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ASSET BUBBLES IN A MONOPOLISTIC COMPETITIVE MACRO MODEL

by

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ABSTRACT

I look at the existence of asset bubbles in a monopolistically competitive dynamic macroeconomic model. The positive predictions of the model are very similar to Tirole's competitive model. But the welfare effects are very different-- in that as capital gets crowded out welfare falls. The monopolistically competitive sector contracts and the wage rate falls, lowering welfare.

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1. Introduction

Asset prices are often believed to have a "bubble" component. This component causes the price of these assets to be greater than warranted by "fundamentals". A lot of research has been done on whether such bubbles can exist in a model with perfect foresight.

Tirole (1985) showed that under certain conditions an asset which does not yield a return or utility may be held by agents in a general equilibrium model. He considers an economy consisting of overlapping generations of individuals with two period lives as in Diamond (1965). He showed that in such a setting if the economy is dynamically efficient, in that the rate of return to capital is greater than the population growth rate, an intrinsically useless asset will not be held. Capital plays a dual role in these models. It is the sole store of value and one of the two factors of production. People in a bid to provide for their old age may save so much that the rate of return is pushed below the population growth rate.[1] If this happens the "bubbly" asset has a socially useful role of providing another store of value. This asset, which vies for saving, crowds out capital. This process continues until the economy reaches a steady state at the "golden rule" level of capital stock. Weil (1987) extends Tirole's analysis to the case of stochastic bubbles. The conditions for the existence of bubbles do not change drastically in such an economy. Since I am concerned with a world without uncertainty, I shall refer to the the existing results as Tirole's, although more correctly it should be referred to as the Tirole-Weil result.

In this paper, I modify the Tirole framework by looking at an economy with two goods. The consumption good is assumed to be a differentiated good with a monopolistically competitive market

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[1] A two-period overlapping generations model may have competitive equilibria which are not Pareto Optimal because of the "double infinity" property (see Shell (1971)).

structure. In this set-up the existence of a bubble equilibrium again requires that the market interest rate be less than the population growth rate. And the new steady state that the economy reaches is again where these two are equal to each other. In terms of positive analysis, the predictions of my model are very similar to Tirole's.

The welfare implications are however very different. As capital gets crowded out welfare falls. Utility is positively related to the stock of capital even though the interest rate is less than the population growth rate. Moreover the output of the monopolistically competitive sector contracts. But it was already a suboptimal level from a social point of view and this get exacerbated by the fall in the capital stock.

There is, by now, a growing literature on dynamic models with imperfect competition (see e.g., Kiyotaki (1988), Startz (1990), Pagano (1990), Gali (1994), Chaterji and Cooper (1993) and Rotemberg and Woodford (1993)). These point to the possibility of the inefficient nature of an equilibrium attained by a <u>laissez-faire</u> system.

In the analysis below I want to distinguish between static and dynamic inefficiencies since the model has both of these. The presence of fixed costs in the monopolistically competitive sector implies pricing above marginal cost. An omniscient social planner with access to lump-sum taxes would subsidize the firms for the fixed cost through lump-sum taxation. The planner would thus raise welfare by getting rid of the static inefficiency.

On the other hand in an overlapping generations model there is the possibility of dynamic inefficiency i.e., the economy may accumulate too much capital in a competitive situation. The planner in such a situation can take the economy to the golden rule equilibrium. Or, as in the Tirole model, a bubble asset could take us there. It is the possibility of dynamic inefficiency that is my primary concern in this paper.[2]

In the model below capital accumulation (if a certain condition is satisfied) is always welfare-improving. The condition which decides whether we have welfare-improving accumulation or not depends on technology and preferences but not on the market structure. The interesting point is that it does not depend on whether the interest rate exceeds or falls short of the population growth rate. Thus, if this condition is satisfied then the process of accumulating capital does not exacerbate the static inefficiency. In this set-up we shall see that the bubble asset would reduce capital accumulation and take the economy to an equilibrium where the rate of interest is equal to the population growth rate and hence is the "golden rule" equilibrium of standard models. Here, however, the crowding out of capital reduces welfare and the "golden rule" equilibrium yields lower utility than the initial bubble-free equilibrium.

