ISSN No. 2454 - 1427

CDE
October 2024

Optimal Fiscal Debt Management Strategy During Financial Crises: A Behavioral New Keynesian DSGE Approach for the US

Arpan Chakraborty
Email: arpan.ms97@kgpian.iitkgp.ac.in,
Department of Humanities and Social Sciences,
Indian Institute of Technology Kharagpur

Siddhartha Chattopadhyay
Email: siddhartha@hss.iitkgp.ac.in,
Department of Humanities and Social Sciences,
Indian Institute of Technology Kharagpur

Mausumi Das
Email: mausumi@econdse.org
Department of Economics,
Delhi School of Economic,
University of Delhi

Sohini Sahu
Email: sahu@iitk.ac.in,
Department of Economics,
Indian Institute of Technology Kanpur

Working Paper No. 352

Centre for Development Economics
Delhi School of Economics
Delhi- 110007

Optimal Fiscal Debt Management Strategy During Financial Crises: A Behavioral New Keynesian DSGE Approach for the US[*]

Arpan Chakraborty[†] Siddhartha Chattopadhyay[‡] Mausumi Das[§]
Sohini Sahu[¶]

September 24, 2024

[*]We are indebted to Professor Betty C. Daniel for helpful comments. We also acknowledge the comments of the seminar participants at the University of Bamberg, Germany; University of Calcutta, India; Presidency University, India; St. Xavier's University, India; Indira Gandhi Institute of Development Research, India; and Department of Economics - IIT Bombay, India. The usual disclaimer applies.

[†]Research Scholar, Indian Institute of Technology Kharagpur, Department of Humanities and Social Sciences, Kharagpur, West Bengal, India. email: arpan.ms97@kgpian.iitkgp.ac.in

[‡]Corresponding author. Associate Professor, Indian Institute of Technology Kharagpur, Department of Humanities and Social Sciences, Kharagpur, West Bengal, India. email: siddhartha@hss.iitkgp.ac.in

[§]Professor, Delhi School of Economics, University of Delhi, New Delhi; and Shiv Nadar University, Delhi NCR, India. email: mausumi@econdse.org

[¶]Associate Professor, Indian Institute of Technology Kanpur, Department of Economic Sciences, Kanpur, Uttar Pradesh, India. email: ssahu@iitk.ac.in

Abstract:

Data shows that, the US Treasury has increasingly issued more long-term bonds to finance expenditures in the periods of financial crisis, whose duration is comparatively higher than the pre-crisis periods. Using ad-hoc, partial equilibrium models, existing literature explain this as a strategic debt management approach of the Treasury to mitigate their fiscal risk. By extending the model of De Grauwe and Foresti (2023) to incorporate bonds with varying maturity structures, we develop a behavioral New Keynesian model to explain debt management strategy of the Treasury by manipulating the duration of the long-term debts. The Treasury in our model optimally chooses the duration of its debt along with the tax rate to minimize the welfare loss. We show that increasing the duration of Treasury debt is optimal in the periods of financial crisis when the traditional monetary policy is unproductive, as it generates an optimal trade-off between the number of newly issued debt, and providing enough incentives to hold them by the banking sectors.

Keywords: Fiscal Debt Management, Behavioral Expectation, Zero Lower Bound

JEL Classifications: E12, E37, E52, E62, E71

1 Introduction

The financial crisis of 2008 necessitated an unprecedented fiscal intervention to stabilize the deteriorating global financial system. The US Department of the Treasury (henceforth "the Treasury") undertook important measures by allocating approximately \$475 billion to safeguard the vulnerable financial institutions of the US. Such fiscal intervention lead to a significant increase in the Treasury's total outstanding debt, escalating from \$4,537 billion in December 2007 to \$12,163 billion by July 2014 (Greenwood et al., 2014). This phenomenon was not quite unique to the US; fiscal authorities in numerous countries accumulated substantial sovereign debt during this period, often at levels that proved unsustainable in the long term.

Consequently, a considerable body of post-crisis literature on fiscal policy has been dedicated to examine the stability, solvency, and the strategic defaults of sovereign debt[1]. Furthermore, given the intrinsic relationship between fiscal limits and tax revenue, another strand of literature has focused on assessing the optimality of various tax rates across different countries through Laffer curve[2]. Along with this, with policy rates at the zero lower bound (ZLB), a string of post-crisis monetary policy literature has investigated the role of optimal forward guidance in stimulating economic activity through the manipulation of individual expectations[3]. Bernanke, Reinhart, and Sack (2004) examine the impact of the unconventional monetary policies, such as Quantitative Easing (QE), and

[1] Extending Bohn's (1998) methodology, Daniel and Shiamptanis (2013, 2022) analyze fiscal limits, long-run sustainability, and solvency of fiscal debt for EU countries and nine high-debt European countries, respectively. Ghosh et al. (2013) and Shiamptanis (2023) examine the interrelationships among fiscal fatigue, tax austerity, and fiscal solvency. For comprehensive discussions on fiscal solvency and fiscal limits, refer to Daniel and Shiamptanis (2012) and Bi (2012). For analyses of strategic and excusable defaults, see Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), and Daniel and Nam (2022). Moreover, using a two country general equilibrium model, Enders et al. (2011) demonstrate that fiscal policy shocks, particularly government spending increases, lead to significant real exchange rate depreciation.

[2] Trabandt and Uhlig (2011, 2012) evaluate the optimality of consumption, capital, and labor income tax rates for the US, EU-14, and various European countries. Daniel and Gao (2015) extend this analysis to calculate optimal tax rates and educational subsidies under productive government spending for the US.

[3] Using rational expectation, while Jung, Teranishi, and Watanabe (2005) and Adam and Billi (2006, 2007) examine optimal forward guidance policies under discretion and commitment at the zero lower bound (ZLB), Eggertsson and Woodford (2001), Nakov (2008), and Chattopadhyay and Daniel (2018) investigate the roles of inflation targeting and the Taylor rule in implementing optimal forward guidance policies. On the other hand, Proaño and Lojak (2020) analyze the optimal forward guidance policy using the behavioral expectation formation among economic agents. The other strand of literature addresses banking sector fragility, with Vinogradov (2011) exploring how a production shock in a closed economy can precipitate the collapse of a competitive banking system, underscoring the paradoxical role of bank credibility.

the resultant changes in the Federal Reserve's balance sheet when the policy rate is at the ZLB. They posit that such unconventional monetary policies yield effective results when the Federal Reserve and the Treasury implement well-coordinated and non-offsetting sets of policies.

Greenwood et al. (2014) identifies that, the unconventional monetary policies undertaken by the Federal Reserve fail to yield desirable outcome in the periods of the financial crisis due to an unprecedented coordination failure between the Federal Reserve and the Treasury. Their findings reveal that while the Federal Reserve was implementing QE through the targeted purchases of long-term securities to stimulate the economy, the Treasury was simultaneously engaged in what could be termed a "reverse QE." At the backdrop of growing liabilities during financial crisis as mentioned above, this reverse QE allows the Treasury to finance its expenditures by issuing bonds of relatively longer durations than those of pre-crisis periods to mitigate fiscal/interest rate risk.

Note, mitigating interest rate risk of a financial organizations (financial intermediaries, fiscal authorities, etc) is a strategy to insulate its net worth from the fluctuations of interest rate by manipulating the duration of its debt. We know that, the percentage change in net worth of a financial organization for a unit change in interest rate depends on the duration gap - difference in duration of assets and liabilities of the organization. Hence, a strategy of issuing liabilities, whose duration is comparatively higher than the assets, insulates the net worth of the financial organizations from the risk of interest rate fluctuations by reducing the duration gap[4]. While quantifying the rise in duration of fiscal debt in the periods of the financial crisis, Greenwood et al. (2014) show that the average duration of Treasury debt increased from 3.9 years to 4.6 years between December 2007 and July 2014 for the US[5]. Since, the standard economic theory provides limited guidance to model the fiscal debt management strategies by manipulating the duration of Treasury debt as described above, Greenwood et al. (2015) use an ad-hoc, partial equilibrium model based on the principle of corporate finance to analyze it.

[4]Note, the duration gap analysis gives, $\frac{\Delta NW}{A} \approx -D_{gap}\frac{\Delta i}{1+i}$; where, i is the interest rate/yield to maturity, $NW = A-L$ is the net worth, $D_{gap} = D_A - \left(\frac{L}{A}\right) D_L$ is the Macaulay duration gap between assets (A) and liabilities (L); and D_A, and D_L are the Macaulay duration of assets, and liabilities of a financial institution respectively. Hence, to insulate the net worth from the interest rate fluctuations/interest rate risk, a financial institution would like to reduce D_{gap} by raising D_L relative to D_A. The net worth of the institution is fully insulated from the interest rate fluctuations, when $D_{gap} = 0 \Rightarrow D_L = \frac{A}{L} D_A$. See; Mishkin (2021) for details.

[5]Greenwood et al. (2014) shows that, the extent of Quantitative Easing (QE) rises from \$0 to \$2,901 billion between December 2007 and July 2014. The time-path of the QE, and the fiscal debt for the US are depicted in Figure 1 in the appendix. It shows the significant rise in QE as well as the amount of the Treasury debt in the periods of the financial crisis for the US. Figure 2 on the other hand shows the significant rise of the duration of the fiscal debt in the periods of the financial crisis (2008-2014).

