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Pollution Abatement Cost Function: Methodological and Estimation Issues

Bishwanath Goldar
and
Badal Mukherji*

Working Paper No. 56

Centre for Development Economics
Delhi School of Economics
Delhi 110 007 INDIA
Tel : 7257005, 7256533-35
Fax : 011-7257159
E-mail : office@cdedse.ernet.in

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ABSTRACT

Formulation and estimation of a correctly specified abatement cost function would be the corner stone of sound policy regarding imposing taxes or user fees as well as of sharing social cost in the presence of environmental pollution.

Often in recent research the output of an abatement activity for water pollution appears to us not to have been clearly specified. This activity is very distinct from that of whatever is the actual product of the plant. We propose in this paper that the output is only and exclusively the reduction in the level of the pollutant (per litre) in the inflow to the plant and demonstrate that with this definition a well behaved cost function can be derived and estimated.

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* Institute of Economic Growth, Delhi, and Delhi School of Economics, Delhi University, Delhi, respectively. We have benefited much from the comments of an anonymous referee on an earlier version of this paper.

1. Introduction

In a number of studies, cost function for water pollution abatement has been estimated for Indian industries (Mehta, Mundle and Sankar 1993, James and Murty 1996, Roy and Ganguli 1997, Pandey 1997, and Goldar and Pandey 1997) with a view to analysing the relationship of abatement cost with the volume of water treated, the characteristics of influent and effluent streams (i.e. the extent of pollution abatement done) and the prices of inputs used in the pollution abatement activity (labour, capital, energy, etc). A major object of these studies was to work out marginal abatement cost from the estimated cost function which could be used as a guide for determining pollution tax.

The Cobb-Douglas functional form was commonly used in these studies. In some studies, the translog functional form was also tried, but the results were not found to be satisfactory. Cost of abatement was taken as the dependent variable, while the water volume treated, characteristics of influent and effluent streams in terms of concentration levels of pollutants (BOD, COD, etc.) and input prices were taken as explanatory variables. There was not much discussion on the basis for (and implications of) the specification chosen for the cost function, nor on the underlying production function.

The focus of this paper is on methodological and estimation issues for water pollution abatement cost functions. The need for such an exercise dawned on us as a closer examination of various existing works showed that they all faced, either at a theoretical level or with their respective estimates, anomalies and exceptions or implications not consistent with intuition. In Section 2, we take a look at the functional specifications used in some earlier studies and point out their inadequacies. We then suggest in Section 3 an alternative approach to specifying the production function for abatement activity that avoids all these problems and derive the associated cost function. In the Appendix, we discuss for what values of the cost function parameters, the marginal abatement cost curve for the suggested mathematical model will have the appropriate shape.

2. Specifications Used in Earlier Studies

Econometric studies on abatement cost function have mostly followed the general econometric literature on cost function, and thus taken for granted the form the cost function should take. Only a few studies have paid attention to the underlying production function and made an attempt to derive the cost function from the production function. However, a closer look at these attempts to derive abatement cost function brings out the problems associated with the derivation, as also with the cost function specification finally adopted.

One of the early studies on pollution abatement cost was undertaken by Rossi, Young and Epp (1979).[1] They specified the production function associated with water pollution abatement activity as

$$O = f(N, I) \tag{1}$$

where O is the vector of flow and quality characteristics of effluent stream (from the treatment plant), N the vector of flow and quality characteristics of influent stream (going to the treatment plant) and I the vector of factors of production such as land, labour and capital (also other inputs, e.g. materials). Given this production function, a cost function could be derived (under the assumption of cost minimization) as

$$C = g(O, N, P) \tag{2}$$

where C denotes cost and P is the prices of factors of production.

There are some problems of interpretation with the above formulation. First, N cannot be treated as an input in the same way as I, since labour or capital cannot substitute the volume and characteristics of the influent stream. The firm minimizes the cost in respect of labour, capital, energy etc and takes N as given. This is the reason why in the cost function prices corresponding to I are included, but it is not possible to treat N likewise. Secondly, treating the volume and characteristics of effluent steam as output of the abatement plant does not seem right. How would one interpret the partial derivatives of O with respect to I and N? Obviously, there are difficulties in treating these partial derivatives as marginal productivities.

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[1] This was a study of municipal sewerage treatment.