In the endogenous growth model of Grossman and Yanagawa (1993) a bubbly equilibrium, much like our model, implies a reduction in welfare. But theirs is a model with no transitional dynamics. The system jumps to a new steady state following the introduction of the bubble asset with a lower rate of growth that is welfare-reducing. In their model the shock could be a bubbly asset whose market value is determined endogenously or public debt. In our model with a fixed debt per capita, the dynamics can be represented by a scalar system while with a bubbly asset the model gives rise to a planar system.

2. The Model

The economy consists of overlapping generations of individuals (or households) who live for two periods. Individuals

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[2] There is always the possibility of immiserizing growth since we are in a second-best world. In our model capital accumulation is always welfare improving (if condition (18) below is satisfied).

supply one unit of labour in the first period of their lives and in the second period consume the saving from the first period plus the return on these savings. There are no bequests or inheritances. The population is growing at a constant rate.

There are two goods produced by the economy- a consumption good which is a differentiated good produced under conditions of increasing returns to scale and the investment good which is homogeneous and is produced under constant returns to scale. The investment good is the numeraire.

The representative household of generation t (i.e., the cohort born in t) maximizes the following utility function

$$U_t = \log C_t^1 + (1+\delta)^{-1} \log C_t^2 \tag{1}$$

where C_t^i is the real consumption in period i of a household born in t and δ is the rate of time preference.

Its lifetime budget constraint in terms of the numeraire good is

$$W_t = P_t \cdot C_t^1 + P_{t+1} \cdot R_{t+1}^{-1} \cdot C_t^2 \tag{2}$$

where W_t is the product wage rate in the capital goods sector in time period t, P_{t+i} is the relative price of the consumption good in period t+i and R_{t+1} the interest factor between t and t + 1.

This yields

$$C_t^1 = [(1+\delta)/(2+\delta)]W_t/P_t$$

and $$C_t^2 = [(R_{t+1} \cdot P_t/p_{t+1})/(2+\delta)]W_t/P_t$$

The consumption good C_t^i ,an index for any period i, is given by

$$C_t^i = n_i^{\frac{1}{1-\sigma}} \left(\sum_{j=1}^{n_i} c_t^{ij \frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1 \tag{3a}$$

and the associated price index is

$$P_i = \frac{1}{n_i} \left(\sum_{j=1}^{n_i} p_i^{j\,1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{3b}$$

The demand for each brand is given by

$$c_t^{ij} = (p_i^j / P_i)^{-\sigma} (C_t^i / n_i) \tag{3c}$$

where n_i is the number of brands per worker of the differentiated good produced, c_t^{ij} is the consumption of the j^{th} brand in the i^{th} period of an individual born at t, and p_i^j is the price of the j^{th} brand (all in period i). σ is the elasticity of substitution between brands (which is the elasticity of demand facing a firm in 3(c)).

Note that in (3a) and (3b) the consumer does not have any love-for-variety per se (i.e., an increase in variety by itself does not yield any utility).

Since we shall be concerned with a symmetric equilibrium, (3a) and (3b) reduce to

$$C_t^i = n_i c_t^i$$

$$P_i = p_i$$

The indirect utility function is given by

$$V_t = m + [(2+\delta)\log(W_t/p_t) + \log(p_t \cdot R_{t-1}/p_{t-1})] / (1+\delta) \tag{4}$$

where m is a constant, W_t/p_t is the consumption wage at t and $p_t \cdot R_{t-1}/p_{t-1}$ is the one period consumption interest factor.