Given this backdrop, our paper's primary objective is to develop a general equilibrium model with optimizing agents to analyze the fiscal debt management strategy of the Treasury through manipulating its duration, and matching it with the US data. To do it, we use a New Keynesian model with behavioral expectation formation among economic agents, as it better fits the US data[6]. Specifically, we have extended the model of De Grauwe and Foresti (2023) by introducing bonds with varying maturity structures as given in Woodford (2001). These bonds have a geometrically decaying coupon payments, whose decay rate determines their duration[7]. These bonds are issued by the Treasury are purchased by the banking sectors in our model. This eliminates the possibilities of the coordination failure between the Federal Reserve and the Treasury as highlighted in Greenwood et al. (2014). Households in our model only hold deposits with the commercial bank, and they do not hold bonds issued by the Treasury. As a result, the deposit rate, determined by the Taylor rule affects the output gap though the aggregate demand equation. On the other hand, the return of the long-term bonds (determined by the duration of the bonds and their prices) affects the output gap, and the inflation rate by the aggregate demand curve, and the supply curve through the current, and expected government spending. The government spending satisfies the government budget constraint, and is determined by a standard fiscal rule in our model.

The Treasury in our model has two instruments - (i) the duration of the long-term bonds, and (ii) the tax rate. Their objective is to optimally choose its instruments to minimize a loss function that depends on the weighted average of the unconditional standard deviation of the inflation rate, and the output gap (see; De Grauwe and Ji, 2019). In our model, the Treasury implements its optimal policy under two distinct economic conditions - 1) when traditional monetary policy is either fully effective or occasionally ineffective, characteristic of pre-crisis periods. In this case, the policy rate of the Federal Reserve, which is the deposit rate in our model, is determined by a truncated Taylor rule, and 2) when the traditional monetary policy is persistently ineffective due to the federal funds rate being at the zero lower bound (ZLB) for an extended period, representative of the 2008 global financial crisis. Note, data shows that, the ZLB was binding for the US

[6]Unlike rational expectation, the forward looking NK model has endogenous persistence under the behavioral expectations of De Grauwe (2012), and De Grauwe and Ji (2019). Hence, a Gaussian demand and supply shock in an NK model with behavioral expectations among economic agents produces -(i) fat-tailed non-Gaussian distributions of the output gap and the inflation rate, and (ii) hump-shaped response of the output gap and the inflation rate, as observed in the US data.

[7]Sims et al. (2023), Sims and Wu (2019, 2020), and Cardamone et al. (2023) have used such bond structure in the New Keynesian models to evaluate the effectiveness and optimality of QE under rational expectations.

from 2008 to 2015. As a result, we keep the deposit rate fixed at the ZLB for the initial $\widehat{t} = 30$ quarters in our optimal policy analysis. Furthermore, we also conduct an optimal policy analysis by keeping the deposit rate at the ZLB for the initial $\widehat{t} = 40$ quarters. After the initial $\widehat{t}$ periods, the economy exits the ZLB; and the deposit rate is determined by the truncated Taylor rule, where the ZLB can be occasionally binding depending on the magnitude of the Shocks. Note, such a policy set-up is a close representation of the forward guidance policy undertaken by the Federal Reserve to stimulate the economy in the periods of the financial crisis.

As previously noted, financing expenditures primarily by manipulating the duration of the long-term bonds was a key component of the Treasury's debt management strategy in the periods of the financial crisis. Hall and Sargent (2011) report that the average duration of the US Treasury debt between 2000-2007 (pre-crisis periods) was approximately 3.5 years. It is increased to 4.2 years for the period 2008-2014 (crisis periods) as reported in Greenwood et al. (2014)[8]. This increase in debt maturity reflects the Treasury's strategic shift in fiscal debt management in response to the financial crisis and subsequent economic challenges when the traditional monetary policy is ineffective due to the binding ZLB constraint. Not only the duration, even the average proportion of long-term bonds in the fiscal portfolio rose from 25% between October 2005 - December 2007 to 43% between January 2008 - July 2014. Along with this, Trabandt and Uhlig (2011, 2012) find that the labour income tax rate, which maximizes the tax revenue of the Treasury should decrease from 28% in the pre-crisis periods to 22% in the periods of the financial crisis.

Matching with the trend of the US data, our model also forecasts a rise in the optimal duration of the long-term bonds in the portfolio of the Treasury in the periods of the financial crisis than the pre-crisis periods. We find, given 25% long-term bonds in the fiscal portfolio, it is optimal for the Treasury to issue long-term bonds with duration 3 years in the pre-crisis periods[9]. On the other hand, we find that given the 43% long-term bonds in the fiscal portfolio, it is optimal for the Treasury to issue long-term bonds with duration 3.25 years when the ZLB is binding till $\widehat{t} = 30$ quarters. Along with this, a income tax rate of 25% is found to be optimal in the pre-crisis periods. However, our model predicts a reduction in the income tax rate as a part of the expansionary fiscal policy from 25% to 5% in the periods of the financial crisis, when the monetary policy is

[8] Also see; Figure 1, and Figure 2 in the appendix.

[9] Note, we define bonds, whose duration is at least as large as 3 months as short-term bonds. Bonds whose duration is more than 3 months are the long-term bonds in our analysis. Figure 3 in the appendix shows the significant rise in the proportion of long-term bonds in the portfolio of the Treasury in the periods of the financial crisis.

unproductive due to the persistently binding ZLB constraint on the deposit rate. We also find a further increase in the duration of the debt from 3.25 years to 5 years, and a rise in the optimal income tax rate from 5% to 25% when the ZLB is binding till, $\widehat{t} = 40$ quarters. Our analysis shows, as a part of the expansionary fiscal policy, while the duration of the debts rises, the optimal tax rate falls with the persistence of the ZLB.

To explain our findings note that, the expansionary fiscal policy with higher government spending and/or lower taxes is required, especially when the traditional monetary policy is unproductive due to the binding ZLB constraint on the policy rate. At this backdrop, to mitigate the fiscal risk, the Treasury needs to issue new debts with higher duration to finance additional spending. However, to incentivize the banking sectors to hold more debts with higher duration, the risk associated with them needs to be appropriately compensated with higher returns.

In our model, the return associated with the long-term bonds rises with its duration by construction. However, the duration cannot be infinitely raised due to the trade-off between the number of newly issued debt, and the return of the long-term bonds in our model - higher duration of bonds increases their return, but simultaneously reduces number of new issuance of debt. The reduction in the number of newly issued bonds constraints the borrowing of the Treasury, which is required to finance the additional spending to stimulate the economy when the traditional monetary policy is unproductive. Such a trade-off between the return, and the number of newly issued debts yields an optimal duration of long-term bonds in our model.

Data shows that, the average duration of bonds in the periods of the financial crisis is higher than the pre-crisis periods. Greenwood et al. (2014) explains it as a debt management strategy of the Treasury to mitigate their fiscal/interest rate risk at the backdrop Treasury's growing liabilities during financial crisis. Although, we also find identical results from our optimal policy analysis based on a full-blown New Keynesian DSGE model, the intuition of our results is quite different than that of Greenwood et al. (2014, 2015). Instead of a duration gap analysis, our results relies on generating an optimal trade-off between the return of the fiscal portfolio, and the number of newly issued debts by the Treasury. In other word, our results rely on achieving an optimal trade-off between the number of newly issued debt by the Treasury to stimulate the economy, and providing enough incentives to hold them by the banking sectors.

The rest of the paper is organized as follows. Section 2 describes the model, Section 3 describes the behavioral expectation formation. Section 4 describes the loss function, and the analysis of the optimal policy. Section 5 describes the results, and Section 6 concludes.

2 The Model

De Grauwe and Foresti (2023) used a forward looking New Keynesian model, and solved it under the behavioral expectation to analyze the interaction of various types of fiscal and monetary policy rules. The aggregate demand curve, and the aggregate supply curve, derived by De Grauwe and Foresti (2023) by using the quadratic adjustment cost of prices (Rotemberg, 1982) is given by equations (1) and (2) respectively.

$$\hat{y}_t = \widetilde{E}_t(\hat{y}_{t+1}) - a_2(\hat{\imath}_t - \widetilde{E}_t(\pi_{t+1})) - a_3(\widetilde{E}_t(\hat{g}_{t+1}) - \hat{g}_t) + \varepsilon_t;\ 0 < a_j < 1, j = 1, 2, 3 \tag{1}$$

$$\pi_t = b_1 \widetilde{E}_t(\pi_{t+1}) + b_2 \hat{y}_t - b_3 \hat{g}_t + \eta_t; 0 < b_1 < 1, 0 < b_3 < 1 \tag{2}$$

where, $\widetilde{E}_t(.)$ is not a rational expectation. It denotes behavioral expectation (see; Section 3 for details).

