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Optimal Growth With Variable Rate of Time Preference

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ABSTRACT

In this paper we develop a continuous time infinite horizon optimal growth model with identical households, where the households' rate of time preference is endogenously determined. However, unlike the existing literature, we assume here that the instantaneous discount rate of the representative household is negatively related to its current consumption. With this assumption, we analyze the long run dynamic behaviour of the economy. We show that contrary to the general belief, a negative relationship between the instantaneous discount rate and the household's current consumption does not necessarily result in instability of the dynamic system. We derive a set of sufficient conditions for stability and instability in this context. We also show the possible existence of a poverty trap such that if an economy starts with a per capita income below a certain critical minimum value, then it optimally chooses a consumption-accumulation path such that it faces economic retrogression over time.

Key Words: Variable time preference, Long run dynamics, Poverty trap.
JEL Classification: D90, C61, O41.

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1. Introduction:

This paper develops a continuous time infinite horizon optimal growth model with identical households, where the households' rate of time preference is endogenously determined. To be more specific, we assume that the instantaneous discount rate of the representative household is *negatively* related to its current consumption. With this assumption, we analyze the long run dynamic behaviour of the economy. We show that contrary to the general belief, a negative relationship between the instantaneous discount rate and the household's current consumption does not necessarily result in instability of the dynamic system. We derive a set of sufficient conditions for stability and instability in this context. We also show the possible existence of a poverty trap such that if an economy starts with a per capita income below a certain critical minimum value, then it optimally chooses a consumption-accumulation path such that it faces economic retrogression over time.

In the standard optimal growth literature (Ramsey [24]; Cass [3]; Koopmans [15]) the rate of time preference is typically assumed to be a constant, independent of the stream of past and present consumption. The assumption of a constant exogenous rate of time preference, however, is based on analytical convenience rather than strong economic intuition. Koopmans [14] developed a discrete time recursive but non-additive preference structure which allowed the rate of time preference to be endogenously determined. Uzawa [26] provided a continuous time parallel, which was later extended and clarified by Epstein and

Hynes [12]. The recursive preference structure, of which the Uzawa-Epstein utility functional is a special case, has subsequently been applied to a variety of models.[1]

While it is generally accepted that the rate of time preference should depend upon the levels of past and present consumption, there is considerable disagreement over whether the rate of time preference should increase or decrease as the consumption level rises. Both Uzawa [26] and Epstein and Hynes [12] postulated a positive relationship between the rate of time preference and the stationary level of consumption (increasing marginal impatience), though neither of them offered any economic rationale for this assumption. While it may be argued that one with a history of high consumption will be less willing to accept a lower consumption profile as he or she gets used to the higher consumption level, it seems equally plausible to argue that one with a history of low consumption level is likely to be more impatient and will be more willing to consume now rather than in future, simply because poverty makes his immediate needs more important. This argument has been put forward most forcefully by Irving Fisher:

> "In general, it may be said, other things being equal, that the smaller the income, the higher the preference for present over future income. ... It is true that a small income implies a keen appreciation of future

[1] For example, the existence of intertemporal optima under recursive preferences has been analysed by Nairay [19]; Becker, Boyd and Sung [1]; and Palivos, Wang and Zhang [23]. Stability conditions for many consumers and many capital goods were studied by Lucas and Stokey [17] and Epstein [11] respectively. Other applications of this framework include analysis of the effects of changes in terms of trade on consumption (Obstfeld [21]), effects of changes in income and interest rate on consumption and investment (Obstfeld [22]), effects of changes in government purchases on interest rate (Devereux [7]), effects of changes in government spending on consumption and current account (Chang, Tsai and Liu [4]). More recently the recursive preference structure has been applied to study the long run dynamics of growth models with increasing returns in production (Drugeon [8]) and with externalities in consumption due to consumption standard (Drugeon [9]).