I now turn to the production side of the economy. The investment good is produced under conditions of constant returns to scale with a Cobb-Douglas technology. Each brand of the consumption good requires a fixed amount of overhead capital. This fixed cost is a recurring cost in each period and not a sunk cost. Except for this fixed cost output in this sector (ie., the variable cost) is produced by a linear homogeneous Cobb-Douglas technology. The fixed cost element makes for increasing returns to scale in the consumption goods sector as a whole. The market structure in this industry is monopolistically competitive. I assume that entry within the period drives profits down to zero (the Chamberlinian "large group" case). In such an equilibrium a proportion σ^{-1} of the revenue of a firm goes to cover fixed cost and the rest (a proportion $((\sigma-1)/\sigma)$ of revenue) to cover variable costs. I also assume that factors of production are mobile between sectors within the period and that capital depreciates completely in the process of production.[3]

$$a_{LI} \cdot W_t + a_{KI} \cdot R_t = 1 \tag{5a}$$

$$\frac{\sigma}{\sigma-1}\,(a_{LC} \cdot W_t + a_{KC} \cdot R_t) = p_t \tag{5b}$$

$$R_t \cdot K_F = \frac{1}{\sigma}\,(p_t \cdot c_t) \tag{5c}$$

where a_{ij} is the requirement of the i^{th} input (i = K, L) in the j^{th} line of production (j = C, I). Equation (5a) is the price equal to marginal (and average) cost in the investment goods sector. Price is a mark-up $(\sigma/\sigma-1)$ on marginal cost in the consumption goods sector (equation (5b)). Finally in (5c) we have σ^{-1} of revenue of each brand going to cover fixed costs - K_F being the overhead capital.

[3] The assumption about depreciation is an innocuous assumption because most of my analysis is concerned with the steady state where there is no capital gains term in the return to capital for households.

In any period there are two goods markets and two factor markets. By Walras Law if three of these are in equilibrium then so is the fourth one. We thus have three market-clearing equations in (6a), (6b) and (6c)

$$a_{LC} . n_t c_t + a_{LI} . I_t = 1 \tag{6a}$$

$$a_{KC} . n_t c_t + a_{KI} . I_t + K_F . n_t = k_t \tag{6b}$$

$$b_t + I_t = S_t . p_t = (2 + \delta)^{-1} W_t \tag{6c}$$

Equations (6a), (6b) and (6c) are the market clearing conditions for the labour, capital and investment goods markets respectively. The variable I_t is the output per worker of the investment good, S_t is the saving of the young per worker (in units of consumption), b_t is the stock of the bubble asset per worker and k_t is the capital stock per worker (all in time period t). Given the logarithmic form of the utility function we have a proportion $1/(2+\delta)$ of labour income saved by the young in (6c). These savings must be held either as capital or in the form of the bubble asset.

Finally, the dynamics of the economy is given by

$$(1+g) . b_{t+1} = R_{t+1} . b_t \tag{7a}$$

$$(1+g)\, k_{t+1} = I_t \tag{7b}$$

where g is the population growth rate.

Equation (7a) is a portfolio balance equation which says that the bubble asset is held in equilibrium only if it pays the same return R_{t-1} as capital i.e., its price in terms of the numeraire falls at a rate R_{t+1}. Equation (7b) equates the capital stock in the next period to this period's investment per worker.

Logarithmic differentiation of equation (5a), (5b) and (5c) yields

$$\theta_{LI} \cdot \hat{W}_t + \theta_{KI} \cdot \hat{R}_t = 0 \tag{8a}$$

$$\theta_{LC} \cdot \hat{W}_t + \theta_{KC} \cdot \hat{R}_t = \hat{p}_t \tag{8b}$$

$$\hat{R}_t = \hat{p}_t + \hat{c}_t \tag{8c}$$

where θ_{ij} is the share of the i^{th} input in the j^{th} sector marginal cost (j=C,I) and a hat over a variable denotes a percentage change. Since in (5c) fixed cost consists entirely of capital the share of capital in fixed cost is unity in 8(c).