We have augmented the model of De Grauwe and Foresti (2023) by introducing bonds with varying maturities to do our analysis. However, to keep the model simple, we assume that such bonds are issued by the Treasury, and purchased by the commercial bank and the central bank. The commercial bank purchases bonds by using the deposits held with them by the households. The central bank on the other hand, uses the reserves of the commercial banks held with them to purchase the bonds from the Treasury. Since, the households do not purchase bonds, and they only hold deposits with the commercial banks, equations (1) and (2) continue to represent the aggregate demand curve, and the aggregate supply curve of our model respectively. Moreover, since the bonds are issued by the Treasury, and held by the banking sectors in our model, it takes care of the issue of coordination failure arising from the conflict of QE (implemented by the Federal Reserve), and the reverse QE (implemented by the Treasury) that adversely affect the effectiveness of the alternative monetary policies in the periods of the financial crisis as mentioned in Section 1.

Note, since households only hold deposits, and do not hold bonds; the monetary policy affects the output gap from the aggregate demand curve through the deposit rate, which in turn affects the inflation rate from the aggregate supply curve. The deposit rate is determined by the Taylor rule given in equation (3).

$$\hat{\imath}_t = \max(-i, c_1\pi_t + c_2\hat{y}_t + c_3\widehat{\imath}_{t-1} + u_t); \tag{3}$$
$$\hat{\imath}_t = i_t - i; \quad i = \frac{1}{b_1} - 1$$

where, $u_t \sim IIDN(0, 0.5)$ in the shock to the deposit rate, and i is the interest rate at steady state[10].

The fiscal policy affects the output gap, and the inflation rate from the aggregate demand curve, and the aggregate supply curve through the current, and expected government spending. The government spending in turn endogenously determined by a fiscal rule. Next section gives a detailed description of the fiscal policy used in our paper.

2.1 The Fiscal Policy

Along with a proportional income tax rule, the Treasury in our model issues bonds with varying maturities to finance their expenditure. Following Woodford (2001), we model the bonds as perpetuities with geometrically decaying coupon payments at a constant rate. The rate of decay of the coupon payments is $\omega \in [0, 1]$. One new issuance of the bonds at time t, promises to pay \$1 next period, $t+1$. Therefore, the total coupon liability (B_t) of the Treasury at time, t is represented in equation (4)

$$B_t = NB_t + \omega NB_{t-1} + \omega^2 NB_{t-2} + ... \tag{4}$$

where, the new issuance of bonds at time t is given in equation (5)

$$NB_t = B_t - \omega B_{t-1} \tag{5}$$

Suppose, the price of one unit of bond issued by the Treasury at time t is Q_t dollars. Due to the geometrically decaying structure of the coupon payments, the price of the bonds issued at $t - j$ periods ago is $\omega^j NB_{t-j}$. Therefore, the total value of the bond portfolio can therefore conveniently be written as $Q_t B_t$.

We introduce two types of bonds in our model - (a) the short-term bonds (with maturity 3 months; $\omega = 0$), and (b) the long-term bonds (with maturity more than 3 months or more; $0 < \omega \leq 1$) in our model. The prices of the short-term bonds, and the long-term bonds are given by Q_t^S, and Q_t^L respectively. The nominal returns of the short-term and

[10] Note, the deposite rate, $i_t = 0$ at the ZLB. This implies, $\hat{\imath}_t = -i$; when the ZLB is binding.

long-term bonds are denoted by $R_t^{n,S}$ and $R_t^{n,L}$ respectively. Additionally, we assume government issues μ proportion of long-term bonds, and $(1-\mu)$ proportion of short-term bonds, where $0 \leq \mu \leq 1$. Hence, the number of short-term bonds, and long-term bonds in our model are given by, $(1-\mu) B_t$, and μB_t respectively. Therefore, the nominal return (R_t^n) and the price (Q_t) of the fiscal portfolio are given by,

$$R_t^n = (1-\mu) R_t^{n,S} + \mu R_t^{n,L}; \tag{6}$$

$$Q_t = (1-\mu) Q_t^S + \mu Q_t^L \tag{7}$$

where,

$$R_t^{n,S} = \frac{1}{Q_{t-1}^S}; \tag{8}$$

$$R_t^{n,L} = \frac{1+\omega Q_t^L}{Q_{t-1}^L} \tag{9}$$

To execute log-linearization of our model; we define a risk free zero inflation steady state, where the nominal return of the short-term and the long-terms bonds are identical. Note, at the risk free zero inflation steady state, the prices of the long-term and the short-term bonds are given by;

$$Q_{ss}^S = \frac{1}{1+i}; Q_{ss}^L = \frac{1}{1+i-\omega}$$

Log-linearization of equations (6), (7), (8), and (9) at the risk free steady state defined above yield[11],

$$\widehat{r}_t^n = (1-\mu)(-\widehat{q}_{t-1}^S) + \mu(\varpi \widehat{q}_t^L - \widehat{q}_{t-1}^L); \tag{10}$$

$$\widehat{q}_t = \frac{(1-\mu) Q_{ss}^S \widehat{q}_t^S + \mu Q_{ss}^L \widehat{q}_t^L}{(1-\mu) Q_{ss}^S + \mu Q_{ss}^L}; \tag{11}$$

$$\widehat{q}_t^S = -\hat{\imath}_t; \tag{12}$$

$$\widehat{q}_t^L = -\hat{\imath}_t + \varpi \widetilde{E}_t(q_{t+1}^L) \tag{13}$$

where, $R_{ss}^{n,S} = R_{ss}^{n,L} = \frac{1}{b_1}$ is the return of the short-term as well as the long-term bonds at the steady state, and $\varpi = \frac{\omega}{1+i}$.

Given the above described bond structure, and the number of newly issued bonds

[11] See appendix for detailed derivation and the log-linearization.

(equation (5)), the government budget constraint becomes[12];

$$P_tG_t + B_{t-1} = P_tT_t + Q_tNB_t;$$

$$P_tG_t + B_{t-1} = P_tT_t + Q_t\left(B_t - \omega B_{t-1}\right) \tag{14}$$

Dividing both sides of equation (14) by P_t , and defining $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ gives,

$$G_t + \frac{R_t^n}{(1+\pi_t)}V_{t-1} = T_t + V_t; \tag{15}$$

where, $V_t = \frac{Q_tB_t}{P_t}$ is the real government debt/real value of the portfolio debt of the Treasury; and G_t is the real government expenditure. Log-linearization of equation (15) around the steady state defined above gives,

$$\widehat{r}_t^n + \widehat{v}_{t-1} = \psi_1\widehat{y}_t + \psi_2\widehat{v}_t - \psi_3\widehat{g}_t + \pi_t + h_t; \tag{16}$$

where, $h_t \sim N(0, 0.5)$ is the shock to the government budget constraint.

2.1.1 The Fiscal Rule

Following De Grauwe and Foresti (2023), we use the following fiscal rule that endogenously determines the government spending in our model.

$$\hat{g}_t = f_1\hat{g}_{t-1} - f_2\hat{y}_{t-1} - f_3\hat{v}_{t-1} + \varphi_t;\ \ 0 < f_j < 1,\ \ j = 1, 2, 3;\ \ f_2 > f_3; \tag{17}$$

where, $\varphi_t \sim N(0, 0.5)$ is the shock to the fiscal rule.

2.2 The Banking Sector

As mentioned above, bonds issued by the Treasury are purchased by the commercial bank, and the central bank in our model. While, the commercial bank uses the deposits of the households to accommodate the bonds in their balance sheet, the central bank uses the reserves held by the commercial banks with them to accommodate the bonds issued by the Treasury in their balance sheet. The balance sheets of the commercial bank, and the central bank are given in equations (18), (19) respectively.

[12] See; Sims et al. (2023), and Sims and Wu (2019, 2020) for details.

$$Q_t B_t^{FI} + RE_t^{FI} = S_t; \tag{18}$$

$$Q_t B_t^{CB} = RE_t^{FI} \tag{19}$$

The profit of the banking sector, $Q_{t-1}B_{t-1}\{R_t^n - (1+i_{t-1})\}$ is transferred to the households in lumpsum fashion. Note, the profit of the banking sectors arises primarily from the risk premium - interest differential between the long-term bonds, and the deposit rate in our model.

3 The Behavioral Expectation Formation

We have used the behavioral expectation formation among economic agents to solve our model. This heuristics of the behavioral expectation formation is proposed by De Grauwe and Ji (2019) by assuming that, economic agents like others, have their own cognitive limitations. As a result, unlike rational expectations, they make systematic forecast errors each period. However, as explained by De Grauwe and Ji (2019), economic agents following the behavioral expectation formation are rational in the sense, they evaluate their performance each period by correcting their past mistakes. De Grauwe and Ji (2019) have shown, unlike rational expectation, a New Keynesian model, solved under the behavioral expectation formation have endogenous persistence, producing the hump-shaped impulse response of the output gap, and the inflation rate under the Gaussian demand shocks and the supply shock. Moreover, a New Keynesian model, solved under the behavioral expectation with Gaussian demand and supply shocks can produce distributions of the output gap and the inflation rate, which is non-Gaussian with fat tails, as observed in the US data. Hence, we use the behavioral expectation formation in our analysis.

