Centre for Development Economics

WORKING PAPER SERIES

Prices, Profits and Resource Mobilisation in a Capacity Constrained Mixed Economy

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Working Paper No. 48
(Revised: January 1998)

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CDE
August, 1997

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ABSTRACT

A mixed economy is one in which the state is actively engaged in many production sectors. Profits of these state owned firms, if any, go into the Revenue side of the state budget. In a general equilibrium model with demand feedback from the households and a Leontief type production system, we study situations of capacity constraints and rationing. A strong condition emerges as necessary and sufficient to provide non-negative prices. We call it the solvency condition. If requires the value of all sales by the state to the private producing sector plus its budgetary deficit to strictly exceed the value of its purchases from the private sector.

But even a positive product price (for the state sector) does not necessarily imply positive profits without further and more restrictive conditions. Finally, in a model of subsidized rationing (at below market clearing price) by the state of its own products we establish that the profits of the state sector are bounded above and of the private sector bounded below. This too would be important in understanding the inner dynamics of a mixed economy.

ACKNOWLEDGEMENT

It is a pleasure to record our gratitude to K. Sundaram, it would be painful to mention the quantum of our indebtedness to him.

1. Introduction

The last few years have witnessed a wave of new economic policies aimed at liberalisation and structural adjustment, being undertaken in many developing economies including India. The centrepiece of the guiding philosophy of these policies has been to allot a wider space and role to market forces. Consequently, the so called "commanding heights of the public sector" and more widely the role of the state as an economic agent in these economies are seen to be de-emphasized. These developments have given rise to a debate on the role of the state (versus that of the market) in shaping the destinies of economies and societies.[1] Whatever may be the outcome of this debate, it is premature to assume away the continued role of the government and more particularly of the public sector in most developing economies over some decades to come. For, the public sector in countries like India, China and Russia is still very large and in command of some basic goods crucial for both production and consumption. India's recent experience with regard to pricing of petroleum products, or foodgrains under the public distribution system or power tariffs or coal prices - all bear witness to this fact. Importantly, the decisions in this regard are not simple in so far as they have on the one hand important welfare effects and on the other complex fiscal implications. Some of these issues which remain relevant even under the new economic policy regime were discussed by us earlier.[2]

As a sequel to that paper the present exercise focusses on a mixed economy which faces explicit capacity constraints in one or more sectors. The economy is mixed in the sense that one commodity producing sector (of two) is owned by the state and its profits are a net revenue of the state. But if one assumes as we do that the economy is indecomposable then a capacity constraint even in one sector constrains output in all sectors. For a less developed economy it is quite natural to assume that the constraints on capacity are imposed by inadequate capital stock, i.e., plant and machinery.

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[1] See Datta-Chaudhuri (1990).

[2] Mukherji, Pandit, and Sundaram (1992).

In the earlier paper we had outlined an analytical framework within which one can answer questions relating to the determination of outputs and prices when the government of a mixed economy tries to mobilise resources by acquisition of commodities (or services) in specific physical magnitudes. The proposed framework highlighted an interaction between the production system, income generation, levels of demand and price adjustments on the one hand and government's expenditure decisions subject to its budget constraint and implied variations in money supply on the other. It was assumed all along that capacity in all sectors was unconstrained so that the system was primarily driven by demand. The paper also analysed how factor shares of income varied under alternative regimes. This paper will subsequently be referred to as MPS.

The questions considered in this and the earlier paper have been the subject of debate in India since the early eighties. The main contributions in this debate, summarised earlier as (in MPS) were either based on faulty analysis or not focussed on the main issues[3]. We established the important proposition under a regime of excess capacity that it is preferable to finance increased government expenditure by a budgetary deficit than by raising administered prices of public sector goods and services not only because the former leaves prices unchanged but also because the latter depresses output.

2. The Model

For reasons of continuity we shall consider a miniature economy with two commodities and one service. Commodity 1 is produced in the public sector. We stick to the earlier notation which is as follows:

x_i (i=1,2)	Gross levels of output
c_i (i=1,2,3)	Total consumption or demand
c_{ip}(i=1,2)	Private consumption or household demand.
g_i (i=1,2)	Government purchases
p_i (i=1,2,3)	Unit prices
w	Wage rate

[3] See for example, Chetty and Ratha (1987), Dasgupta (1992), Jha and Mundle (1987), Panda and Sarkar (1990), Pandit and Bhattacharya (1987) and Sundaram and Tendulkar (1987).

θ_i (i=1,2)	Unit mark up rates
W_0	Government expenditure on wages and salaries for administration etc.
m^s, m^d	Stock of money supplied, demanded
Y	Total Household (private) income

It should also be useful to recall the assumptions underlying this as well as the earlier exercise. These are:

(i) The production structure is Leontief type and indecomposable specified by the intermediate input coefficients $a_{ij}(i,j=1,2)$ and the labour coefficients $a_{oi}(i=1,2)$.

(ii) Utility function of households is Cobb-Douglas type yielding constant budget share demand functions. These shares are denoted by α, β, γ and δ for the two commodities, one service and money.