From (8a), (8b) and (8c) we have

$$\hat{W}_t / \hat{c}_t = -\theta_{KI} / \theta_{LC} \tag{9a}$$

$$\hat{R}_t / \hat{c}_t = \theta_{LI} / \theta_{LC} \tag{9b}$$

$$\hat{p}_t / \hat{c}_t = (\theta_{LI} - \theta_{LC}) / \theta_{LC} \tag{9c}$$

Note that (9a), (9b) and (9c) imply that the indirect utility of a 't' period household (equation (4)) depends only on c_t and c_{t-1}.

Further from (6a), (6b) and (6c) we have (by differentiating logarithmically).

$$\beta_{LC} \cdot \hat{c}_t + \beta_{LI} \cdot \hat{I}_t + \beta_{LC} \cdot \hat{n}_t = [\hat{W}_t - \hat{R}_t](\beta_{LC} \cdot \theta_{KC} + \beta_{LI} \cdot \theta_{KI}) \tag{10a}$$

$$\beta_{KC} \cdot \hat{c}_t + \beta_{KI} \cdot \hat{I}_t + (1-\beta_{KI}) \cdot \hat{n}_t = -[\hat{W}_t - \hat{R}_t][\beta_{KC} \cdot \theta_{LC} + \beta_{KI} \cdot \theta_{LI}] + \hat{k}_t \tag{10b}$$

$$\eta \hat{b}_t + (1-\eta) \hat{I}_t = \hat{W}_t \tag{10c}$$

where β_{ij} is the share of the j^{th} sector in the total employment of the i^{th} input. In (10a) and (10b) the elasticity of substitution between inputs in sectors C and I have been assumed to be unity i.e., the technology is Cobb-Douglas.

If we substitute for $\hat{W}_t$ and $\hat{R}_t$ from (9a) and (9b) into (10a), (10b) and (10c), we can solve for $\hat{c}_t$, $\hat{I}_t$ and $\hat{n}_t$ in terms of $\hat{k}_t$ and $\hat{b}_t$. The parameter η is the steady state share of b in saving.

3. The Economy without Bubbles

Let us look at the version of the model where there is no bubble asset i.e., $b_{t+i} = 0$ for all i. I refer to this as the MCD version (for Monopolistic Competition - Diamond).

The dynamics of this economy can be represented in terms of k alone. This is due to the logarithmic preference structure.

Equation (9c) and (10c) gives us

$$\hat{I}_t = \hat{W}_t = (-\theta_{KI} / \theta_{LC})\, \hat{c}_t \tag{12}$$

Substituting this in (10a) and (10b) we can solve for $\hat{c}$ and $\hat{n}$ in terms of $\hat{k}$

$$\hat{c}_t / \hat{k}_t = -\theta_{LC} \tag{13}$$

$$\hat{n}_t / \hat{k}_t = 1 \tag{14}$$

From (12) and (13) then

$$\hat{I}_t / \hat{k}_t = \theta_{KI} \tag{15}$$

The dynamics is given by (linearizing around the steady state)

$$dk_{t+1} = \theta_{KI}\, dk_t \tag{16}$$

In equation (16) $dk_{t+i} \equiv (k_{t+i} - k^*)$ where k^* is the steady state value of k (an asterisk denotes a steady state value).

Since $0 < \theta_{KI} < 1$, convergence is monotonic and the steady state is stable.

Note that so far we have not made any assumptions about the relative capital intensities of the two lines of production which use both capital and labour. This is due to the Cobb-Douglas production technology combined with the logarithmic utility function.[4]

Next we do welfare analysis <u>across steady states.</u> Substituting (9a), (9b), (9c) and (13) in (4) we have

$$dV^* = -[(2+\delta)\theta_{KC} - \theta_{LI}]/(1+\delta).\theta_{LC}.\hat{c}^* \quad (17a)$$

$$=[(2+\delta)\theta_{KC} - \theta_{LI}]/(1+\delta).\hat{k}^* \quad (17b)$$

Note that all the θ_{ij}'s are constants in (17) because of the Cobb-Douglas technology. So capital accummulation increases welfare iff

$$(2+\delta)\theta_{KC} > \theta_{LI} \quad (18)$$

If this condition is satisfied capital accumulation is welfare improving no matter what R^* is relative to (1 + g). At the MCD steady state if the expression in (18) is positive then we cannot have capital overaccumulation from "the individual point of view".