The behavioral expectation formation, proposed by De Grauwe and Ji (2019) has two types of individuals with their own cognitive limitations - (i) the fundamentalists, and (ii) the extrapolators. Under behavioral expectation formation, the expectation of a generic variable $\widehat{x}_t$ is denoted by $\widetilde{E}_t(\widehat{x}_{t+1})$ and it is determined by the following heuristics,

$$\alpha_{f,t}^{\widehat{x}} \widetilde{E}_t^f(\widehat{x}_{t+1}) + \alpha_{e,t}^{\widehat{x}} \widetilde{E}_t^e(\widehat{x}_{t+1}) = \widetilde{E}_t(\widehat{x}_{t+1}) \tag{20}$$

Here, the fundamentalists, who mostly plays a role of a pacifier always forecast steady state[13]. Since, $\widehat{x}_t$ denotes the deviation from the steady state, the heuristics of the fundamentalists in our model yields,

$$\widetilde{E}_t^f(\widehat{x}_{t+1}) = 0 \tag{21}$$

The heuristics of the extrapolator on the other hand is given by equation (22),

$$\widetilde{E}_t^e(\widehat{x}_{t+1}) = \widehat{x}_{t-1}; \widehat{x} = \widehat{y}, \pi, \widehat{g}, \widehat{q} \tag{22}$$

The proportion of fundamentalist and extrapolators in the behavioral expectation formation, $\alpha_{f,t}^{\widehat{x}}$ and $\alpha_{e,t}^{\widehat{x}}$ are determined by the their relative forecast performance. The relative forecast performance on the other hand depends on their mean square forecast error as given below of the corresponding group of people as given below.

$$U_{f,t} = -\sum_{k=0}^{\infty} \varrho_k [\widehat{x}_{t-k-1} - \widetilde{E}_{f,t-k-2}(\widehat{x}_{t-k-1})]^2; \tag{23}$$

$$U_{e,t} = -\sum_{k=0}^{\infty} \varrho_k [\widehat{x}_{t-k-1} - \widetilde{E}_{e,t-k-2}(\widehat{x}_{t-k-1})]^2; \tag{24}$$

where, $\varrho_k = (1-\rho)\rho^k$ and $0 < \rho < 1$ represents the memory parameter of both fundamentalists and extrapolators. An individual in this model switches from extrapolators to fundamentalists when,

$$P[U_{f,t} + \vartheta_{f,t} - U_{e,t} - \vartheta_{e,t}] > 0$$

Therefore, the proportion of fundamentalists in this model is determined by the following equation,

$$\alpha_{f,t}^{\widehat{x}} = P[U_{f,t} + \vartheta_{f,t} > U_{e,t} + \vartheta_{e,t}] \tag{25}$$

Similarly, the proportion of extrapolators in this model is determined by the following equation,

$$\alpha_{e,t}^{\widehat{x}} = P[U_{e,t} + \vartheta_{e,t} > U_{f,t} + \vartheta_{f,t}]; \tag{26}$$

where, the random variables $\vartheta_{f,t}$ and $\vartheta_{e,t}$ represent the emotional state of mind/the cognitive dissonance of the individuals. Following De Grauwe and Ji (2019), we assume that $\vartheta_{f,t}$ and $\vartheta_{e,t}$ are independently and identically distributed random variables and they follow a logistic distribution. As a result following equation (27), the proportion of fundamentalist

[13] De Grauwe and Foresti (2023) defined them as "steady-state" forecasters.

is calculated as,

$$\alpha^{\widehat{x}}_{f,t} = \frac{\exp(\gamma U_{f,t})}{\exp(\gamma U_{f,t}) + \exp(\gamma U_{e,t})} \tag{27}$$

Similarly, following equation (26); the proportion of extrapolators is calculated as,

$$\alpha^{\widehat{x}}_{e,t} = \frac{\exp(\gamma U_{e,t})}{\exp(\gamma U_{f,t}) + \exp(\gamma U_{e,t})} = 1 - \alpha^{\widehat{x}}_{f,t}; \tag{28}$$

where, $\gamma \in [0, \infty)$ is the intensity of choice parameter of the logistic distribution. It represents individual's willingness to learn from their past mistakes in this model. Since, $\vartheta_{f,t}$ and $\vartheta_{e,t}$ are i.i.d., the property of the logistic distribution gives, $\gamma = var(\vartheta_{f,t})^{-1} = var(\vartheta_{e,t})^{-1}$. Therefore, the willingness to learn of an individual tends to zero in this model when $var(\vartheta_{f,t})^{-1}$ and $var(\vartheta_{e,t})^{-1}$ tend to infinity and *vice-versa*.

Note that, the heuristics of output gap ($\widehat{y}_t$), inflation (π_t), government expenditure gap ($\widehat{g}_t$) and bond-price gap ($\widehat{q}_t$) in our model are determined using the behavioral expectation as described above. Consequently, following De Grauwe and Ji (2019), we calculate a measure of animal spirits/market sentiments by using the proportions of fundamentalists, and extrapolators associated with the expectation formation of the output gap, as given in from equations (29). De Grauwe and Ji (2019) show that, the animal spirits governs the business cycle by generating an endogenous waves of optimism and pessimism in the behavioral New Keynesian model[14].

$$\begin{aligned} AS_t &= \alpha^{\widehat{y}}_{e,t} - \alpha^{\widehat{y}}_{f,t} = 2\alpha^{\widehat{y}}_{e,t} - 1 \text{ if } \widehat{y}_{t-1} > 0 \\ &= -\alpha^{\widehat{y}}_{e,t} + \alpha^{\widehat{y}}_{f,t} = -2\alpha^{\widehat{y}}_{e,t} + 1 \text{ if } \widehat{y}_{t-1} < 0 \end{aligned} \tag{29}$$

4 The Loss Function

Following De Grauwe and Ji (2019), we use the loss function given in equation (30) to calculate the optimal financial portfolio of the Treasury, and it's relationship with the extent of inflation targeting of the Federal Reserve.

$$L = \frac{1}{2}(sd(\pi) + \lambda sd(y)); \ \lambda \in [0, \infty) \tag{30}$$

[14]Similarly, we can calculate a measure of the central bank's credibility index C^i_t by using the proportion of fundamentalist, and extrapolators associated with the expectation formation of the inflation rate as follows, $C^i_t = 1 - \alpha^{\pi}_{e,t}$ where, $\alpha^{\pi}_{e,t}$ (see; De Grauwe and Ji, 2019) is the proportion of inflation extrapolators.

Here, λ measures the importance of the output gap relative to the inflation rate. Higher λ implies that, the Federal reserve dislike the fluctuations of the output gap more than that of the inflation rate and vice-versa. Literature defines pure inflation targeting when $\lambda = 0$, and pure output targeting when $\lambda = \infty$. In our paper, the Treasury optimally chooses her instruments - (i) the duration of the long-term bonds, and (ii) the income tax rate. The Treasury optimally chooses its instruments to numerically minimize the loss function given in equation (30) subject to the equations (1), (2), (3), (17), (16), (12), (13), (10) and (11) under two distinct economic conditions 1) when traditional monetary policy is either fully effective or occasionally ineffective, characteristic of pre-crisis periods. In this case, the policy rate of the Federal Reserve, which is the deposit rate in our model, is determined by a truncated Taylor rule, and 2) when the traditional monetary policy is persistently ineffective due to the federal funds rate being at the zero lower bound (ZLB) for an extended period, representative of the 2008 global financial crisis. Note, data shows that, the ZLB was binding for the US from 2008 to 2015. As a result, we kept the deposit rate at the ZLB for the initial $\widehat{t} = 30$ quarters in our optimal policy analysis. To check the robustness, we have also conducted an optimal policy analysis by keeping the deposit rate at the ZLB for the initial $\widehat{t} = 40$ quarters. After that, the economy exits the ZLB, and the deposit rate is determined by the truncated Taylor rule, where the ZLB can be occasionally binding depending on the magnitude of the Shocks.. Note, such a policy set-up is a close representation of the forward guidance policy undertaken by the Federal Reserve to stimulate the economy in the periods of the financial crisis.

Intuitively, the optimal policy yields an optimal combination of the fiscal instruments, duration of debt, and the tax rate by minimizing the possibilities of extreme events (by extenuating the fat-tail of the distributions of the output gap, and the inflation rate, as explained in De Grauwe and Ji, 2019). To find the optimal policy, we set $\omega \in (0, 1]$ and $\tau \in (0, 1]$. Then, for a particular combinations of ω and τ , we solve the model 2000 time periods, and calculate the loss using the standard deviation of the output gap and the inflation rate. We repeat the above mentioned steps for each possible combinations of ω and τ. Among these combinations, we choose the optimal combination of ω and τ, denoted by ω^* and τ^* that produce the minimum value of the loss function, given in equation (30) both under the economic conditions 1) and 2) mentioned above. We did the calculation of optimal policy under the parametric settings given in Table 1 below.