(iii) All savings are held as money because no interest - bearing financial assets exist.

(iv) Unit mark-ups are additive over prime production costs.

(v) Nominal wage rate w and government expenditure on wages and salaries W_0 are exogenously fixed.

(vi) Government imposes no direct or indirect taxes.

(vii) There are no external transactions.

For the present analysis we impose the restriction that productive capacity in the two commodity producing sectors is subject to an effective ceiling. Thus $x_1 \leq x_1^*$ and $x_2 \leq x_2^*$ which implies that net outputs c_1 and c_2 are subject to upper limits c_1^* and c_2^* respectively. The problem for the state then is to choose the mark-up rate θ_1 and money supply m consistently with overall equilibrium. It should be obvious that the introduction of effective capacity constraints makes the problem far more difficult to handle but equally more relevant and meaningful to a developing mixed economy. Our treatment is purely analytical.

After some manipulations meant to weed out the unnecessary relationships the formal model simplifies to:

$$c = (I - A) x \qquad x \leq x^* \tag{2.1}$$

$$p = (I - A')^{-1} \{ w a_o + \theta \} \tag{2.2}$$

$$p_3 = \lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 w \tag{2.3}$$

$$m^s = W_o + \bar{p}' \bar{g} - \theta_1 x_1 + m_b \tag{2.4}$$

$$Y = W_o + w a_o' x + p_3 c_3 + \theta_2 x_2 \tag{2.5}$$

$$\underline{c} = \begin{pmatrix} \alpha / p_1 \\ \beta / p_2 \end{pmatrix} (Y + m_b) + \underline{g} \tag{2.6}$$

$$c_3 = \gamma (Y + m_b) / p_3 \tag{2.7}$$

$$m^d = \delta (Y + m_b) \tag{2.8}$$

variables in italics are vectors.

It is easy to see that (2.1) gives the production system; (2.2) and (2.3) price formation; (2.4) follows from the government budget constraint; (2.5) determines private income; (2.6) and (2.7) demand for the two commodities and the service and (2.8) is a disguised saving function.

3. Balancing the Budget

Before we proceed any further it will be useful to take note of the following implication of the budget constraint. (2.4) gives us:

$$m - m_b + \theta_1 x_1 = W_o + p_1 g_1 + p_2 g_2$$

Substituting $\theta_1 = (1 - a_{11}) p_1 - a_{21} p_2 - w a_{01}$, we get

$$(m - m_0) + [(1 - a_{11})x_1 - g_1]p_1 = W_0 + wa_{01}x_1 + (a_{21}x_1 + g_2)p_2$$

Notice that $(1 - a_{11})x_1 - g_1 \equiv S_1$ is the quantity of product 1 sold by the public sector to the private sector. Similarly, $a_{21}x_1 + g_2 \equiv S_2$ is the quantity of product 2 sold by the private sector to the public sector. Now denoting public sector's total wage bill on both production and administration by $W = W_0 + a_{01}x_1$ and assuming without loss of generality that $m_0 = 0$ we have, $m - p_2S_2 = W - p_1S_1$ which can be expressed as:

PROPOSITION 1: The excess (or shortfall) of increased money supply over the public sector's purchase (of product 2) from the private sector must be exactly equal to the excess (or shortfall) of public sector's total wage bill over its sales (of commodity 1) to the private sector.

We shall see an important consequence of this in proposition 2 below. Note that if $W > p_1 S_1$ then $m > p_2 S_2$ and vice versa.

Two distinct adjustments are possible in such a capacity constrained set up. These are:

(a) Mark-up rates θ_1 and θ_2 are endogenously determined so as to clear markets. This necessarily implies that suitable adjustments in m, satisfying the budget constraint (2.4), must accompany changes in θ_1 and θ_2. This implies that monetary policy is in this case passive.

(b) A second possibility is that the government supplies the available output of sector 1 at a fixed price p_1^*. This is indeed the case closest to what occurs for several public sector products in a planned economy. In the present discussion we assume p_1^* to be well below its market clearing level so that we are dealing with a case in which output of the public sector is rationed to consumers, without assuming any particular rationing scheme.

It is useful now to recall proposition 1 from our earlier paper which says that no matter what regime is considered and which variables are fixed and which ones are left free to adjust and how, money demand will always equal its supply. In other words, government will always be able to induce households to hold exactly as much money as it decides to supply. Hence $m^s = m^d = m$ always holds. Bearing this in mind, let us consider the two cases as follows.