Another problem with the production function in equation (1) above is that given N and I, the vector O is not unique, since abatement activity may be directed more at one pollutant rather than another.[2]

The Rossi et al. framework has been taken as the basis for specifying the abatement cost function in several subsequent studies (for example, Fraas and Munley, 1984). These studies, needless to say, are subject to the same problems of interpretation pointed out above.

Reflecting on the problems of interpretation with the Rossi et al. framework, one would realise that these problems arise because the output of abatement activity is not clearly defined. In cost function studies for manufacturing activity, cost is taken as a function of output of the activity and prices of inputs. It seems to us that a similar approach needs to adopted also for specifying the cost function for pollution abatement activity. Thus, the first step in deriving the cost function for pollution abatement is to define the output of the abatement activity. Since the output of a abatement plant is given by the flow and characteristics of influent and effluent streams, the output may accordingly be taken as a function of them. Therefore, one may write output (Y) as:

$$Y = Y(O, N) \tag{3}$$

The production function corresponding to this definition of output may be written as:

$$Y = f(I) \tag{4}$$

and the cost function may be derived under the assumption of cost minimization as:

$$C = g(Y, P) \tag{5}$$

This way of writing the production function and cost function avoids the problems of interpretation with the Rossi et al. framework.

One question that arises here is how should one define the function Y so as to have a meaningful interpretation of the output measure formed by the function. This aspect is taken up in the next section.

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[2] One way of getting over this difficulty is to write the production function in the implicit form:

$$H(O, N, I) = 0$$

from which the cost function given in equation (2) can be derived.

In this context, it is important to mention the paper by Hartman, Wheeler and Singh (1993) in which implicitly they have the same formulation, as ours, of the output of an abatement activity, namely the quantum of abatement done. However, the theoretical framework used in their paper is different from ours in that it allows for up-stream abatement (i.e. pollution abatement other than end-of-pipe treatment through modifications in process technology, input use, etc.). Thus, in their framework, production costs and abatement costs are combined, and the model of cost behaviour is cast on the basis of a more general technology transformation set.

Although Hartman et al. use a framework different from ours, there is similarity in the approaches taken: they define explicitly the output of abatement activity and incorporate it as an argument in the cost function, which is the approach suggested in this paper. In their paper, Hartman et al. begin with a KLEM (capital, labour, energy and materials) model of a four-input production function in which they incorporate pollution abatement activity, and derive a relationship between total abatement cost (ABC) and the outputs of abatement activities. The model is applied empirically to study cost of air pollution abatement. The cost function is specified as:

$$ABC = \alpha + \Sigma\ \beta_j\ POLL_j + \Sigma\ \beta_{jj}\ POLL_j^2 \tag{6}$$

where ABC is the abatement cost and $POLL_j$ is the quantum of abatement of j'th pollutant (e.g. sulphur oxides, nitrogen oxides, and carbon monoxide). Given the estimate of the above equation, the marginal and average costs of abatement are computed separately for different pollutants.

<u>Studies for Indian Industries</u>

Turning to studies on abatement cost function for Indian industries, the following two specifications have been commonly used.

$$\ln C = a + \alpha\ \ln Q + \beta\ \ln\ [q_I/q_E] + \Sigma\ b_i\ \ln P_i \tag{7a}$$

$$\ln C = a + \alpha \ln Q + \beta \ln q_I + \gamma \ln q_E + \Sigma\, b_i \ln P_i \tag{7b}$$

In these equations, C denotes total cost of abatement, Q the volume of water treated (one may distinguish between the volumes of influent and effluent stream), q_I is the level of a particular pollutant (say, BOD) in the influent stream and q_E that in the effluent stream. The input prices (labour, capital, energy, etc.) are denoted by P_i. In some studies, the prices of inputs have not been included in the estimated function (e.g. Mehta, Mundle and Sankar, 1993).

The equations given above (7a and 7b) use a Cobb-Douglas functional specification. Attempts have been made in some studies to use the translog functional specification which is more general.

Compared to equation (7a) used by James and Murty (1996), equation (7b) used by Roy and Ganguli (1997), and Goldar and Pandey (1997) is less restrictive as it allows q_I and q_E to have different coefficients. But, both forms 7a and 7b suffer from one problem. If $q_I = q_E$, i.e. there is no abatement of pollution, then the cost should be zero, but one can easily see that this is not so in either of the two equations.[3] Indeed, even at zero abatement, the estimated cost may turn out to be quite high. The implication is that an estimated abatement cost function using specifications in 7a or 7b will overstate to some extent the cost of abatement at low levels of abatement. Clearly, this will affect the estimates of marginal cost of abatement.