Table 1 : Parameters of the Model	
$a_1 = 1$	Coefficient of expected output in aggregate demand
$a_2 = 0.5$	Interest elasticity of output demand (McCallum and Nelson, 1999)
$a_3 = 0.2658$	Coefficient of public expenditure in aggregate demand
$b_1 = 0.99$	Coefficient of expected inflation in aggregate supply
$b_2 = 0.2$	Coefficient of output in inflation equation in aggregate supply
$b_3 = 0.02658$	Coefficient of government expenditure in aggregate supply
$c_1 = 1.5$	Coefficient of inflation in Taylor rule (Taylor, 1993)
$c_2 = 0.5$	Coefficient of output in Taylor rule (Taylor, 1993)
$c_3 = 0.5$	Interest smoothing parameter (Blattner and Margaritov, 2010)
$\pi^{ss} = 0$	Steady state inflation rate
$f_1 = 0.5$	Public expenditure smoothing in fiscal rule
$f_2 = 0.6$	Fiscal feedback of output gap (De Grauwe and Foresti, 2023)
$f_3 = 0.05$	Fiscal feedback of debt (De Grauwe and Foresti, 2023)
$i = 0.01$	Steady state interest rate (De Grauwe and Ji, 2019)
$\widetilde{\psi}_1 = -0.06$	$\frac{sp_{ss}}{sp_{ss}+v_{ss}}$
$\psi_2 = 1.06$	$1-\frac{sp_{ss}}{sp_{ss}+v_{ss}}$
$(\frac{G}{Y})_{ss} = 0.21$	Government expenditure to GDP ratio (De Grauwe and Foresti, 2023)
$T = 2000$	Simulation for 500 years (De Grauwe, 2012)
τ	Income tax rate (endogenously determined)
$\gamma = 2$	Willingness to learn parameter (Kukacka et al, 2018)
$\rho = 0.5$	Memory parameter (De Grauwe, 2012)
$\lambda = 0.0074$	Inflation targeting parameter (Adam and Billi, 2006)

5 Results

Table 2 reports the average duration, and the proportions of the long-term bonds of the US Treasury in the pre-crisis periods, and also in the periods of the financial crisis. Hall and Sargent (2011) calculates the average duration of the long-term bonds for the US as 3.5 years in the pre-crisis periods (2000 - 2007). On the other hand, Table 2 reports that, the average duration of the long-term bonds increased to 4.2 years in the period of financial crisis (January 2008- July 2014), as calculated by Greenwood et al. (2014).

Along with this, we calculate the average proportion of long-term bonds in the portfolio of the Treasury in the pre-crisis periods, and also in the periods of the financial crisis, and reported them in Table 2. Our calculation shows that, the proportion of long-term bonds in the Treasury's portfolio significantly rises from 25% in the pre-crisis periods (2005 - 2007) to 43% in the periods of the financial crisis (2008 - 2014). Moreover, Table 2 also reports that the revenue maximizing labour tax rate should fall from 28% in the pre-crisis periods to 22% in the periods of the financial crisis, as calculated by Trabandt and Uhlig (2011), and Trabandt and Uhlig (2012) respectively[15].

Table 2: The US Data

Pre-crisis Periods		
October 2005- December 2007	$\mu = 0.25$	Own calculation
2000- 2007	3.5 years	Hall and Sargent (2011)
Optimal Tax rate	$\tau = 0.28$	Trabandt and Uhlig (2011)
Crisis Periods		
January 2008- July 2014	$\mu = 0.43$	Own calculation
2008- 2014	$\omega = 4.2$ years	Greenwood et al. (2014)
Optimal Tax rate	$\tau = 0.22$	Trabandt and Uhlig (2012)

Table 3 reports the results of our optimal policy, which closely aligns with the corresponding data of the US reported in Table 2. Table 3 shows, given 25% long-term bonds in the fiscal portfolio, it is optimal for the Treasury to issue long-term bonds with duration 3 years in the pre-crisis periods. On the other hand, we find that, given the 43% long-term bonds in the fiscal portfolio, it is optimal for the Treasury to issue a long-term bonds of duration 3.25 years when the ZLB is persistently binding till, $\widehat{t} = 30$ quarters[16]. Beside this, a income tax rate of 25% is found to be optimal in the pre-crisis periods.

[15] Note, in our model, the years of maturity, and duration, ω of the long-term bonds are related by the following formula, $\omega = 1 - (4 \times D_M)^{-1}$, where, D_M is the Macaulay duration of the debt. See; Sims et al. (2023), Sims and Wu (2019, 2020), and Mishkin (2021) for details.

[16] Note, fiscal policy of the US suffers from a long inside lag, delaying its implementation and changes due to procedural, political and administrative delays. Due to the inside lag, the proportion of the long-term bonds in the portfolio of the Treasury remains high even after the US exits from the ZLB by 2016 (see; Figure 3 in the appendix). As a result, we also kept the percentage of the long-term bonds at fixed at 43% (the average proportion of the long-term bonds in the periods of the financial crisis as mentioned in text) to calculate the optimal instruments of the Treasury in the periods of the financial crisis (described in the economic conditions mentioned in point 2) of the text), and did not change it even if the economy exits from the persistent ZLB after either 30 quarters or 40 quarters.

However, our model predicts that a reduction in the tax rate as a part of the expansionary fiscal policy to 5% in the periods of the financial crisis when the monetary policy is unproductive due to the persistently binding ZLB constraint on the deposit rate.

Table 3: The Optimal Policy	
Pre-crisis Periods	
$(\omega^*, \tau^* \mid \mu = 0.25)$	$[0.9167 \ \text{(3 years)}, 0.25]$
Crisis Periods (Persistent ZLB)	
$(\omega^*, \tau^* \mid \widehat{t} = 30Q,\ \mu = 0.43)$	$[0.9231 \ \text{(3.25 years)}, 0.05]$
$(\omega^*, \tau^* \mid \widehat{t} = 40Q,\ \mu = 0.43)$	$[\ 0.95 \ \text{(5 years)}, 0.25]$

Along with this, we also analyze the optimal policy by keeping the deposit rate fixed at the ZLB for, $\widehat{t} = 40$ quarters to stimulate the economy in the periods of the financial crisis through monetary policy. Commensurate with the extended persistence of the ZLB, our model predicts a higher duration of the long-term bonds, issued by the Treasury to stimulate the economy through the fiscal policy when the ZLB is more persistent. We find that, the optimal duration of the long-term bonds increases from 3.25 years to 5 years when the persistence of the ZLB rises from $\widehat{t} = 30$ quarters to $\widehat{t} = 40$ quarters. In our model, higher duration of the long-term bonds automatically increases their nominal return $\left(R_t^{n,L} = \frac{1+\omega Q_t^L}{Q_{t-1}^L}\right)$ to incentivize the banking sectors to hold them. However, the higher duration of the long-term bonds simultaneously reduces their number of new issuance $(NB_t = B_t - \omega B_{t-1})$, forcing the Treasury to increase the optimal income tax rate to from 5% to 25% to finance the additional fiscal stimulus in the periods of the financial crisis.

To explain our results note that, the expansionary fiscal policy with higher government spending and/or through lower taxes is required, especially when the traditional monetary policy is unproductive due to the binding ZLB constraint on the nominal interest rate. At the backdrop of lower taxes, the Treasury needs to issue long-term debts with higher duration to mitigate its fiscal risk for financing the additional spending in the periods of the financial crisis when the traditional monetary policy is unproductive. However, to incentivize the banking sectors to hold more long-term bonds, the risk associated with them needs to be appropriately compensated with higher returns.

Note, Greenwood et al. (2014, 2015) explain the rising duration of the long-term debt in the periods of the financial crisis as a strategy for minimizing the interest rate risk of the Treasury debt by influencing the duration gap between their assets and the liabilities.

Our analysis on the other hand shows that, raising the duration of the Treasury debt is the outcome of an optimal policy, yielding an optimal trade-off between the nominal return, and the number of newly issued debt when the Treasury optimally choose their instruments - (i) the duration of the long-term bonds, and (ii) the income tax rate at the time of the financial crisis when the traditional monetary policy is unproductive due to the binding ZLB constraint on the policy rate.

6 Conclusion

This study posits that, analogous to other financial institutions, extending the duration of liabilities constitutes an optimal debt management strategy for the US Treasury during periods of financial crisis, particularly when conventional monetary policy is unproductive due to the binding Zero Lower Bound (ZLB) on policy rates. While extant literature has examined such debt management strategies across various financial institutions using ad hoc, partial equilibrium models, and explain it as a tool of mitigating the interest rate risk by manipulating the duration gap between their assets and liabilities, our research employs a more comprehensive approach. We develop a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model to analyze this phenomenon.

To do it, we augment the behavioral New Keynesian model of De Grauwe and Foresti (2023) by introducing bonds with varying maturities. In our framework, the fiscal authority optimally determines their instruments - (i) bond duration, and (ii) income tax rate to minimize a loss function, which depends on the weighted average of the standard deviation of the output gap, and the inflation rate. We use the behavioral expectation formation proposed by De Grauwe and Ji (2019), as it better matches the US data.

Greenwood et al. (2014, 2015) explain the rising duration of the long-term debt in the periods of the financial crisis is a debt management strategy of the Treasury to minimize the interest rate risk. Our model demonstrates that increasing the duration of Treasury debt is optimal in the periods of the financial crisis when the traditional monetary policy is unproductive due to the binding ZLB constraint on the deposit rate, as it generates an optimal trade-off between the number of newly issued debt, and providing enough incentives to hold them by the banking sectors. Along with this, our results also show that a reduction of the income tax rate from 25% in the pre-crisis periods to 5% in the periods of the financial crisis is optimal to stimulate the economy by an expansionary fiscal policy when the traditional monetary policy is unproductive due to the ZLB constraint on the policy rates.