By this I mean that since there is a static distortion due to the presence of mark-up pricing in the monopolistically competitive sector, an omniscient planner could definitely do better than the laissez-faire equilibrium, if he/she had access

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[4] See e.g., Azariades (1993) p. 264 for a discussion of the "triple Cobb-Douglas" economy.

to lump-sum taxes. The planner would produce only one variety since there is no love for variety per se, and raise the fixed cost RK_F through lump-sum taxes. The pricing of the consumption good would then be at marginal cost.

I show the case where equation (18) has been satisfied in Figure 1 with the steady state values of log W and log R on the axes. The curve VV is the steady state (indirect) is outility locus from equation (4). FF is the factor-price frontier i.e., equation (5a). AA is the accumulation equation in the $\log R^* - \log W^*$ space[5]. The economy is dynamically efficient since VV is flatter than FF (see Matsuyama(1991)).

4. Bubbles

Suppose that we now have a bubble asset in the economy. How these are introduced need not detain us here (because our primary interest is in the steady-state). In equation (8a) we see that these assets will be held if the price of these assets rises (relative to the numeraire) at the rate of return on capital. In the competitive model of Tirole (1985), if R < 1+g, then the bubble asset, by crowding out investment, raises welfare. The steady state of the economy with bubbles is when $R^* = 1+g$, i.e., the "golden rule" capital stock. This is precisely what an omniscient planner would have chosen. Bubbles cannot exist if R > 1+g because this implies it would grow faster than the economy and in finite time become larger than the wage bill. At the outset perfect foresight would rule out the movement of the economy along such a path.

The original MCD steady state, by assumption, has $R^* < (1+g)$. Even at this capital stock, individual welfare is increasing in the steady capital. So while the bubbly asset takes us towards the "golden rule" capital stock with $R^* = 1+g$, welfare

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[5] We have $(1+g)\,k_{t+1} = (2+\delta)^{-1} W_t$. Next use $R_{t+1} = R_{t+1}(c_{t+1}(k_{t+1}))$. In the steady state we have AA in the R-W space.

falls. There is nothing golden about the "golden rule" in this model.

If the bubble asset is held by agents it must grow at the rate of interest and in per capita terms the dynamics is given by (7a) and (7b)

$$(1+g)\cdot b_{t+1} = R_{t+1}\cdot b_t$$

$$(1+g)\cdot k_{t+1} = I_t$$

We can linearize them around a steady-state (b^*,k^*) and write it in matrix form as

$$\begin{bmatrix} db_{t+1} \\ dk_{t+1} \end{bmatrix} = A\cdot\begin{bmatrix} db_t \\ dk_t \end{bmatrix} \tag{19}$$

where $dx_{t+i} \equiv x_{t+i} - x^*$ (x = b, k)

The elements of the matrix A are given in the Appendix. It is also shown there that A has two roots λ_1 and λ_2 such that

$$0 < \lambda_1 < 1 < \lambda_2 \tag{20}$$

if the following two sufficient conditions are met

$$\beta_{LI} > \beta_{KI} \tag{21}$$

and

$$\theta_{LI} > \eta/(1-\eta) \tag{22}$$

where $\eta \equiv b^*/S^*$ from (10c).

Condition (21) loosely speaking, says that the investment good is labour-intensive. This is not exact because there are three uses capital can be put to, i.e., in I, C, and F. Condition (22) requires that b as a proportion of saving is not "too large" since θ_{LI} is the share of labour in the labour-intensive sector.