Attention needs to be drawn here to the fact that there are several pollutants in waste water and the cost of abatement depends on the extent of abatement done in respect of the various pollutants. One may according specify the cost function (extending 7a) as:

$$\begin{aligned}\ln C = a + \alpha \ln Q &+ \beta_1 \ln (BOD_I/BOD_E) \\ &+ \beta_2 \ln (COD_I)/COD_E) + \beta_3 \ln (SS_I/SS_E) \\ &+ \beta_4 \ln (DS_I/DS_E) + \Sigma\, b_i \ln P_i\end{aligned} \tag{8}$$

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[3] One would similarly expect cost to be zero if the volume of water treated is zero or if the prices of inputs are zero. This property holds for equations 7a and 7b.

BOD (biological oxygen demand), COD (chemical oxygen demand), SS (suspended solids) and DS (dissolved solids) are the four pollutants/characteristics considered. This specification has the advantage that the marginal cost of abatement can be computed for each pollutant separately[4] and pollutant specific taxes may be worked out. On theoretical grounds, the above specification is attractive, but there are problems in applying it empirically because the ratios BOD_I/BOD_E, COD_I/COD_E, SS_I/S_E and DS_I/DS_E are often found to be highly correlated to one another, with the consequence that the regression results are affected by the problem of multicollinearity if all these ratios are included in the same regression equation. This is basically a reflection of jointness of pollution abatement activity. Goldar and Pandey (1997) note this jointness of pollution abatement activity and therefore take BOD levels before and after treatment as an index of the extent of pollution abatement done. A similar approach is taken in other studies, for example James and Murty (1996) and Roy and Ganguli (1997). Thus, in the estimated equation only the BOD levels before and after treatment (or the ratio of BOD_I to BOD_E) are taken as explanatory variables. In a separate equation, COD levels before and after treatment are taken as explanatory variables (in place of BOD).

Since the extent of reduction achieved in different pollutants are highly correlated, the reduction in respect of any one of the pollutants, say BOD or COD, may be taken as an index of the pollution abatement done. This is a valid econometric procedure if one is interested only in estimating the cost function for the purpose of working out estimates of cost of abatement corresponding to different levels of waste water volume treated, the extent of abatement done (overall), and the prices of inputs. However, it needs to be pointed out that the coefficient of BOD ratio will capture the combined effect of reduction of different pollutants on cost and not the effect of BOD reduction only. As a result, one cannot work out the marginal abatement cost in respect of BOD reduction (or COD reduction) from the estimated cost function, as some studies for Indian industries have done. Since marginal abatement costs cannot be computed separately for BOD, COD, etc. (unless all these are included in the estimated regression), one cannot work out properly the optimal pollutant specific tax rates.

[4] This equation may be contrasted with equation (6) above which also permits derivation of marginal costs of abatement for different pollutants.

3. An Alternative Framework

It seems to us that to define properly the production function and the associated cost function for a pollution abatement plant, it should be viewed as a service activity. The plant gets a given volume of polluted water. The job of the plant is to reduce the impurities. Thus, the output of the plant is given by the extent of pollution reduction achieved for the given volume of waste water treated.[5] Since there are more than one pollutant, one may take reduction in each pollutant as one type of output, and thus think the plant as having multiple outputs.

For a given abatement plant, let the volume of waste water be denoted by Q. The pollution level of influent water is denoted by q_I and that of effluent water by q_E. For the sake of simplicity, we assume that there is only one type of pollutant, say BOD. Alternatively, we may assume that the process of treatment of waste water reduces the levels of various pollutants by and large proportionately so that any one of the pollutants can be taken as an indicator of the level of pollution.[6]

The work done by the pollution abatement plant is indicated by the reduction in the pollution level that is achieved by the process of treatment. Since the volume of water treated is fixed, the output (Y) of the effluent treatment plant (ETP) may be written as:

$$Y = Qy = Q(q_I - q_E) \tag{9}$$

This measure of output is easy to interpret. If q represents BOD, then y measures the reduction in BOD (in mg) per litre of waste water treated, i.e. the reduction in pollution

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[5] It should be mentioned that pollution standards in India are concentration-based and not load-based. Once load-based standards are established, the output measure will by definition be the quantum of pollution load removed. It may be pointed out that the economic literature on pollution control (as contrasted to the econometric one) has recognized the importance of load reduction as the object of abatemen. Targetting the concetration level as such would allow the total pollution to vary with the volume of water treated.See McConnell and Schwarz (1992) for a discussion.