> wants as well as of immediate wants. Poverty bears down heavily on all portions of a man's expected life, both that which is immediate and that which is remote. But it enhances the utility of immediate income *even more* than the future income. This influence of poverty is partly rational, because of the importance, by supplying present needs, of keeping up the continuity of life and the ability to cope with the future; and partly irrational, because the pressure of present needs blinds one to the needs of the future. .... We see, then, that a low income ... tends to produce a high rate of impatience, partly from the thought that provision for the present is necessary both for the present itself and for the future as well, and partly from lack of foresight and self control." [2]

Writing more recently, even Koopmans [16] observed that a majority of economists would make an introspective argument in favour of decreasing impatience. While there might be a case for increasing marginal impatience when the actual consumption profile is very high, it is the second argument which seems intuitively more compelling - definitely so when the level of actual consumption happens to be very low. However, while the Uzawa-Epstein formulation has been widely applied to various fields of Economics (see footnote 1), there has not been any attempt so far to characterise the long run dynamics associated with the assumption of *decreasing* marginal impatience. Obstfeld [22] does mention in a footnote that such a negative relationship may give rise to multiple steady states, but he does not analyse this case. Our work differs from the existing literature in that we postulate a *negative* relationship between the level of present consumption and the

[2] Fisher [13], pp. 72-73.

instantaneous rate of time preference. The lower is the level of consumption today, the greater is the unwillingness of the people to cut down their present consumption for sake of higher consumption tomorrow. This unwillingness gets reflected in the rate at which future utilities are discounted. We assume that at any point of time, the instantaneous discount rate is a decreasing function of the current consumption. With this assumption we characterise the optimal path(s) for the representative household and analyse the long run dynamic behaviour of the economy.

It has often been argued that a positive relationship "appears necessary" for stability. This was the justification given by Epstein.[3] Blanchard and Fischer [2] also maintained that while this assumption "is difficult to defend *a priori*, ...(it) is however needed for stability".[4] In this paper, on the contrary, we show that a positive relationship between the discount rate and the stationary level of consumption is *not* necessary for stability of the long run dynamics in the variable time preference framework. In the model that follows, we have shown that if the production conditions are such that for low levels of per capita capital stock, the corresponding marginal product of capital is sufficiently high, then the long run dynamic behaviour of the economy is qualitatively similar to the standard Cass-Koopmans result and it approaches *some* steady state over time. However, if for low values of k, the marginal product of capital is relatively low, then various interesting possibilities arise. In this case, if an economy happens to start with a low level of

[3] Epstein [10], pp. 73.
[4] Blanchard and Fischer [2], pp. 73.

per capita capital stock, then it may optimally choose a consumption and accumulation path such that it approaches the zero production point over time. This case is somewhat analogous to the one discussed by Skiba [25] in the context of a convex-concave production function.

The structure of the paper is as follows. Section 2 lays out the basic framework of the model. In section 3 we characterise the different steady state equilibria and discuss their local stability property. In section 4, the long run dynamics have been analysed with the help of phase diagrams. Section 5 offers the concluding remarks.

2. The Model

Consider a perfectly competitive decentralised economy where the households maximise the discounted value of their dynastic utility over an infinite horizon. The households are identical in tastes and preferences as well as initial endowments. A single commodity is produced using two factors of production - capital and labour and at every point of time, there is full employment of both the factors. Each household maximises its infinite-time utility subject to a budget constraint such that in any period, its consumption and investment expenditure cannot exceed its total income in that period. The households take the wage rate and the rate of interest as given. However, they have perfect foresight; so they can correctly guess the time path of the wage rate and the rental rate. The population grows at a constant rate n. There is no depreciation of the existing capital stock.