To get an idea about the dynamic movement of the variables, we draw a phase diagram for the following system

$$\begin{bmatrix} \Delta b_{t-1} \\ \Delta k_{t-1} \end{bmatrix} = B . \begin{bmatrix} db_t \\ dk_t \end{bmatrix} \tag{23}$$

where $\Delta x_{t-1} = dx_{t+1} - dx_t \qquad (x = b, k)$

and $\quad B \equiv A - I$

It can be shown (the details are in the Appendix) that the determinant of B is negative so we have a saddle-point structure. The $\Delta b_{t+1} = 0$ locus is upward sloping in k - b space in Figure 2 and $\Delta k_{t+1} = 0$ locus is downward sloping. The stable arm is upward sloping and the long run equilibrium is at (k_G^*, b^*).

The steady state with the bubble asset requires $b_{t+1} = b_t = b^*$ Hence $R^* = 1 + g$ i.e., the capital stock is at its "golden rule" value (k_G^*). As before

$$k^*(1+g) = I^* \tag{24}$$

But now

$$I^* = (2+\delta)^{-1} . W^* - b^* \tag{25}$$

Starting from a capital-labour ratio where $R^* < (1+g)$, the economy has reached the "golden rule" steady state with a higher interest rate and a lower wage rate(associated with a lower capital per worker) and $b^* > 0$.

From equation (17a) we know that the effect of the bubble asset on welfare can be calculated from its effect on c^*. In the Appendix it is shown that c^* rises, both because of $db^* > 0$ and $dk^* < 0$.

Welfare in the new steady state falls if (18) is satisfied i.e., if

$$(2+\delta)\,\theta_{KC} > \theta_{LI}$$

$$\text{then} \qquad dV^*/db^* < 0 \tag{26}$$

As Figure 2 shows that the MCD steady state at k_D^* exceeds k_G^*. The introduction of the bubble takes us to k_G^* just as in Tirole's model, by crowding out investment and lowering saving along the adjustment path. But in our model k_G^* does not have the same normative connotation that it does in Tirole's analysis. This is because at k_D^* a steady state increase in k still increases welfare. In Figure 1 the new steady state is at point B which is on a lower isoutility locus than the initial equilibrium (at A).

Why was the original steady state, where R^* had been driven down below (1 + g), not dynamically inefficient? This is a two sector model and we may have other factors at play, in addition to the rates of profit and population growth, such as sectoral capital intensities, the logarithmic form of the utility function, the difference between the consumption and the product wage etc.

Across steady states capital accumulation reduces the size of the monopolistically competitive sector. In the new steady state

$$n^*.c^* = [(1+\delta)(2+\delta)^{-1}.(W^*/p^*) + R^*.(2+\delta)^{-1}.(W^*/p^*)]$$

$$= W^*/p^* \tag{27}$$

i.e., at the "golden rule" value of k^* all wages are consumed. Thus if c^* rises and W^*/p^* falls at k_G^* compared to k_D^* then, from (27), n^* falls. Note the model has been specified in such a way that the fact that n^* falls does not affect utility directly. What matters is that $n^*.c^*$ falls . This shrinkage of the consumtion goods sector accentuates the static inefficiency. This monopolistically competitive sector whose output was "too low" from a social standpoint contracts even more in the new steady state.

To sum up then, the introduction of the bubble asset crowds out capital and lowers welfare by reducing the consumption wage rate which more than offsets the gain in welfare due to a high interest rate. With the fall in the capital stock we have a fall in the output of the monopolistically competitive sector which further lowers welfare.

5. Conclusions

In a two sector overlapping generations model having a monopolistically competitive sector I analyzed conditions under which a bubbly equilibrium would exist. These conditions turn out to be very similar to those in Tirole (1985). In particular, the economy can support an equilibrium with a bubble asset only if the interest rate is less than the growth rate of the economy. The new steady state with the bubble asset is where the interest rate is the same as the exogenously given growth rate.