[6] As noted earlier, empirical studies do find high correlation between the extent of abatement done in respect of different pollutants providing justification for this assumption.

concentration level, and output Y is the reduction in pollution load.[7] As q_E is lowered, there is greater abatement of water pollution and the output of the plant goes up. Higher levels of output would require a higher amount of input. One may accordingly define a production function, relating input used for abatement activity and the output obtained in terms of reduction in pollution load. To take a simple case, let L denote labour used and K capital input for treatment of waste water. As explained above, Y denotes the reduction in pollution load, say BOD in grams. Then, assuming further a Cobb-Douglas functional specification,[8] the production function may be written as:[9]

$$Y = AL^{\alpha}K^{\beta} \tag{10}$$

Let w be the wage rate and r the rental of capital, then the cost of treatment of waste water is given by

$$C = wL + rK \tag{11}$$

We assume further that the firm minimizes the cost of treatment, given the prices of inputs and the volume of waste water. Therefore, from the conditions of cost minimization, the cost function may be derived as:[10]

$$C = B\, w^{\gamma}\, r^{1-\gamma}\, Y^{\delta} \tag{12}$$

where

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[7] Consider in this context, the cost function specification in equation 7a. The measure of output, consistent with the equation is:

$$Y = Q^{a}\,(q_I / q_E)^{b}$$

The interpretation of this expression is not clear, although one can see that output will go up if more water is treated or if there is greater reduction in the pollution concentration level.

[8] The Cobb-Douglas form is used here for convenience of exposition. One can also work with more general forms of the production function and derive the corresponding cost function.

[9] One may easily extend this to a case of more than two inputs. Also, one may allow for more than one type of output in which case Y may be interpreted as an aggregate of the different types of output.

[10] The derivation of cost function from the Cobb-Douglas production function is available in many textbooks and therefore not given here.

$\gamma = \alpha/(\alpha+\beta)$ and $\delta = 1/(\alpha+\beta)$

The Cobb-Douglas functional specification requires that α and β be positive fractions. Accordingly, γ should also be a positive fraction. The coefficients of w and r must add up to unity (as they do in equation 12), since the cost function must be homogeneous of degree one in input prices.

There are reasons to expect δ to be greater than one. As q_E goes down and Y increases the input required to produce this output will also increases. But at lower levels of q_E further reductions become more and more difficult. This should be reflected in the production function specified above by diseconomies of scale (since increase in output will require more than proportionate increase in inputs). Thus, $\alpha+\beta$ should be less than one, and δ should be greater than one.

Studies on water pollution abatement cost function for Indian industries have used the estimated cost function to obtain the marginal cost of abatement from which pollution tax rates have been worked out. On theoretical grounds, one would require a convex downward slopping curve relating the cost of abatement to the post-treatment pollutant concentration level. Thus, the derivatives of C with respect to q_E should be of the following sign:

$$\frac{\partial C}{\partial q_E} < 0 \qquad \frac{\partial^2 C}{\partial q_E^2} > 0$$

It can be shown that if δ is more than one, then the second-order partial derivatives of C with respect to q_E will have the correct sign, otherwise this property is not satisfied for the cost function given in equation (12).[11]

For applying the above model empirically to cross section data for firms, some modifications become necessary. One must in the abatement cost function allow for economies of scale in

[11] For a modified version of this cost function, given in equation (15), the condition that δ should be greater than one is sufficient but not necessary for the second order derivative to be positive. This is discussed in the Appendix.

treating a large volume of water.[12] Thus, the cost function in equation (12) needs to be changed to:

$$C = B\, Q^{\epsilon}\, w^{\gamma}\, r^{1-\gamma}\, Y^{\delta} \tag{13}$$

In this equation, the exponent of Q should be negative to signify that given other things the same a firm which treats a larger volume of waste water should have a lower cost of abatement per litre. Using equations (9) and (13), the total cost function may be written as:

$$C = B\, Q^{\eta}\, w^{\gamma}\, r^{1-\gamma}\, (q_I - q_E)^{\delta} \tag{14}$$

where $\eta = 1+\epsilon$. Since there are economies of scale in treatment of waste water and the coefficient of Q in equation (13) is negative, the coefficient of Q in equation (14) should be less than one (it should be a positive fraction).