References

[1] Adam, K., and Billi, R. M. (2006), Optimal monetary policy under commitment with a zero bound on nominal interest rates, *Journal of Money, Credit, and Banking*, 38(7), 1877–1905. https://doi.org/10.1353/mcb.2006.0089

[2] Adam, K., and Billi, R. M. (2007), Discretionary monetary policy and the zero lower bound on nominal interest rates, *Journal of Monetary Economics*, 54(3), 728–752. https://doi.org/10.1016/j.jmoneco.2005.11.003

[3] Aguiar, M. and Gopinath, G. (2006), Defaultable debt, interest rates and the current account, *Journal of International Economics,* 69(1), 64-83.

[4] Arellano, C. (2008), Default risk and income fluctuations in emerging economies, *American Economic Review,* 98(3), 690-712

[5] Bernanke, B., Reinhart, V., & Sack, B. (2004), Monetary policy alternatives at the zero bound: An empirical assessment, *Brookings Papers on Economic Activity*, 2004(2), 1–100. https://doi.org/10.1353/eca.2005.0002

[6] Bi, H. (2012), Sovereign default risk premia, fiscal limits, and fiscal policy, *European Economic Review,* 56(3), 389-410.

[7] Binder, M., and Pesaran, H. (2000), Solution of finite-horizon multivariate linear rational expectations models and sparse linear systems, *Journal of Economic Dynamics and Control*, 24(3), 325–346. https://doi.org/10.1016/S0165-1889(99)00008-1

[8] Blattner, T. S., and Margaritov, E. (2010), Towards a robust monetary policy rule for the euro area, *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.1617252

[9] Bohn, H. (1998), The behavior of U. S. Public debt and deficits, *The Quarterly Journal of Economics*, 113(3), 949–963. https://doi.org/10.1162/003355398555793

[10] Cardamone, D., Sims, E., and Wu, J. C. (2023), Wall street qe vs. Main street lending, *European Economic Review*, 156, 104475. https://doi.org/10.1016/j.euroecorev.2023.104475

[11] Chattopadhyay, S., and Daniel, B.C. (2018), Taylor-rule exit policies for the zero lower bound, *International Journal of Central Banking,* 14(5), 1-53.

[12] Cochrane, J. H. (2022), The fiscal roots of inflation, *Review of Economic Dynamics*, 45, 22–40. https://doi.org/10.1016/j.red.2021.06.002

[13] Daniel, B. C., and Gao, S. (2015), Implications of productive government spending for fiscal policy, *Journal of Economic Dynamics and Control*, 55, 148–175. https://doi.org/10.1016/j.jedc.2015.04.004

[14] Daniel, B. C., and Nam, J. (2022), The Greek debt crisis: Excusable vs. strategic default, *Journal of International Economics*, 138, 103632. https://doi.org/10.1016/j.jinteco.2022.103632

[15] Daniel, B. C., and Shiamptanis, C. (2012), Fiscal risk in a monetary union, *European Economic Review*, 56(6), 1289-309

[16] Daniel, B. C., and Shiamptanis, C. (2013), Pushing the limit? Fiscal policy in the european monetary union, *Journal of Economic Dynamics and Control*, 37(11), 2307–2321. https://doi.org/10.1016/j.jedc.2013.06.003

[17] Daniel, B. C., and Shiamptanis, C. (2022), Identifying countries at risk of fiscal crises: High-debt developed countries, *Canadian Journal of Economics*, 55, 828-867. https://doi.org/10.1111/caje.12600

[18] De Grauwe, P. (2012), *Lectures on behavioral macroeconomics*, Princeton University Press.

[19] De Grauwe, P., and Ji, Y. (2019), *Behavioural Macroeconomics: Theory and Policy*, Oxford University Press, UK.

[20] De Grauwe, P., and Foresti, P. (2023), Interactions of fiscal and monetary policies under waves of optimism and pessimism, *Journal of Economic Behavior & Organization*, 212, 466–481. https://doi.org/10.1016/j.jebo.2023.05.024

[21] Eaton, J. and Gersovitz, M. (1981), Debt with potential repudiation: Theoretical and empirical analysis, *The Review of Economic Studies,* 48(2), 289-309.

[22] Eggertsson, G. B., and Woodford, M. (2003), Zero bound on interest rates and optimal monetary policy, *Brookings Papers on Economic Activity*, 2003(1), 139–233. https://doi.org/10.1353/eca.2003.0010

[23] Enders, Z., Müller, G. J., and Scholl, A. (2011), How do fiscal and technology shocks affect real exchange rates? *Journal of International Economics*, 83(1), 53–69. https://doi.org/10.1016/j.jinteco.2010.08.005

[24] Ghosh, A. R., Kim, J. I., Mendoza, E. G., Ostry, J. D., and Qureshi, M. S. (2013), Fiscal fatigue, fiscal space and debt sustainability in advanced economies, *The Economic Journal*, 123(566), F4–F30. https://doi.org/10.1111/ecoj.12010

[25] Greenwood, R., Hanson, S. G., Rudolph, J. S., and Summers, L. H. (2014), Government debt management at the zero lower bound, *Hutchins Center on Fiscal & Monetary Policy at Brookings Institution*, https://www.brookings.edu/research/government-debt-management-at-the-zero-lower-bound

[26] Greenwood, R., Hanson, S. G., Rudolph, J. S., and Summers, L. H. (2015), The optimal maturity of government debt, In: *The $13 trillion question: How America manages its debt*, 1-41

[27] Hall, G. J., and Sargent, T. J. (2011), Interest rate risk and other determinants of post-wwii us government debt/gdp dynamics, *American Economic Journal: Macroeconomics*, 3(3), 192–214. https://doi.org/10.1257/mac.3.3.192

[28] Jung, T., Teranishi, Y., and Watanabe, T. (2005), Optimal monetary policy at the zero-interest-rate bound, *Journal of Money, Credit, and Banking*, 37(5), 813–835. https://doi.org/10.1353/mcb.2005.0053

[29] Kukacka, J., Jang, T., and Sacht, S. (2018), On the estimation of behavioral macroeconomic models via simulated maximum likelihood, *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.3299450

[30] McCallum, B.T., and Nelson, E. (1999), *Performance of operational policy rules in an estimated semiclassical structural model*, In: Monetary Policy Rules, University of Chicago Press, 15–56.

[31] Mishkinn, F. S. (2021), *Economics of Money, Banking, and Financial Markets*, 13th edition, Pearson.

[32] Nakov, A. A. (2008), Optimal and simple monetary policy rules with zero floor on the nominal interest rate, *International Journal of Central Banking,* 4 (2): 73–128.

[33] Proaño, C. R., and Lojak, B. (2020), Animal spirits, risk premia and monetary policy at the zero lower bound, *Journal of Economic Behavior & Organization*, 171, 221-233. https://doi.org/10.1016/j.jebo.2020.01.016

[34] Rotemberg, J. J. (1982), Sticky prices in the united states, *Journal of Political Economy*, 90(6), 1187–1211. https://doi.org/10.1086/261117

[35] Shiamptanis, C. (2023), Tax austerity: Does it avert solvency crises? *Journal of Money, Credit and Banking*, jmcb.13034. https://doi.org/10.1111/jmcb.13034

[36] Sims, E., Wu, J. C., and Zhang, J. (2023), The four equation new keynesian model, *The Review of Economics and Statistics*, 1–45. https://doi.org/10.1162/rest_a_01071

[37] Sims, E. R., and Wu, J. C. (2019), Evaluating central banks' tool kit: Past, present, and future, *Journal of Monetary Economics*, 118: 135-160.

[38] Sims, E. R., and Wu, J. C. (2020), Are qe and conventional monetary policy substitutable? *International Journal of Central Banking*, 16(1): 195-230.

[39] Taylor, J. B. (1993), Discretion versus policy rules in practice, *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214. https://doi.org/10.1016/0167-2231(93)90009-L

[40] Trabandt, M., and Uhlig, H. (2011), The Laffer curve revisited, *Journal of Monetary Economics*, 58(4), 305–327. https://doi.org/10.1016/j.jmoneco.2011.07.003

[41] Trabandt, M., and Uhlig, H. (2012), How do laffer curves differ across countries? In: Fiscal Policy after the Financial Crisis, *NBER Chapters*, pp. 211-249.