4. Market Clearing Prices and Passive Money Supply

From (2.2) we see that

$$\theta_1 = (1-a_{11})p_1 - a_{21}p_2 - wa_{o1} \ ; \ \theta_2 = -a_{12}p_1 + (1-a_{22})p_2 - wa_{o2} \qquad (4.1)$$

But from (2.6) we have, on setting $Y + m_o = m/\delta$

$$p_1 = \frac{\alpha m}{\delta(c_1 - g_1)} \text{ and , } p_2 = \frac{\beta m}{\delta(c_2 - g_2)} \qquad (4.2)$$

Thus

$$\left.\begin{aligned} \theta_1 &= \left(\frac{(1-a_{11})}{c_1 - g_1} - \frac{\beta a_{21}}{c_2 - g_2}\right) m/\delta - wa_{01} \\ \theta_2 &= \left(\frac{\alpha a_{12}}{c_1 - g_1} + \frac{\beta(1-a_{22})}{c_2 - g_2}\right) m/\delta - wa_{02} \end{aligned}\right\} \qquad (4.3)$$

However, m is not a free variable because it must satisfy (2.4) i.e.,

$$W_o + p_1 g_1 + p_2 g_2 = m - m_o + \theta_1 x_1 \qquad (4.4)$$

Notice that any changes in θ_1 and θ_2 would alter p_1 and p_2 and therefore the LHS of (4.4); of course, a change in θ_1 also changes the RHS of (4.4). As stated earlier we assume $m_o = 0$.

The solution (4.3) does not reveal some of the interesting features of the problem, for, if prices clear markets as well as satisfy mark-ups on costs then p_1 ,p_2 , θ_1 and θ_2 need to be jointly determined. In this exercise, however, in the present context, θ_2 is a passive adjuster. To appreciate this, let us look at the following basic system:

$$\left.\begin{aligned}
p_1 &= \frac{\alpha}{\delta c_{1p}} m = \frac{\alpha}{\delta c_{1p}} (W_0 + p_1 g_1 + p_2 g_2 - \theta_1 x_1) \\
p_2 &= \frac{\beta}{\delta c_{2p}} m = \frac{\beta}{\delta c_{2p}} (W_0 + p_1 g_1 + p_2 g_2 - \theta_1 x_1) \\
\theta_1 &= p_1 - a_{11} p_1 - a_{21} p_2 - w a_{o1} \\
\theta_2 &= p_2 - a_{22} p_2 - a_{12} p_1 - w a_{o2}
\end{aligned}\right] \qquad (4.5)$$

where $c_{1p} = c_1 - g_1$ and $c_{2p} = c_2 - g_2$

The first three equation of (4.5) determine p_1 ,p_2 and θ_1 while θ_2 is fixed afterwards. It is important to appreciate the fact that this asymmetry between θ_1 and θ_2 is due to the presence of θ_1 in the government budget constraint.

Dropping the fourth equation and rearranging, we have

$$\left.\begin{aligned}
(1 - a_{11}) p_1 - a_{21} p_2 - \theta_1 &= w a_{o1} \\
(\delta c_{1p} - \alpha g_1) p_1 - \alpha g_2 p_2 + \alpha x_1 \theta_1 &= \alpha W_0 \\
-\beta g_1 p_1 + (\delta c_{2p} - \beta g_2) p_2 + \beta x_1 \theta_1 &= \beta W_0
\end{aligned}\right) \qquad (4.6)$$

or, in matrix-vector notation,

$$\begin{bmatrix} 1-a_{11} & -a_{21} & -1 \\ \delta c_{1p} - \alpha g_1 & -\alpha g_2 & \alpha x_1 \\ -\beta g_1 & \delta c_{2p} - \beta g_2 & \beta x_1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} w a_{o1} \\ \alpha W_0 \\ \beta W_0 \end{bmatrix} \qquad (4.7)$$

Equation system (4.6) is the fundamental system for a mixed economy working under capacity constraints. Denoting the determinant of the LHS matrix by Δ we have the solution,

$$p_1 = \frac{\Delta_1}{\Delta}, \quad p_2 = \frac{\Delta_2}{\Delta} \text{ and } \theta_1 = \frac{\Delta_3}{\Delta}$$

where, Δ_i (i=1,2,3) are the appropriate determinants. Direct expansion yields

$$\Delta = \delta[\beta c_{1p}(a_{21}x_1 + g_2) - \alpha c_{2p}(x_1 - a_{11}x_1 - g_1) - \delta c_{1p}c_{2p}]$$
$$= \delta[\beta c_{1p}S_2 + \alpha c_{2p}S_1 - \delta c_{1p}c_{2p}] \tag{4.8}$$

Where S_1 and S_2 are as defined earlier. As it stands, Δ is unsigned. However, it is easy to verify that

$$\left.\begin{array}{l} \Delta_1 = -\alpha\delta c_{2p}(W_o + wa_{o1}x_1) = -\alpha\delta c_{2p}W < 0 \\ \qquad\qquad \text{and} \\ \Delta_2 = -\beta\delta c_{1p}(W_o + wa_{o1}x_1) = -\beta\delta c_{1p}W < 0 \end{array}\right\} \tag{4.9}$$

Thus, for positive prices, a condition $\Delta < 0$ is required.