While in Tirole's analysis this implies that the bubble asset is a panacea for dynamic inefficiency, in my model it is not the case. In the model of this paper welfare was increasing in the steady state capital stock. Crowding out of capital reduces private welfare. Further the crowding out makes the monopolistically competitive sector shrink from its previous sub-optimal level.

Appendix

Substituting (9a) and (9b) into (10a), (10b) and (10c) we can solve for

$$\hat{c}_t = \hat{c}_t(\hat{k}_t, \hat{b}_t)$$

and

$$\hat{I}_t = \hat{I}_t(\hat{k}_t, \hat{b}_t)$$

We have $\quad \hat{c}_t / \hat{k}_t = -(1-\eta)\beta_{LC}/\Omega$

$$\hat{c}_t / \hat{b}_t = \eta(\beta_{LI}-\beta_{KI})/\Omega$$

$$\hat{I}_t / \hat{k}_t = \theta_{KI}\beta_{LC}/(\theta_{LC}.\Omega)$$

$$\hat{I}_t / \hat{b}_t = -\eta(\beta_{LC}+\theta_{KI}(\beta_{LI}-\beta_{KI}))/(\theta_{LC}.\Omega)$$

where $\quad \Omega \equiv [(\theta_{LI}-\eta)(\beta_{LI}-\beta_{KI})+(1-\eta)\beta_{LC}]/\theta_{LC}$

we substitute these in equation (19) reproduced below

$$\begin{bmatrix} b_{t-1} \\ k_{t-1} \end{bmatrix} = A \cdot \begin{bmatrix} b_t \\ k_t \end{bmatrix}$$

$$a_{11} = \{1+b^* R_c c_K I_b \cdot (1+g)^{-2}\} / \{1-b^* R_c c_b (1+g)^{-1}\} > 0$$

$$a_{12} = \{b^* R_c c_K I_K \cdot (1+g)^{-2}\} / \{1-b^* R_c c_b (1+g)^{-1}\} < 0$$

$$a_{21} = I_b(1+g)^{-1} < 0$$

$$a_{22} = I_K(1+g)^{-1} > 0$$

We have used (21) and (22) in the text to sign the a_{ij}'s.

$$|A| = I_K / \{(1+g) - b^* R_c c_b\} > 0$$

The trace of A $\equiv Tr(A) = a_{11} + a_{22} > 0$

The characteristic polynomial of A is

$$p(\lambda) = \lambda^2 - Tr(A).\lambda + |A|$$

Since a_{21} and a_{12} are both negative the roots are real.[6]

$$p(0) = |A| > 0$$

we can show $p(1) = 1 - Tr(A) + |A| < 0$ (This is $|B|$ below)

Hence $0 < \lambda_1 < 1 < \lambda_2$.

The matrix $B \equiv A - I$ where I is the (2×2) identity matrix.

$$b_{11} = \{\bar{b}R_c c_b I_b.(1+g)^{-2} + \bar{b}R_c c_b.(1+g)^{-1}\}/\{1 - \bar{b}R_c C_b(1+g)^{-1}\} > 0$$

$$b_{12} = a_{12} < 0$$

$$b_{21} = a_{21} < 0$$

$$b_{22} = \{I_K - (1+g)\}/(1+g) < 0$$

Therefore the determinant of B < 0.

In Figure 2 the $\Delta b_{t-1} = 0$ schedule in upward sloping with the vertical arrows pointing away from it. The $\Delta k_{t-1} = 0$ schedule is downward sloping and the horizontal arrows pointing towards it. The saddle path is upward sloping and flatter than the $\Delta b_{t-1} = 0$ line.