Since the households are identical, we can carry out the analysis in terms of a representative household. The infinite-time utility of the representative household is given by:

$$W = \int_0^\infty u(c_t)\, e^{-\int_0^t \delta(c_v)\, dv}\, dt\,, \tag{1}$$

where c_t is the per capita consumption in period t, $u(c_t)$ is the household's instantaneous utility function and $\int_0^t \delta(c_v)\, dv = \rho(t)$ is the time preference term which depends on the past and present consumption through the function δ. The function $\delta(c_t)$ denotes the discount rate or the instantaneous rate of time preference and it depends on c_t alone.

The household maximises (1) subject to the following budget constraint:

$$\frac{dk}{dt} = w_t + r_t\, k_t - n\, k_t - c_t\,, \tag{2}$$

where w_t and r_t denote the wage rate and interest rate respectively and k_t is the per capita capital stock in period t.

The production function satisfies the standard assumptions about continuity, concavity and constant returns to scale. Let $f(k)$ denote the per capita output. Then

$$f(0) = 0\,;\ f'(k) > 0 \text{ and } f''(k) < 0\,. \tag{3}$$

Moreover, due to perfect competition, the wage and rental rates are equal to the marginal products of labour and capital respectively:

$$w = f(k) - kf'(k) \ ; \ r = f'(k). \tag{4}$$

The per capita capital stock lies in the closed interval $[0, \bar{k}]$, where $\bar{k}$ is the maximum attainable capital stock (per capita) and is defined as

$$\bar{k} : f(\bar{k}) = n\bar{k}. \tag{5}$$

On the other hand, per capita consumption c lies within the range $[0, \bar{c}]$, where $\bar{c}$ denotes the highest possible level of per capita consumption and is defined as follows. We assume that people can consume the existing capital stock so that investment can be negative. However, there is an exogenously given finite upper bound on the *rate* at which capital can be consumed. Let this bound be α. Then in any period investment cannot be more negative than $-\alpha$. Therefore, the maximum possible consumption in any period is given by

$$\bar{c} = f(\bar{k}) + \alpha. \tag{6}$$

The instantaneous utility function satisfies the following conditions:

$$\text{for all } c \in [0, \bar{c}], \ u(0) = 0 \ ; \ u(c) > 0 \ ; \ u'(c) > 0 \ ; \ u''(c) < 0. \tag{7}$$

As was mentioned before, in this model we assume that δ, i.e., the rate at which future utility is discounted at any period, is a decreasing function of the current consumption level. We impose the following conditions on the δ function:

$$\text{for all } c \in [0, \bar{c}], \ \delta(c) \geq 0 \ ; \ \delta'(c) < 0 \ ; \ \delta''(c) = 0. \tag{8}$$

Assumption (8) implies that when consumption level remains stationary, a higher stationary level of consumption is associated with a lower value of δ, i.e., greater patience. Also the last condition in (8) implies that δ is a linear function of c. This has been assumed to keep the long run dynamics of the model analytically tractable. We may have given up some generality in doing so, but we gain in expositional and mathematical simplicity. In any case, there is no compelling economic reason to assume either $\delta'' > 0$ or $\delta'' < 0$. We should also mention here that the results of our paper do not crucially depend on the linearity of the δ function. We have shown elsewhere[5] that exactly same conclusions will emerge if we take a utility function of the form: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}; 0 < \sigma < 1$ and an instantaneous time preference function of the form: $\delta(c) = \frac{A}{c}; A > 0$.

Since $\delta(c)$ is a linear function of c, we assume that it takes the following functional form:

$$\left.\begin{aligned} \delta(c) &= A - Bc \quad \text{for } c < \frac{A}{B}, \text{ where } A, B > 0 \\ &= 0 \quad \text{for } c \geq \frac{A}{B}. \end{aligned}\right\} \tag{9}$$

We assume that $\bar{c} < \frac{A}{B}$ so that for the relevant c values $\delta(c)$ is strictly positive.