This condition ($\Delta < 0$) is obviously necessary as well as sufficient for the existence of positive prices p_1 and p_2 . It can be expressed and interpreted in alternative but related ways as follows. First, dividing the RHS in (4.8) by c_{1p} c_{2p} ,$\Delta < 0$ implies

$$\delta > \beta\frac{S_2}{c_{2p}} - \alpha\frac{S_1}{c_{1p}} \tag{4.10a}$$

which sets a lower bound to the propensity to save for given values of parameters relating to capacity (x_1 and x_2), technology (a_{ij} 's), policy (g_1 and g_2) and behaviour (α and β). Second, substituting $\alpha m/\delta p_1$ for c_{1p} and $\beta m/\delta p_2$ for c_{2p} in (4.8) $\Delta < 0$ implies

$$m > p_2S_2 - p_1S_1 \tag{4.10b}$$

which means that the private sector must hold as much money as the government is compelled to supply (at positive prices). This is obviously possible only if the propensity to save is adequately high. Third, we can rewrite (4.10a) as:

$$g_2 < \frac{\alpha S_1c_2 + (\delta c_2 - \beta a_{21}x_1)c_{1p}}{\alpha S_1 + (\beta + \delta)c_{1p}} \tag{4.10c}$$

which sets an upper limit to g_2 for given values of all other parameters.

Let us summarise the foregoing results as follows:

PROPOSITION 2a: Market determined prices for the two products will be positive only if the propensity to save is above a critical magnitude so that the private sector is inclined to hold as much money as the government is constrained to supply (at positive prices) to maintain its solvency.

Conditions (4.10) we call the <u>solvency condition</u> in as much as it requires the value of total sales of the public sector (to the private sector) plus budgetary deficit (≥ 0) to strictly exceed the value of total purchases from the private sector. A moment's reflection reveals why. In reality, in any mixed economy, the public sector has a very large presence in major infrastructures and elsewhere and the <u>sheer volume of intra-public sector trade is enormous</u>. Taken singly, a lot of them can show a profit but only at each other's or of the private sector's expense. Analytically, the solvency condition requires us to set out the value of all these intra-public sector trades. Quantitatively, it would force a substantial rewriting the national income accounts of a mixed economy.

Considering the difficulty of enturing positive profits in the regulated sectors, even given positive prices (proposition 2b below) our worries would seem to be justified. Thus, the solvency condition holds, there is nowhere for the government to pay for its own precommitment to employees, i.e., (W_o+ wa_{o1} x_1) from. This is indirectly meant to ensure that the inter sectoral terms of trade do not turn <u>too much</u> against the public sector. What should be obvious is that given other things, government cannot induce the households to hold more than a certain amount of m. Thus the saving propensity assumes importance. Hence non negative solutions in equilibrium will not be assured if the solvency condition breaks down. This condition appears to be analogous to the Keynesian multiplier being positive.

Turning now to the profit margins θ_1 and θ_2 let us first reiterate that conditions (4.10) only ensure the positivity of the two prices. This, by itself does not ensure that margins in the two sectors (θ_1 and θ_2) will be positive. Consider θ_1 first. Since $\theta_1 = \Delta_3/\Delta$, $\theta_1 > 0$ holds if and only if $\Delta_3 < 0$, given that p_1 and p_2 are positive, i.e. $\Delta < 0$. To this end, note that

$$\begin{aligned} \Delta_3 &= \delta[\, wa_{o1}(\delta c_{1p}c_{2p} - \beta c_{1p}g_2 - \alpha c_{2p}g_1) + W_0\{a_{21}\beta c_{1p} - (1-a_{11})\alpha c_{2p}\}] \\ \Delta &= \delta[\, (\alpha c_{2p}g_1 + \beta c_{1p}g_2 - \delta c_{1p}c_{2p}) + x_1\{\beta a_{12}x_{1p} - \alpha(1-a_{11})c_{2p}\}] \end{aligned} \quad (4.11)$$

$$\frac{\partial\theta_2}{\partial g_2} = \frac{\delta p_1^2}{\alpha\beta mW}\left[(1-a_{22})(\alpha+\beta+\delta)c_{1p}+\alpha(a_{12}(1-a_{22})x_2-a_{21}(1-a_{11})x_1)\right] \quad (4.23)$$

A sufficient condition for $\partial\theta_2/\partial g_2 > 0$ is thus

$$\frac{x_2}{x_1} > \frac{a_{21}(1-a_{11})}{a_{12}(1-a_{22})}$$

i.e., the private sector's requirement of the public sector product exceeds the public sector's requirement of the private sector product. This depends clearly on the technological coefficients particularly a_{12} and a_{21} as well as the levels of production in the two sectors. Let us now summarize the foregoing discussion as follow:

PROPOSITION 3: In a regime of constrained capacity and flexible prices in both sectors an increase in the purchase of the private sector product by the government (say, for investment) will raise prices in both sectors. However, an increase in the use of its own product by the government has a depressing effect on the price of the private sector product; the effect on the price of its own (public sector product) is positive if the prevailing purchase of the private sector product is below a certain magnitude and negative if it is above that magnitude - all parameters remaining unchanged.