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6 For λ_1 and λ_2 to be real we require

$$Tr(A)^2 - 4|A| > 0$$

or $(a_{11} - a_{22})^2 + 4a_{12}a_{21} > 0$

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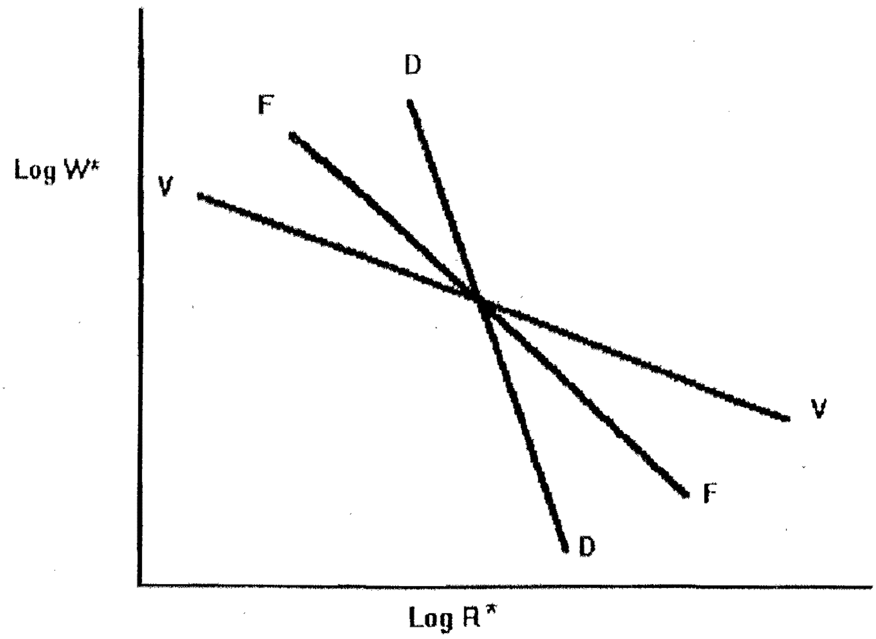

FIGURE 1

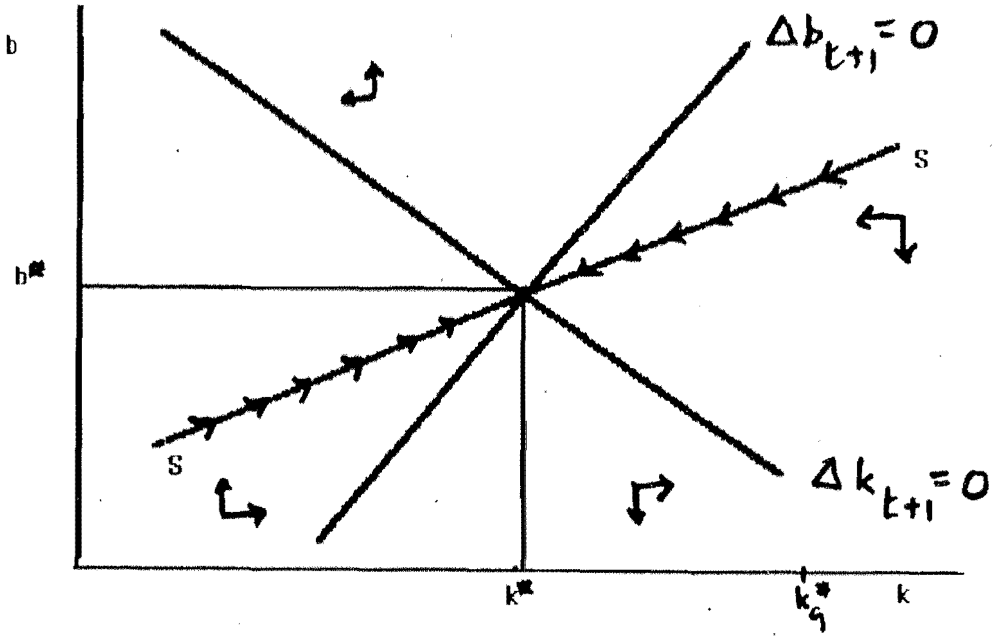

FIGURE 2

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