At any point of time t, the time preference term, $\rho(t)$, is given by the integral $\int_0^t \delta(c_v)\, dv$. Therefore, by the fundamental theorem of integral calculus, we can write

[5] Das [6], chapter 6.

$$\frac{d\rho}{dt} = \delta(c_t) \text{ i.e., } dt = \frac{d\rho}{\delta(c_t)} = \frac{d\rho}{A - B\,c_t}. \tag{10}$$

Also note that whatever be the time path of consumption, as t tends to zero, ρ tends to zero, and as t tends to infinity, ρ also tends to infinity.[6] Therefore the transformation given in (10) allows us to write the intertemporal optimization problem of the representative household in terms of ρ in the following way:

$$\text{Maximise } W = \int_0^\infty \frac{u(c)}{A - B\,c}\, e^{-\rho}\, d\rho \tag{1'}$$

$$\text{subject to } \quad \frac{dk}{d\rho} = \frac{w + rk - nk - c}{A - B\,c}. \tag{2'}$$

The corresponding Hamiltonian and Current-value Hamiltonian functions are given by H and $\overline{H}$ respectively:

$$H = \frac{u(c)}{A - B\,c} e^{-\rho} + \lambda(\rho)\left[\frac{w + rk - nk - c}{A - B\,c}\right], \tag{11}$$

$$\overline{H} = \frac{u(c)}{A - B\,c} + \mu(\rho)\left[\frac{w + rk - nk - c}{A - B\,c}\right], \tag{11'}$$

where $\mu(\rho) = \lambda(\rho)\, e^{\rho}$ and $\overline{H} = H\, e^{\rho}$.

The necessary conditions for optimization are:

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[6] Since $\delta(c)$ is a decreasing function of c and also $c \le \overline{c}$, $\int_0^\infty \delta(c_v)\, dv \ge \int_0^\infty \delta(\overline{c})\, dv = \infty$. Therefore it follows that as t tends to infinity, $\rho = \int_0^t \delta(c_v)\, dv$ tends to infinity as well.

$$\frac{\partial \overline{H}}{\partial c} = 0, \tag{12}$$

$$\frac{\partial \overline{H}}{\partial k} = -\frac{d\mu}{d\rho} + \mu , \tag{13}$$

$$\frac{\partial \overline{H}}{\partial \mu} = \frac{dk}{d\rho} , \tag{14}$$

$$\lim_{\rho \to \infty} H = \lim_{\rho \to \infty} \overline{H} e^{-\rho} = 0 .[7] \tag{15}$$

From these first order conditions, we can derive the optimal consumption and accumulation paths in terms of ρ as:

$$\frac{dc}{d\rho} = \frac{1}{A - B\,c} \left[\frac{(A - B\,c)\, u'(c) + B\, u(c)}{-u''(c)} \right] \left[\frac{r - n}{A - B.(w + rk - nk)} - 1 \right] , \tag{16}$$

$$\frac{dk}{d\rho} = \frac{w + rk - nk - c}{A - B\,c} . \tag{17}$$

Substituting t for ρ (from condition (10)) and using relation (4), the above optimal paths can be expressed in terms of t as:

$$\frac{dc}{dt} = \left[\frac{(A - B\,c)\, u'(c) + B\, u(c)}{-u''(c)} \right] \left[\frac{f'(k) - n}{A - B.(f(k) - nk)} - 1 \right] , \tag{16'}$$

$$\frac{dk}{dt} = f(k) - nk - c . \tag{17'}$$

[7] Condition (15) was derived by Michel [18]. Also see Chiang [5] for a justification and economic interpretation of this condition.

Eq.(16′) and (17′) are the two basic dynamic equations of our model. These are the two equations of motion, which determine the dynamic behaviour of the economy over time. In the next section, we use these two to characterise the steady state equilibrium (or equilibria, as the case may be) of the system.