If all firms in a cross section data set have the same (or similar) initial level of water pollution concentration q_I then equation (14) may be estimated by regressing ln C on ln Q, ln $(q_I - q_E)$ and the logarithm of prices of inputs. In such a situation, the estimate of δ is expected to be greater than one which will ensure that the second order partial derivative of C with respect to q_E has the correct sign. But, if there is significant variation among firms in regard to the initial and post-treatment levels of pollution concentration (say, because they produce different products and do not bring down the pollution concentration in the effluent stream to the same level), then the initial level of pollution concentration or the post-treatment level of pollution concentration, q_I or q_E, should be introduced as an additional variable in the cost function.[13] [14] It is better to

[12] Consider two firms which reduce pollution load by the same amount but differ in the volume of waste water treated. The cost of treatment cannot be expected to be the same in the two firms, if there are economies of scale in the treatment of a larger volume of waste water. Indeed, the cost should be lower for the firm which treats a larger volume but reduces pollution concentration by a smaller extent (so that the quantum of pollution load reduction is the same). This provides justification for including the volume of wastewater treated as an additional argument in the cost function.

[13] Even if two firms treat the same volume of waste water and bring down the pollution concentration level by the same amount (say, by 400 mg per litre), the cost of treatment would differ between them if the initial

introduce q_E as an additional variable in the cost function rather than q_I because with the inclusion of q_E in the function a convex relationship will arise between C and q_E even if the estimate of δ is found to be less than one (discussed further in the Appendix). Also, the cost of abatement should be more sensitive to the post-treatment level of pollution concentration than to the pre-treatment of pollution concentration. Again, one would expect the cost of treatment to rise to very high levels as the post-treatment pollution level approaches zero. This can be better represented by including q_E in the equation with a negative exponent. These provide further justification for including q_E as an additional variable in the abatement cost function. Thus, the total cost function for pollution abatement may be specified as:

$$C = B\, w^{\gamma}\, r^{1-\gamma}\, (q_I - q_E)^{\delta}\, Q^{\eta}\, (q_E)^{-\theta} \tag{15}$$

This equation can be generalised to more than two inputs. Also, the translog functional form may be used in place of the Cobb-Douglas functional form.[15]

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and post-treatment levels of pollution concentration are different. The firm for which the post-treatment level of pollution concentration is lower should have a higher cost.

[14] If firms differ in terms of the initial concentration level but have to treat wastewater to bring down pollution concentration to the same level (MINAS), then q_E cannot be taken as a separate argument in the cost function, but q_I can be included.

[15] As a next step to the above discussion, the methodology suggested in the paper for the estimation of pollution abatement cost function should be subjected to rigorous empirical testing. But, this could not be done for want of suitable data. On our request, Dr Rita Pandey (NIPFP, New Delhi), who has been studying cost of pollution abatement in distilleries, has provided us with estimates of abatement cost function for distilleries using the functional form suggested in the paper (equations 14 and 15) and the forms used in earlier studies (equations 7a and 7b). The estimates are based on data for 44 distilleries. The Cobb-Douglas functional specification has been used. Prices of capital input and energy input have been included in the estimated equations. Price of labour input has been excluded as the result were not good. Although these inadequacies of the estimates limit their usefulness, it may be ponted out here that the estimated cost function using the functional form suggested in the paper works quite well and in terms of `goodness of fit' and statistical significance of parameters these estimates are found to be better than the estimates based on the forms commonly used in earlier studies. The estimated cost functions using the suggested functional form are shown below:

$$\ln C = -13.01 + \underset{(19.02)}{0.950} \ln Q + \underset{(8.226)}{1.154} \ln(q_I - q_E) + \underset{(3.97)}{0.492} \ln P_E + \underset{(18.94)}{0.911} \ln P_K$$