[42] Vinogradov, D. (2011), Banks, credibility and macroeconomic evolution after a production shock: Banks, credibility and shocks, *The Manchester School*, 79(3), 480–509. https://doi.org/10.1111/j.1467-9957.2009.02166.x

[43] Woodford, M. (2001), Fiscal requirements for price stability, *Journal of Money, Credit and Banking*, 33(3), 669. https://doi.org/10.2307/2673890

7 Appendix

7.1 Derivation of the Resource Constraint

Budget constraint of the household is,

$$P_tC_t + S_t = W_tL_t + (1 + i_{t-1})S_{t-1} + TR_t - P_tT_t; \tag{A.1}$$

where, C_t is the real consumption, S_t is the deposit to the financial intermediaries, L_t is the labor supply, TR_t is the transfer received from the banking sector, and T_t is the real income tax. The balance sheet of the commercial bank is,

$$Q_tB_t^{FI} + RE_t^{FI} = S_t; \tag{A.2}$$

where, B_t^{FI} is the number of bonds purchased by the financial intermediaries from the fiscal authority, RE_t^{FI} is the reserves held by the financial intermediaries. The balance sheet of the central bank is,

$$Q_tB_t^{CB} = RE_t^{FI}; \tag{A.3}$$

where, B_t^{CB} is the number of bonds purchased by the central bank from the fiscal authority

$$S_t = Q_t\left(B_t^{FI} + B_t^{CB}\right) = Q_tB_t$$

The government budget constraint is,

$$\begin{aligned} P_tT_t &= P_tG_t + (1 + \omega Q_t)B_{t-1} - Q_tB_t \\ &= P_tG_t + R_t^nQ_{t-1}B_{t-1} - Q_tB_t \end{aligned} \tag{A.4}$$

where, Q_t is the price of the bond portfolio, and R_t^n is the nominal return of the bond portfolio.

Note, equations (A.1) to (A.4) with $P_tY_t = W_tL_t$, and the Transfer of the banking sector's profit to the household, $TR_t = Q_{t-1}B_{t-1}\{R_t^n - (1 + i_{t-1})\}$ yields the resource constraint, $Y_t = C_t + G_t$.

7.2 Government Budget Constraint

This section describes the log-linearization of the government budget constraint. Equation (A.5) gives the government budget constraint, where, the primary surplus is defined as; $SP_t = T_t - G_t$

$$\frac{R_t^n}{(1+\pi_t)} V_{t-1} = SP_t + V_t$$

Log-linearization around the steady state gives,

$$\widehat{r}_t^n - \pi_t + \widehat{v}_{t-1} = \frac{sp_{ss}}{sp_{ss} + v_{ss}} \widehat{sp}_t + \frac{v_{ss}}{sp_{ss} + v_{ss}} \widehat{v}_t \tag{A.5}$$

Log-linearization of SP_t around steady state gives,

$$\widehat{sp}_t = \frac{\tau}{\tau - \frac{G}{Y}} \widehat{y}_t - \frac{1}{\frac{Y}{G}\tau - 1} \widehat{g}_t; \tag{A.6}$$

$$\widehat{sp}_t = F_4 \widehat{y}_t - F_5 \widehat{g}_t \tag{A.7}$$

Following De Grauwe and Foresti (2023), we use $\frac{G}{Y} = 0.21$. Substituting, equation (A.7) to equation (A.5) gives,

$$\widehat{r}_t^n - \pi_t + \widehat{v}_{t-1} - \widetilde{h}_t = \psi_1 \widehat{y}_t + \psi_2 \widehat{v}_t - \psi_3 \widehat{g}_t;$$

where, $h_t = -\widetilde{h}_t \sim N(0, 0.5)$ is the random shock to the government budget constraint; $\widetilde{\psi}_1 F_4 = \psi_1$; $\widetilde{\psi}_1 F_5 = \psi_3$.

7.3 Derivation of Steady State Bond Prices

We concentrate on a risk free steady state with identical return of short-run and long-run bonds, $R_{ss}^{n,L} = R_{ss}^{n,s} = (1+i) = \frac{1}{b_1}$. Note, $R_{ss}^{n,L} = R_{ss}^{n,s} = (1+i)$ yields the bond prices at the steady state as,

$$R_{ss}^{n,L} = (1+i) = \frac{1 + \omega Q_{ss}^L}{Q_{ss}^L};$$

$$Q_{ss}^L = \frac{1}{1+i-\omega}; \; Q_{ss}^S = \frac{1}{1+i}$$

where, b_1 is the discount factor in our model.

The return of the fiscal portfolio is,

$$R_t^n = (1-\mu)\,R_t^{n,S} + \mu R_t^{n,L}$$

The log-linearization of the above equation yields the return of the fiscal portfolio as,

$$\widehat{r}_t^n = (1-\mu)(-\widehat{q}_{t-1}^S) + \mu(\varpi \widehat{q}_t^L - \widehat{q}_{t-1}^L)$$

7.4 Algorithm for Solving Bond Prices

This section describes the algorithm of solving the long-term bond price gap.

1. Our model starts from period one. In period one, all the variables are in steady state. Hence, $\widehat{v}_1 = \widehat{q}_1 = \widehat{r}_1^n = \widehat{y}_1 = \pi_1 = \widehat{g}_1 = \widehat{\iota}_1 = \widehat{q}_1^L = \widehat{q}_1^S = 0$; $\widehat{q}_{SS}^L = 0$.

2. In period two, all the five i.i.d. shocks $(\varepsilon_t,\, \eta_t, u_t, \varphi_t, h_t) \sim N(0, 0.5)$ simultaneously hit the model. Hence, the agents start making the expectations. Following the expectation hypothesis, $\widetilde{E}_t(\widehat{r}_{t+1}^n) = \widehat{\iota}_t$, $\widehat{q}_t^L = -(\widehat{\iota}_t) + \varpi \widetilde{E}_t(\widehat{q}_{t+1}^L)$ (see; Cochrane, 2022), we calculate, $\widehat{q}_2^L$ as follows;

$$\widetilde{E}_2(\widehat{q}_3^L) = \frac{(\widehat{\iota}_2) + \widehat{q}_2^L}{\varpi} = \alpha_{f,t}^{\widehat{q}^L} \widehat{q}_{SS}^L + (1-\alpha_{f,t}^{\widehat{q}^L})\widehat{q}_1^L = 0 \tag{A.8}$$

$$\widehat{q}_2^L = -\widehat{\iota}_2$$

3. Similarly, for $t = 4, 5, ...$we calculate the bond prices for $t = 3, 4, 5, ...$ as follows;

$$\widetilde{E}_{t-1}(\widehat{q}_t^L) = \frac{(\widehat{\iota}_{t-1}) + \widehat{q}_{t-1}^L}{\varpi} = \alpha_{f,t}^{\widehat{q}^L} \widehat{q}_{SS}^L + (1-\alpha_{f,t}^{\widehat{q}^L})\widehat{q}_{t-2}^L = (1-\alpha_{f,t}^{\widehat{q}^L})\widehat{q}_{t-2}^L \tag{A.9}$$

$$\widehat{q}_{t-1}^L = [\varpi(1-\alpha_{f,t}^{\widehat{q}^L})\widehat{q}_{t-2}^L - \widehat{\iota}_{t-1}]$$

4. We calculate the short-run and the long-run portfolio prices as follows;

$$\widehat{q}_t^S = -\widehat{\iota}_t$$

$$\widehat{q}_t = \frac{(1-\mu)\,Q_{ss}^S \widehat{q}_t^S + \mu Q_{ss}^L \widehat{q}_t^L}{(1-\mu)\,Q_{ss}^S + \mu Q_{ss}^L} \tag{A.10}$$

using equation (A.10), we can find the values of $\widehat{q}_1, \widehat{q}_2, \widehat{q}_3, ...,$ where $Q^S_{ss} = \frac{1}{1+i}$, $Q^L_{ss} = \frac{1}{1+i-\omega}$

5. We calculate the short-run and the long-run portfolio return as follows;

$$\widehat{r}^{n,S}_t = (-\widehat{q}^S_{t-1})$$

$$\widehat{r}^{n,L}_t = \varpi \widehat{q}^L_t - \widehat{q}^L_{t-1}$$

6. The portfolio return is calculated as follows;

$$\widehat{r}^n_t = (1-\mu)(-\widehat{q}^S_{t-1}) + \mu(\varpi \widehat{q}^L_t - \widehat{q}^L_{t-1})$$

7. Following the above mentioned steps, we calculate $\widehat{q}^S_t, \widehat{q}^L_t, \widehat{r}^{n,S}_t, \widehat{r}^{n,L}_t, \widehat{q}_t$, and $\widehat{r}^n_t$ by setting $\widehat{\iota}_t = -i$, when the ZLB is binding[17].