3. Steady States

In steady state per capita consumption and per capita capital stock both remain constant. Therefore $\frac{dc}{dt} = 0$ and $\frac{dk}{dt} = 0$ together determine the long run equilibrium values of k and c. Now, $\frac{dc}{dt} = 0$ implies

$$\text{either } \frac{(A - B\,c)\,u'(c) + B\,u(c)}{-u''(c)} = 0, \tag{18}$$

$$\text{or } \frac{f'(k) - n}{A - B(f(k) - nk)} - 1 = 0,$$

$$\text{i.e., } f'(k) = A + n - B(f(k) - nk). \tag{19}$$

Let $\sigma(c)$ denote the elasticity of marginal utility with respect to consumption and $\varepsilon(c)$ denote the elasticity of the instantaneous utility, $u(c)$, with respect to consumption. Under the assumption that $\sigma^{-1}(0) \neq 0$ and $\varepsilon < 1$, (18) gives us the trivial equilibrium value of c (i.e., $c = 0$).[8] Eq. (19) on the other hand determines the non-trivial steady state values of k. In the discussion that follows, we are going to analyse the characteristics of the non-trivial equilibria only.

Again $\frac{dk}{dt} = 0$ implies

$$c = f(k) - nk\,. \tag{20}$$

Therefore solving (19) and (20) we can derive the equilibrium values of c and k for the non-trivial steady state.

It is convenient to characterise the equilibrium values of k with the help of diagrams. For this purpose consider equation (19). The LHS of (19) is the marginal product of capital, which is a decreasing function of k. Figure 1 depicts a possible diagrammatic representation of this function with k on the horizontal axis.

On the other hand, the RHS of (19) takes a finite positive value $A+n$ when $k = 0$, falls as k rises, reaches a minimum at the 'golden rule' value of the per capita capital stock (k_g),[9] and increases thereafter. This is shown in Figure 2.

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[8] Note that $\sigma(c) = \left[\frac{-u''(c)\,c}{u'(c)}\right]$ and $\varepsilon(c) = u'(c)\,\frac{c}{u(c)}$. When $c \to 0$, the LHS of equation (18) becomes $\frac{A\,c}{\sigma(c)}$. Therefore if $\sigma^{-1}(0) \neq 0$, then obviously c = 0 is a steady state solution. If $\sigma^{-1}(0) = 0$, then we have to analyse the behaviour of $\frac{c}{\sigma(c)}$ as $c \to 0$. It is generally assumed that $\sigma^{-1}(0) \neq 0$. Also note that under the assumption that $\varepsilon < 1$, $c = 0$ is the only solution to (18). Given our assumptions about $u(c)$, for any non-zero c, the LHS of (18) is always positive.

[9] The 'golden rule' capital stock is defined as $k_g\ :\ f'(k_g) = n$.

(Figure 1 and Figure 2 somewhere here)

The equilibrium values of k are given by the point(s) of intersection of these two curves. We discuss below three cases, which are mutually exclusive and exhaustive.

Case I: $\lim_{k \to 0} f'(k) > A + n$

In this case, the $f'(k)$ curve starts above the $A+n-B\,(f(k) - nk)$ curve. A possible equilibrium configuration is shown in Figure 3. Note that here when k takes very low values, $f'(k) > A + n - B\,(f(k)-nk)$. On the other hand, for $k \geq k_g$, $f'(k) \leq n < A + n - B\,(f(k) - nk)$.[10] This ensures that there is at least one point of intersection. Also, if the two curves intersect each other more than once, then such intersection points will be odd in number. Hence in this case, a non-trivial steady state always exists and either this equilibrium is unique or there are odd numbered multiple equilibria.[11]

(Figure 3 somewhere here)

Case II: $\lim_{k \to 0} f'(k) < A + n$

Here there are two possibilities: either there does not exist a non-trivial steady state at all, or there exist at least two points of intersection. In the latter case, there may

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[10] By assumption, at steady state $c = f(k) - nk \leq \bar{c} < \frac{A}{B}$ for all k. Therefore, $A - B(f(k) - nk) > 0$.