PROPOSITION 4: if capacity is constrained and prices for both sectors are flexible, then an increase in the use of its own (public sector) product by the government depresses the unit mark-up rate for the private sector; and raises its own unit mark-up rate if it increases the price of its own product. However, the corresponding effects associated with an increase in the governments' purchase of the private sector product are indefinite. The unit mark-up rate for the public sector product is raised (lowered) if the magnitude of this sector's product for final use by the government is above (below) a certain level. The effect on the private sector's unit mark-up rate too is positive if the quantum of intermediate requirement of the public sector product by the private sector exceeds the intermediate requirement of the private sector product by the public sector.

5. Fixed Public Sector Price and Rationing

We have so far considered the case in which outputs in the two commodity producing sectors are capacity constrained but prices are assumed to be market clearing for both the commodities. This meant that the two mark-up magnitudes θ_1 and θ_2 were endogenously determined so as to equate supply and demand in all the markets, including those for money and services. The consequences of the government decision to increase investment by raising either g_1 or g_2 or both were then traced.

We now turn to the more likely case in which the first commodity, produced in the public sector, is subject to rationing. To be more specific it is postulated that the price for this (first) commodity is fixed by the government and the available fixed quantity is sold to households at that price. Four points need to be underlined before we proceed further. <u>First</u>, as in the preceding case outputs of both the commodities are capacity constrained. <u>Second</u>, for the rationing to be meaningful the price fixed by the government must be lower than the price that would clear the market. <u>Third</u>, price for the second commodity is left free to find its market clearing level. <u>Fourth</u>, we do not consider any specific rationing scheme for the scarce commodity.

Let us now turn to the nature of adjustments envisaged. As mentioned earlier we must set $p_1 = p_1^*$ as a policy parameter. Since p_2 is market clearing variations in both θ_1 and θ_2 are required so as to ensure that p_1 remains fixed at p_1^*. With the money wage w being exogenous, changes in θ_1 and θ_2 must be in opposite directions. Further, since the government budget must balance, money supply cannot be held fixed. For, it must adjust passively to any changes in θ_1 and θ_2 brought about by changes in g_1 and/or g_2. <u>The question of an exclusive choice between changes in money supply and those in administered prices does not arise</u>. Failure to articulate these issues has frequently led to much confusion in discussions relating to price policy and resource mobilisation.

To formalise the problem let us start with household demand. Following the earlier framework we have

$$\text{Max} \quad u = c_{1p}^{\alpha}\, c_{2p}^{\beta}\, c_{3p}^{\gamma} (m/p_1)^{\delta}$$

Subject to: $Y + m_0 = p_1 c_{1p} + p_2 c_{2p} + p_3 c_3 + m^d$ (5.1)

Also, $p_1 = p_1^*$ which is below the market clearing level so that the entire supply of commodity 1 is purchased. Thus, we have

$$c_{1p} = c_1^* - g_1 = c_{1p}^* \quad (\text{ say })$$

Assuming without loss of generality that $m_o = 0$ the demand functions are:

$$\begin{aligned} c_{2p} &= c_2 - g_2 = \frac{\beta'}{p}(Y - p_1^* c_{1p}^*) \\ c_3 &= \frac{\gamma'}{p_3}(Y - p_1^* c_{1p}^*), \textit{ and}, \\ m^d &= \delta'(Y - p_1^* c_{1p}^*) \end{aligned} \tag{5.2}$$

It must be that $\beta' + \gamma' + \delta' = 1$.

This,in turn implies, as shown in proposition 1, that $m^d = m^s = m$ (say). Thus, we must have from (5.2)

$$m = \delta'(Y - p_1^* c_{1p}^*) \Rightarrow Y - p_1^* c_{1p}^* = m/\delta' \tag{5.3}$$

From the preceding section we note that under market clearing condition we have

$$\begin{aligned} p_1^o &= \frac{\alpha' c_{2p}(W_0 + wa_{01} x_1)}{\beta' c_{1p}(a_{21} x_1 + g_2) - \alpha' c_{2p}(x_1 - a_{11} x_1 - g_1) - \delta' c_{1p} c_{2p}} \\ &= -\alpha' c_{2p} W[\delta' c_{1p} c_{2p} + \alpha' c_{2p} S_1 - \beta' c_{1p} S_2] \end{aligned} \tag{5.4}$$

where $W \equiv W_0 + wa_{01} x_1$ denotes as before the government's total wage bill. Now, if rationing is meaningful p_1 must be set at a level below p_1^o i.e., $p_1^* < p_1^o$ where p_1^* is the administered and p_1^o the market clearing price for the public sector product. This implies that the following condition must hold.

$$\alpha' c_{2p} W > p_1^*[\delta' c_{1p} c_{2p} + \alpha' c_{2p} S_1 - \beta' c_{1p} S_2]$$

$$\Rightarrow \alpha' c_{2p}(W - p_1^* S_1) > p_1^* c_{1p}(\delta' c_{2p} - \beta' S_2) \tag{5.5}$$

Noting that $c_{2p} = \beta' m / (\delta' p_2)$ and by proposition 1 that $W - p_1 S_1 = m - p_2 S_2 > 0$ (5.5) can be written in many alternative forms, e.g.,

$$\alpha' p_2 c_{2p} - \beta' p_1^* c_{1p} > 0$$

$$\text{or ,} \qquad (5.6)$$

$$p_2 > \beta' p_1 c_{1p} / \alpha' c_{2p}$$

Thus, we have:

PROPOSITION 5: If in a regime of rationing and capacity constraints price of the public sector product is set below its market clearing level, then the market clearing level of the price of the private sector product is not only positive but also subject to a lower bound - depending directly on the level of the public sector product price.