$n = 44 \qquad R^2 = 0.962$

$$\ln C = -9.75 + \underset{(15.34)}{0.862} \ln Q + \underset{(6.059)}{0.929} \ln(q_I - q_E) - \underset{(-2.750)}{0.046} \ln(qE) + \underset{(3.25)}{0.392} \ln P_E + \underset{(16.84)}{0.847} \ln P_K$$

4. Conclusion

Econometric studies on pollution abatement cost function have mostly followed the general econometric literature on cost function and thus taken for granted the form the abatement cost function should take. Very few studies have paid attention to the underlying production function and made an attempt to derive the cost function from the production function. In this paper, we have dealt with these issues. We have pointed out certain problems of interpretation with the framework of Rossi, Young and Epp (1979) which has been taken as a basis for the abatement cost function estimated in several subsequent studies. We underscore the need to define explicitly the output of abatement activity and incorporate it as an argument in the abatement cost function. We have suggested an alternative framework in which output of abatement activity is defined in terms of the pollution load reduction and have derived a form of abatement cost function consistent with that framework.

$n = 44 \quad R^2 = 0.968$

where C = total cost of abatement; Q = volume of water treated; q_I = BOD level before treatment; q_E = BOD level after treatment; P_E = price of energy; and P_K = price of capital input (Details available with the authors on request)

Appendix

Relationship between Abatement Cost and the post-treatment pollution concentration level

As the post-treatment level of pollution concentration (q_E) is reduced (i.e. more abatement is done), the cost of abatement (C) should increase. At lower levels of q_E, further reductions will be more and more difficult and therefore the marginal cost of abatement should increase. Accordingly, the partial derivatives of C with respect to q_E should have the following signs:
In this context, let us compare the following two functional forms for the abatement cost function:

$$C = B\, w^{\gamma}\, r^{1-\gamma}\, (q_I - q_E)^{\delta}\, Q^{\eta}\, (q_I)^{-\theta}$$

$$\frac{\partial C}{\partial q_E} < 0, \quad \frac{\partial^2 C}{\partial q_E^2} > 0$$

$$C = B\, w^{\gamma}\, r^{1-\gamma}\, (q_I - q_E)^{\delta}\, Q^{\eta}\, (q_E)^{-\theta}$$

To simplify the above expressions, we take $M = B\, w^{\gamma}\, r^{1-\gamma}\, Q^{\eta}$. Thus, the first equation may be written as:

$$C = M\, (q_I - q_E)^{\delta}\, (q_I)^{-\theta}$$

The partial derivative of C with respect to q_E is

$$\frac{\partial C}{\partial q_E} = -M\delta (q_I - q_E)^{\delta-1} q_I^{-\theta}$$

which is clearly negative. The second order partial derivative is

$$\frac{\partial^2 C}{\partial q_E^2} = M\delta(\delta-1)(q_I - q_E)^{\delta-2} q_I^{-\theta}$$

Only if the estimate of δ is more than one, this derivative will be positive, and the estimated cost function will yield well behaved marginal cost curve.

Turning now to the second functional form, it may be written as

$$C = M\, (q_I - q_E)^{\delta}\, (q_E)^{-\theta}$$

The partial derivative of C with respect to q_E is

$$\frac{\partial C}{\partial q_E} = -\delta M(q_I - q_E)^{\delta-1} q_E^{-\theta} - \theta M(q_I - q_E)^{\delta} q_E^{-\theta-1}$$

This is negative. The second order partial derivative is

$$\frac{\partial^2 C}{\partial q_E^2} = \delta(\delta-1)M(q_I - q_E)^{\delta-2} q_E^{-\theta} + \delta\theta M(q_I - q_E)^{\delta-1} q_E^{-\theta-1}$$
$$+ \delta\theta M(q_I - q_E)^{\delta-1} q_E^{-\theta-1} + \theta(\theta+1)M(q_I - q_E)^{\delta} q_E^{-\theta-2}$$

The last three terms of the above expression are positive. The first term is positive if δ is more than one. Let z be equal to $(q_I - q_E)/q_E$. Then the above expression may be simplified to

$$\frac{\partial^2 C}{\partial q_E^2} = \frac{C}{(q_I - q_E)^2}[\delta(\delta-1) + 2\delta\theta z + \theta(\theta+1)z^2]$$

This shows that even if δ is less than one, for certain values of θ and z the second order partial derivative will be positive and the marginal abatement cost curve will have the right shape.

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