[11] We are ignoring the possible tangency points. It can be easily shown that the dynamics remains qualitatively similar even if we allow for such tangency cases.

exist more than two intersection points, but the equilibrium points will always appear in even numbers. These two possibilities are depicted in Figure 4. Figure 4(a) shows the no equilibrium case whereas a possible equilibrium configuration in the second case is shown in Figure 4 (b).

(Figure 4 somewhere here)

Case III: $\lim_{k \to 0} f'(k) = A + n$

In this case, both the curves start from the same point. Therefore, $k = 0$ is an equilibrium. However, for positive k values either there does not exist any equilibrium, or the equilibrium points exist in odd or even numbers. Since these possibilities are similar to the cases already discussed in I and II, we do not analyse this case separately.[12]

To analyse the local stability property of these steady states, we look at the Jacobian matrix of the two differential equations given by (16′) and (17′) and evaluate the matrix at different equilibrium points.

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[12] Note that if the production function satisfies the Inada condition $\lim_{k \to 0} f'(k) = \infty$, then cases II and III do not arise. This happens here due to the linear nature of the δ function. However we have shown elsewhere (Das [6], chapter 6) that with a specific nonlinear δ function and a specific utility function, these cases will arise even when the production function satisfies the Inada conditions. The specific forms of the utility function and the δ function have been mentioned earlier in page 8 of the present paper.

Let $\dfrac{(A-Bc)\,u'(c)+B\,u(c)}{-u''(c)}=\phi(c)$ and $\dfrac{f'(k)-n}{A-B(f(k)-nk)}-1=\psi(k)$.

Then from (16′):

$$\frac{dc}{dt}=\phi(c)\ \psi(k)=G(c,k),\ \text{say.} \qquad (16'')$$

Again from (17′):

$$\frac{dk}{dt}=f(k)-nk-c=H(c,k),\ \text{say.} \qquad (17'')$$

So the Jacobian of (16″) and (17″) is :

$$J=\begin{bmatrix} G_c & G_k \\ H_c & H_k \end{bmatrix} \qquad (21)$$

Now, $G_c=\phi'(c)\psi(k)$. At the non-trivial steady state, $\psi(k)=0$ and $\phi'(c)\neq 0$. Therefore when evaluated at the equilibrium point,

$$G_c=0. \qquad (22)$$

Again, $G_k=\phi(c)\psi'(k)$, where $\psi'(k)=\dfrac{[A-B(f(k)-nk)]f''(k)+B\,[f'(k)-n]^2}{[A-B(f(k)-nk)]^2}$.

In equilibrium, $f'(k)-n=A-B(f(k)-nk)$. Hence, when evaluated at the equilibrium point,

$$G_k=\phi(c)\left[\frac{f''(k)}{A-B(f(k)-nk)}+B\right]. \qquad (23)$$

On the other hand,

$$H_c=-1, \qquad (24)$$

and $H_k = f'(k) - n$. (25)

From (21), using (22) – (25), the Jacobian evaluated at equilibrium is :

$$\hat{J} = \begin{bmatrix} 0 & \phi(c)\left\{\dfrac{f''(k)}{A - B(f(k) - nk)} + B\right\} \\ -1 & f'(k) - n \end{bmatrix}. \tag{21'}$$

Determinant of the Jacobian:

$$\text{Det}\,\hat{J} = \frac{\phi(c)}{A - B(f(k) - nk)}\left[B(f'(k) - n) + f''(k)\right].$$

Det $\hat{J} >$ or < 0 according as $B(f'(k) - n) >$ or $< -f''(k)$ at the equilibrium point. Note that $B(f'(k) - n)$ is nothing but the slope of the $A + n - B(f(k) - nk)$ function (RHS of equation (19)) where as $-f''(k)$ is the absolute value of the slope of $f'(k)$ (LHS of equation (19)). Therefore the sign of Det $\hat{J}$ depends on whether the curve representing the LHS of equation 19 intersects the curve representing the RHS of (19) from above or from below.

