732\alpha^5 +\right.\right.$$

$\textbf{ClearAll}[a, c, t, \alpha]$

$\textbf{Manipulate}\left[\textbf{Plot}\left[\left\{\frac{1}{24}(-a + 12c - t), \frac{1}{33}(-5a + 18c), \frac{-10a+36c+15a\alpha-96c\alpha-9a\alpha^2+144c\alpha^2+2a\alpha^3-108c\alpha^3+48c\alpha^4}{6(11-14\alpha+11\alpha^2)},\right.\right.\right.$
$\left.\left.\left. -\frac{2a+24c-21a\alpha-124c\alpha+38a\alpha^2+224c\alpha^2-33a\alpha^3-264c\alpha^3+14a\alpha^4+164c\alpha^4-64c\alpha^5}{6(-1+\alpha)(16-16\alpha+13\alpha^2)}\right\}, \{\alpha, .01, .7\}, \text{AxesLabel->}\{\alpha, q\},\right.$
$\left.\textbf{PlotLabels->}\left\{\text{"}q_1^{1*}\text{"}, \text{"}q_1^{2*}\text{"}, \text{"}q_1^{3*}\text{"}, \text{"}q_1^{4*}\text{"}\right\}, \textbf{PlotLabel->}\text{“Panel 1}\rightarrow\text{Firm 1”}\right], \{c, 1, 4\}, \{a, 2, 5\}, \{t, 1.21, 9\}\right]$

$a = 2;$

```
c = 1;

t = 1.21;

Table [{N [1/24 (-a + 12 c - t)], 1/12 (5 a - 12 c + 5 t), N [1/33 (-5 a + 18 c)], "0.0606061" ("7." a - "12." c),
(-10 a + 36 c + 15 a α - 96 c α - 9 a α^2 + 144 c α^2 + 2 a α^3 - 108 c α^3 + 48 c α^4)/(6 (11 - 14 α + 11 α^2)),
(14 a - a α (21 + (-18 + α) α) - 24 c (1 + (-1 + α) α)^2)/(33 - 42 α + 33 α^2),
-(2 a + 24 c - 21 a α - 124 c α + 38 a α^2 + 224 c α^2 - 33 a α^3 - 264 c α^3 + 14 a α^4 + 164 c α^4 - 64 c α^5)/(6 (-1 + α) (16 - 16 α + 13 α^2)),
(a (-1 + α) (14 + α (-7 + α (4 + 7 α))) - 4 c (1 + (-1 + α) α) (-6 + α (11 + α (-11 + 8 α))))/(3 (-1 + α) (16 + α (-16 + 13 α)))},

{α, .04, .2, .02}]//TableForm


Table [{{N [1/11 (3 a - 2 c)], (6 a - 4 c - 5 a α + 12 c α + 4 a α^2 - 12 c α^2 + 8 c α^3)/(2 (11 - 14 α + 11 α^2)) (1 - α),
-(-2 a - 24 c - 11 a α - 4 c α + 26 a α^2 + 32 c α^2 - 25 a α^3 - 96 c α^3 + 12 a α^4 + 68 c α^4 - 40 c α^5)/(6 (-1 + α)^2 (16 - 16 α + 13 α^2)) (1 - α),
(-4 c (1 + (-1 + α) α) (6 + α (-7 + α (7 + 2 α))) + a (-1 + α) (-14 + α (35 + 2 α (-16 + 9 α)))) / (3 (-1 + α)^2 (16 + α (-16 + 13 α)))},

{α, .04, .2, .02}]//TableForm
```