Under this regime we have x_1, x_2, c_1 and c_2 fixed as in the preceding case. g_1 and g_2 are policy variables. But once these are fixed c_{1p} and c_{2p} are also fixed. The endogenous variables are: θ_1, θ_2, p_2 and m. Note that both θ_1 and θ_2 must adjust so as to ensure that p_1 is rigidly fixed and p_2 is market clearing. Thus, with p_2 and θ_1 both endogenous, m also must adjust as required. Thus, we have a basic system of <u>four</u> equations as:

$$\begin{array}{ll} \text{(i)} & \theta_1 + a_{21} p_2 = (1 - a_{11}) p_1 - wa_{o1} \\ \text{(ii)} & \theta_2 - (1 - s_{22}) p_2 = - a_{12} p_1 - wa_{o1} \\ \text{(iii)} & \beta' x_1 \theta_1 + (\delta' c_{2p} - \beta' g_2) p_2 = \beta' (W_0 + p_1 g_1) \\ \text{(iv)} & m = W_0 + p_1 g_1 + p_2 g_2 - \theta_1 x_1 \end{array} \qquad (5.7)$$

(note that from now on we drop the asterisk (*) on p_1)

Of these four equation (i) and (iii) can be solved for θ_1 and p_2. These can then be substituted in (ii) to obtain θ_2 and in (iv) to get m. Equations (i) - (iii) can be written as:

$$\begin{bmatrix} a_{21} & 1 & 0 \\ -(1 - a_{11}) & 0 & 1 \\ \delta' c_{2p} - \beta' g_2 & \beta' x_1 & 0 \end{bmatrix} \begin{bmatrix} p_2 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} (1 - a_{11}) p_1 - wa_{o1} \\ -a_{12} p_1 - wa_{o2} \\ \beta' (W_0 + p_1 g_1) \end{bmatrix} \qquad (5.8)$$

so that $p_2 = \Delta_1/\Delta$, $\theta_1 = \Delta_2/\Delta$ and $\theta_2 = \Delta_3/\Delta$

Where Δ, Δ_1, Δ_2, Δ_3 are the appropriate determinants

Direct calculations show that

$$\left.\begin{aligned}
\Delta &= \delta' c_{2p} - \beta'(a_{21}x_1 + g_2) \equiv \delta' c_{2p} - \beta' p_2 \\
\Delta_1 &= -\beta' x_1[(1 - a_{11})p_1 - wa_{01}] + \beta'(W_0 + p_1 g_1) \\
&= \beta'(W - p_1 S_1) \\
\Delta_2 &= -a_{21}\beta'(W_0 + p_1 g_1) + (\delta' c_{2p} - \beta' g_2)[(1 - a_{11})p_1 - wa_{01}] \\
&= [(1 - a_{11})p_1 - wa_{01}]\Delta - \beta a_{21}(w - p_1 S_1] \\
\Delta_3 &= \beta' x_1[-(1 - a_{22})\{(1 - a_{11})p_1 - wa_{01}\} + a_{21}(a_{12}p_1 + wa_{02})] \\
&\quad -\beta'(W_0 + p_1 g_1)[-(1 - a_{22})] - (\delta' c_{2p} - \beta' g_2)(a_{12}p_1 + wa_{02}) \\
&= \beta'(1 - a_{22})(Wp_1S_1) - (wa_{02} + a_{12}p_1)\Delta
\end{aligned}\right\} \tag{5.9}$$

Thus,

$$\left.\begin{aligned}
p_2 &= \frac{\beta(W - p_1 S_1)}{(\delta c_{2p} - \beta p_2)} \\
\theta_1 &= \frac{(1 - a_{11})p_1 - wa_{01} - \beta a_{21}(W - p_1 S_1)}{\delta c_{2p} - \beta p_2)} \\
\theta_2 &= -\frac{(a_{12}p_1 + wa_{02}) + \beta(1 - a_{22})(W - p_1 S_1)}{\delta c_{2p} - \beta p_2)} \\
&\text{Further} \\
m &= \frac{\delta c_{2p}(W - p_1 S_1)}{(\delta c_{2p} - \beta p_2)}
\end{aligned}\right\} \tag{5.10}$$

Since $p_2 > 0$ it is obvious that $m > 0$. Of particular concern in the present context are the signs and magnitudes of θ_1 and θ_2. Let us turn to these. From (5.7) and (5.10) we have

$$\left.\begin{aligned}
\theta_1 &\lesssim (1 - a_{11})p_1 - wa_{01} - \beta a_{21}p_1 c_{1p}/(\alpha c_{2p}) \\
\theta_2 &\gtrsim -a_{12}p_1 - wa_{02} + \beta(1 - a_{22})p_1 c_{1p}/(\alpha c_{2p})
\end{aligned}\right\} \tag{5.11}$$