7.5 Algorithm of Optimal Policy

Our objective is to calculate the eight endogenous variables, $(\widehat{y}_t, \pi_t, \widehat{\iota}_t, \widehat{g}_t, \widehat{v}_t, \widehat{q}^L_t, \widehat{r}^n_t, \widehat{q}_t)$ by numerically solving the following eight equations. To do it, we have written the eight equations in matrix notation. Then, following De Grauwe and Ji (2019) we have solved it using the algorithm of Binder and Pesaran (2000).

$$\hat{y}_t = \widetilde{E}_t(\hat{y}_{t+1}) - a_2(\hat{\iota}_t - \widetilde{E}_t(\pi_{t+1})) - a_3(\widetilde{E}_t(\hat{g}_{t+1}) - \hat{g}_t) + \varepsilon_t;\ t = 1, 2, 3, ...;$$

$$\pi_t = b_1 \widetilde{E}_t(\pi_{t+1}) + b_2 \hat{y}_t - b_3 \hat{g}_t + \eta_t;$$

$$\hat{\iota}_t = \max(-i, c_1\pi_t + c_2\hat{y}_t + c_3\widehat{\iota}_{t-1} + u_t);$$

$$\hat{g}_t = f_1\hat{g}_{t-1} - f_2\hat{y}_{t-1} - f_3\hat{v}_{t-1} + \varphi_t;$$

$$\widehat{r}^n_t + \widehat{v}_{t-1} = \psi_1\widehat{y}_t + \psi_2\widehat{v}_t - \psi_3\widehat{g}_t + \pi_t + \widetilde{h}_t;$$

$$\widehat{q}^L_{t-1} = [\varpi(1 - \alpha^{\widehat{q}^L}_{f,t})\widehat{q}^L_{t-2} - \widehat{\iota}_{t-1}];$$

[17] The Matlab code will be made available on request.

$$\widehat{r}_t^n = (1-\mu)(-\widehat{q}_{t-1}^S) + \mu(\varpi\widehat{q}_t^L - \widehat{q}_{t-1}^L);$$
$$\widehat{q}_t = \frac{(1-\mu)\,Q_{ss}^S\widehat{q}_t^S + \mu Q_{ss}^L\widehat{q}_t^L}{(1-\mu)\,Q_{ss}^S + \mu Q_{ss}^L}$$

The above eight equations can be succinctly written in matrix notation as given below,

$$\Theta Z_t = \Xi_1\widetilde{E}_t(Z_{t+1}) + \Xi_2 Z_{t-1} + \Sigma_t$$

where,

$$\Theta = \begin{bmatrix} 1 & 0 & a_2 & -a_3 & 0 & 0 & 0 & 0 \\ -b_2 & 1 & 0 & b_3 & 0 & 0 & 0 & 0 \\ -c_2 & -c_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \psi_1 & 1 & 0 & -\psi_3 & \psi_2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu\varpi & 1 & 0 \\ 0 & 0 & \frac{(1-\mu)Q_{ss}^S}{(1-\mu)Q_{ss}^S+\mu Q_{ss}^L} & 0 & 0 & -\frac{\mu Q_{ss}^L}{(1-\mu)Q_{ss}^S+\mu Q_{ss}^L} & 0 & 1 \end{bmatrix};$$

$$Z_t = \begin{bmatrix} \widehat{y}_t \\ \pi_t \\ \widehat{\iota}_t \\ \widehat{g}_t \\ \widehat{v}_t \\ \widehat{q}_t^L \\ \widehat{r}_t^n \\ \widehat{q}_t \end{bmatrix}; \Xi_1 = \begin{bmatrix} 1 & a_2 & 0 & -a_3 & 0 & 0 & 0 & 0 \\ 0 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

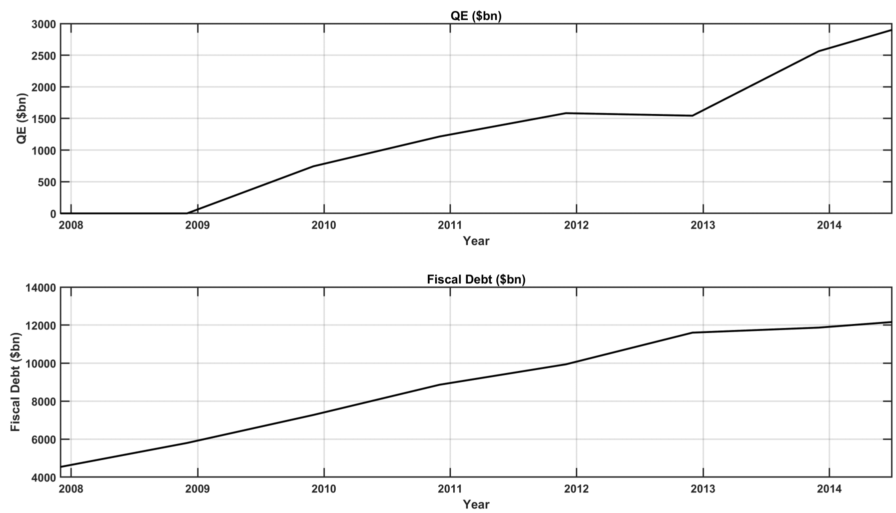
$$\Xi_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_3 & 0 & 0 & 0 & 0 & 0 \\ -f_2 & 0 & 0 & f_1 & -f_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \varpi(1-\alpha_{f,t}^{\widehat{q}^L}) & 0 & 0 \\ 0 & 0 & (1-\mu) & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \Sigma_t = \begin{bmatrix} \varepsilon_t \\ \eta_t \\ u_t \\ \varphi_t \\ h_t \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Intuitively, the optimal policy yields an optimal combination of the fiscal instruments that minimizes the possibilities of extreme events by extenuating the fat-tail of the distributions of the output gap, and the inflation rate (see; De Grauwe and Ji, 2019). To find the optimal policy, we set $\omega \in (0, 1]$ and $\tau \in (0, 1]$. Then, for a particular combination of ω and τ , we solve the model 2000 time periods, and calculate the loss using the standard deviation of the output gap and the inflation rate. We repeat the above mentioned steps for each possible combinations of ω and τ. Among these combinations, we choose the optimal combination of ω and τ, denoted by ω^* and τ^* that produce the minimum value of the loss function both under the economic conditions 1) and 2) as mentioned in the text.

7.6 Figures

This section plots the figures.

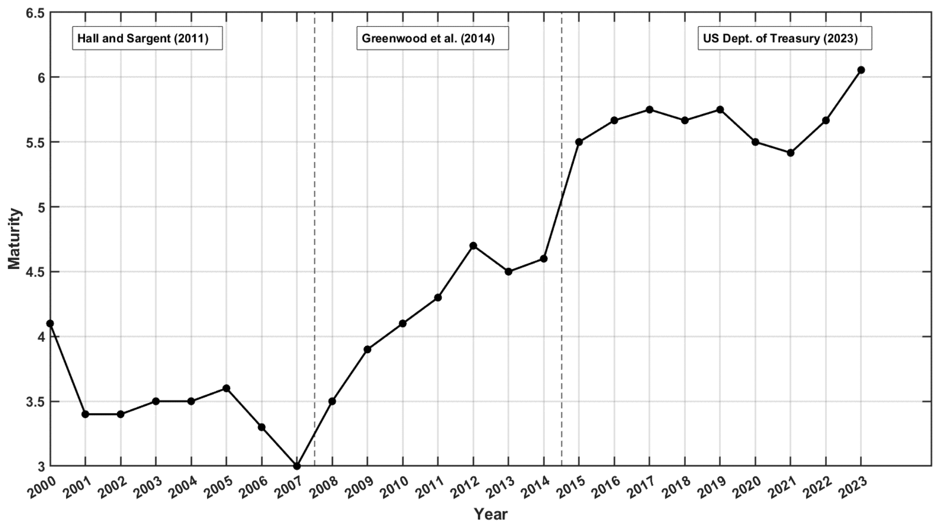
Figure 1: The QE and the Fiscal Debt



This figure plots rising trend in QE and the Treasry debt in the periods of the financial crisis.

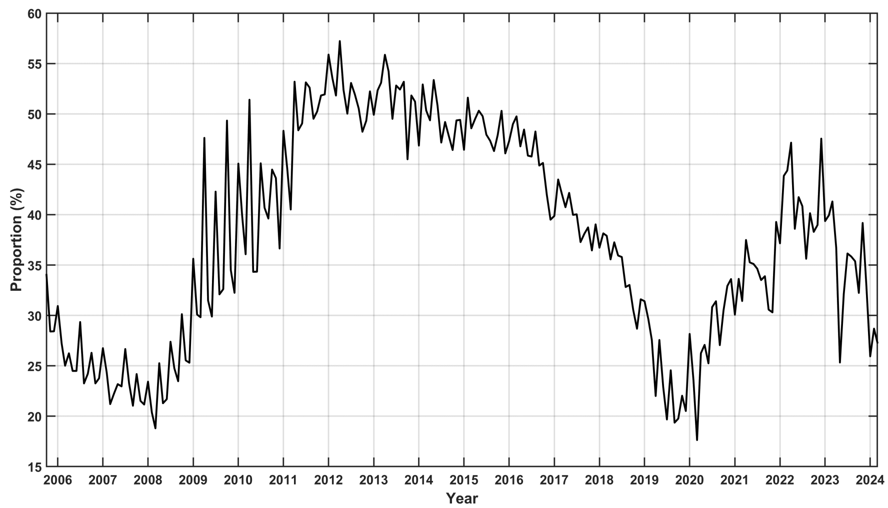
Source: Greenwood et al. (2014)

Figure 2: Average Duration of the Treasury Debt



This figure plots significant the rise in the duration of the Treasury debt in the periods of financial crisis than the pre-crisis periods. Source: Hall and Sargent (2011), Greenwood et al. (2014).

Figure 3: Proportion of Long-term Bonds



This figure depicts the significant rise in the proportion of long-term bonds in the portfolio of the Treasury in the periods of financial crisis than the pre-crisis periods. Source: own calculations from the data of the Fiscal Treasury, obtained from FiscalData.Treasury.gov