Clearly, the signs of both θ_1 and θ_2 are indefinite. However, given the technological parameters θ_2 is more likely to be positive whereas θ_1 is more likely to be negative. It is worth noting that the upper bound for θ_1 and the lower bound for θ_2 do critically depend on the level at which p_1 is set and the relative magnitudes of the net availability of the two commodities, c_{1p} and c_{2p}. We have a proposition quite drastic for its massage to many a mixed economy, which is:

PROPOSITION 6: If the public sector product is subject to rationed distribution at a fixed price (below the market clearing level) then the mark up rate in the public sector is subject to a ceiling and that in the private sector subject to a floor. Both, the ceiling as well as the floor depend critically on the relative net availability of the two products for private consumption.

Turning now to comparative statics let us totally differentiate (5.8), which gives us

$$\begin{bmatrix} 1 & 0 & a_{21} \\ 0 & 1 & -(1-a_{22}) \\ \beta x_1 & 0 & \delta c_{2p} - \beta g_2 \end{bmatrix} \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ dp_2 \end{bmatrix} = \begin{bmatrix} (1-a_{11})\,dp_1 \\ -a_{12}\,dp_1 \\ \beta p_1 dg_1 + (\delta+\beta)\,p_2 dg_2 + \beta g_1 dp_1 \end{bmatrix} \tag{5.12}$$

By Cramer's rule

$$\left.\begin{aligned} d\theta_1 &= [\,\Delta_{11} pg_1 + \Delta_{12} pg_2 + \Delta_{13} dp_1\,]\,/\,\Delta \\ d\theta_2 &= [\,\Delta_{21} dg_1 + \Delta_{22} dg_2 + \Delta_{23} dp_1\,]\,/\,\Delta \\ dp_2 &= [\,\Delta_{31} dg_1 + \Delta_{32} dg_2 + \Delta_{33} dp_1\,]\,/\,\Delta \end{aligned}\right\} \tag{5.13}$$

Where $\Delta \equiv \delta c_{2p} - \beta p_2$

Simple calculations show that

$$\frac{\partial\theta_1}{\partial g_1} = -\beta a_{21} p_1/\Delta\,; \quad \frac{\partial\theta_1}{\partial g_2} = -(\beta+\delta)\,a_{21} p_2/\Delta$$

It follows that

$$\partial\theta_1/\partial g_1 < 0 \ \text{ and } \ \partial\theta_1/\partial g_2 < 0 \ \text{ if } \ S_2/C_{2p} < \delta/\beta \tag{5.14}$$

a condition that has been extensively discussed in Section 4. (See proposition 3)

6. Some Illustrative Simulations

In this section we report the results of a few numerical exercises focusing on the nature of solutions for prices and profit rates for a hypothetical but plausible economy. On the one hand, we verify the cases where prices are positive and their directions of change unambiguous. On the other, we illustrate the kind of parameter combinations which might lead to alternative, in some cases unacceptable solutions. In cases relating to comparative statics the primary focus is on changes in the governments' acquisition of the two goods.

In all exercises the following parameters remain fixed at values indicated.

$a_{11} = 0.36$, $a_{12} = 0.15$, $a_{21} = 0.08$

$a_{22} = 0.80$, $a_{01} = 0.07$, $a_{02} = 0.15$

$\lambda_1 = 0.15$, $\lambda_2 = 0.35$, $\lambda_3 = 0.20$

$m_o = 1000$, $W_o = 3000$, $w = 50$

Values of the behavioural parameters, α, β, γ and δ the capacity outputs levels x_1 and x_2 are fixed in different exercises to ensure that any one or more specific conditions do or do not hold as the case may be. Policy variables g_1 and g_2 and, in some case p_1 are naturally varied across different exercises.

Before we proceed further let us restate the important conditions and label them as follows. Equation numbers at the end refer to those in the text for quick identification, in each case.

$$A: \ \delta > \beta \frac{S_2}{c_{2p}} - \alpha \frac{S_1}{c_{1p}} \tag{4.10a}$$

$$B1: \ \alpha > \frac{c_{1p}}{c_{2p}} \ \frac{wa_{01}(\delta c_{2p} - \beta g_2) + a_{21}\beta W_0}{(1 - a_{11}) W_0 + wa_{01} g_1} \tag{4.12}$$

$$B2: \ \beta > \frac{c_{2p}}{c_{1p}} \ \frac{wa_{02}(\delta c_{1p} - \alpha g_1) + \alpha a_{12} W_0 + \alpha wx_1[a_{02}(1 - a_{11}) - a_{01}a_{12}]}{(1 - a_{22}) W_0 + wa_{02} g_2 + wx_1[a_{01}(1 - a_{22}) + a_{02}a_{22}]} \tag{4.13}$$

$$C: \ \frac{S_2}{c_{2p}} < \frac{\alpha + \delta}{\beta} \tag{4.15}$$

$$D: \frac{x_2}{x_1} > \frac{a_{21}(1 - a_{11})}{a_{12}(1 - a_{22})} \tag{4.24}$$

$$E: \frac{c_{1p}}{x_2} < \frac{\alpha\,|I - A|}{(1 - \gamma)\,a_{21}} \tag{4.21}$$

$$F: \frac{S_2}{c_{2p}} < \frac{\delta}{\beta} \tag{5.14}$$

Recall from section 4 that A which can be expressed in alternative forms and was christened as the 'Solvency Condition' ensures positive prices p_1 and p_2. Assuming A holds, B1 ensures positive θ_1 and B2 ensures positive θ_2. To be more precise,

$$A \Leftrightarrow p_1 > 0\ ,\ \ p_2 > 0,\ \ A,\ B1 \Leftrightarrow \theta_1 > 0,\ \ A,\ B2 \Leftrightarrow \theta_2 > 0$$

A is also necessary and sufficient for $\frac{\delta p_1}{\delta g_2} > 0,\ \ \frac{\delta p_2}{\delta g_1} < 0,\ \ \frac{\delta p_2}{\delta g_2} > 0$

No additional condition is required. However, for $\frac{\delta p_1}{\delta g_1} > 0$ we need condition C in addition to A.

Now consider cases I(a) and I(b) for which we have:

$x_1 = 200,$	$x_2 = 400,$	$c_1 = 68,$	$c_2 = 304$
$\alpha = 0.15,$	$\beta = 0.50,$	$\gamma = 0.10,$	$\delta = 0.25$

However for I(a) we set $g_1 = 18$, $g_2 = 54$ and for I(b)$g_1 = 28, g_2 = 134$. The results are as follows:

Table 1

Case (Ia)

Variable	p_1	p_2	θ_1	θ_2	γ	m
Base Solution	32.05	21.37	15.30	4.79	9683	2670
Deviation ($\Delta g_1 = 10$)	4.29	-1.99	2.90	-2.32	-992	-248
Deviation ($\Delta g_2 = 10$)	2.07	2.33	1.14	1.55	689	172

Case (Ib)

Variable	p_1	p_2	θ_1	θ_2	γ	m
Base Solution	95.72	75.08	51.76	38.20	24533	6389
Deviation ($\Delta g_1 = 10$)	-5.01	-21.71	-1.472	-16.62	-7387	-1849
Deviation ($\Delta g_2 = 10$)	204.16	198.48	115.34	122.57	54471	136

Note that for both I(a) and I(b) A, B1 and B2 all hold giving us positive solutions for prices as well as profit rates. However while C holds for I(a) it is violated in I(b). Hence $\delta p_1/\delta g_1$ is negative, in the latter case and positive in the former. Note also that $\delta p_2/\delta g_1$ is negative in both the cases. We also have $\delta p_1/\delta g_1$ and $\delta p_2/\delta g_2$ positive in both cases, as expected.

Comparative statics for θ_1 and θ_2 is rather more complex. Recall from section 4 that so long as prices are positive we always have $\delta\theta_2/\delta g_1 < 0$. For $\delta\theta_2/\delta g_2 > 0$, D is sufficient but not necessary. For $\delta\theta_1/\delta g_1 > 0$, C is necessary and sufficient if $p_1 > 0$ and for $\delta\theta_1/\delta g_2 > 0$, E is necessary and sufficient. In cases I(a), D as well as E hold. Hence the signs of the deviations given in table 1 above are, as expected viz., $\delta\theta_1/\delta g_1$ is positive in I(a) and positive in I(b); $\delta\theta_1/\delta g_2 > 0$ in both cases; $\delta\theta_2/\delta g_1 < 0$ in both cases and $\delta\theta_2/\delta g_2$ positive in both cases.

Next we take up case II in which A, B1, B2 and E hold, but C and D are violated as we set the capacity output etc. as follows.

$x_1 = 450,\quad x_2 = 100,\quad c_1 = 273,\quad c_2 = 44$

$g_1 = 173,\quad g_2 = 0,\quad c_{10} = 100,\quad c_{2p} = 44$

The results are as given in table 2 below.

Table 2

Case (II)

Variable	p_1	p_2	θ_1	θ_2	γ	m
Base Solution	643.2	4872.4	18.3	3794.5	427678	107031
Deviation ($\Delta g_1 = 40$)	-144.6	-2606	116	-2063.4	-229283	-57259
Deviation ($\Delta g_2 = 10$)	1153.3	3483.6	459.8	2613.3	290290	72975

Since A, B1 and B2 hold both prices as well as profit rates are positive. But as C is violated $\delta p_1/\delta g_1 < 0$. $\delta\theta_2/\delta g_1 < 0$ holds unconditionally. However note that $\delta\theta_1/\delta g_1$ and $\delta\theta_2/\delta g_2$ are both positive even though C and D are violated. This is because both of these are sufficient but not necessary conditions.

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