Trace of the Jacobian :

$$\text{Tr}\,\hat{J} = f'(k) - n.$$

In equilibrium, Tr $\hat{J} > 0$ since all the equilibrium points lie to the left of k_g.

When the equilibrium points are odd in number (Case I), at the first equilibrium point, the $f'(k)$ cuts the $A + n - B(f(k) - nk)$ curve from above. If (c^*, k^*) denote the first equilibrium point, then it follows that at (c^*, k^*), Det $\hat{J} < 0$ while Tr $\hat{J} > 0$. So the first equilibrium is a saddle point. The other equilibria (if they exist) will be alternatively unstable (either unstable *node* or unstable *focus*) and saddle points depending on the sign of Det $\hat{J}$.

In the other case when the equilibrium points exist in even numbers (Case II), at the first equilibrium point, the $f'(k)$ cuts the $A+n-B(f(k)-nk)$ curve from below and at the second equilibrium point. $f'(k)$ cuts the $A+n-B(f(k)-nk)$ curve from above. Therefore, if (c^*, k^*) denote the first equilibrium point and (c^{**}, k^{**}) denote the second equilibrium point, then at (c^*, k^*) Det $\hat{J}$ is positive and at (c^{**}, k^{**}) Det $\hat{J}$ is negative. So (c^*, k^*) is an unstable equilibrium while (c^{**}, k^{**}) is a saddle point. The first equilibrium point will be an unstable *node* or unstable *focus* depending on whether the characteristic roots at that point are real or complex. When the number of equilibria exceeds two, then they will be alternatively unstable and saddle points.

In the rest of the analysis, we are going to deal with only two possibilities, namely, the unique equilibrium situation in Case I and the two equilibria situation in Case II. It can be easily shown that when the number of equilibria is more, the nature of the long run dynamics will be similar to either of the two above-mentioned cases.

4. Long Run Dynamics

The local stability analysis tells us that in Case I, where the equilibrium is unique, it is a saddle point. On the other hand, in Case II, out of the two equilibria, the first one is unstable while the second one is a saddle point. In this section, we analyse the long run dynamics of these two cases using phase diagram technique. For this purpose, consider the two equations of motion in our model given by (16′) and (17′). Using these two, we can determine the direction of movements of c and k over time. The following relations describe the dynamic behaviour of these variables for any c and k such that $c \in (0, \bar{c})$ and $k \in (0, \bar{k})$.

$$\frac{dc}{dt} \gtreqless 0 \text{ according as } f'(k) \gtreqless A + n - B(f(k) - nk), \tag{26}$$

$$\frac{dk}{dt} \gtreqless 0 \text{ according as } c \lesseqgtr f(k) - nk. \tag{27}$$

Relation (26) can be interpreted in lines similar to the standard case. We can write (26) in the following way:

$$\frac{dc}{dt} \gtreqless 0 \text{ according as } f'(k) - n \gtreqless A - B(f(k) - nk),$$

$$\text{i.e., } f'(k) - n \gtreqless \delta(c) + \delta'(c)\,(f(k) - nk - c) \text{ where } \delta(c) = A - Bc.$$

At any point of time, given the per capita capital stock, the household has to decide how much to consume and how much to save. Now consider any point in the c-k plane. At the arbitrarily given consumption level c, whether the household consumes or invests an additional unit of output depends on the *net* cost of a unit of consumption foregone. If the additional unit of output is invested today then the future return (i.e. the extra per capita output that will be available tomorrow) is given by $f'(k) - n$. On the other hand, the cost of foregoing consumption today gets reflected in the discount rate. If $\Delta k = f(k) - nk - c$ is the amount of consumption foregone today, then $\delta(c) + \delta'(c)\,\Delta k$ is the cost associated with that. Clearly, if the future return is greater than the cost of consumption foregone today, then the household will consume less today and more tomorrow; hence $\frac{dc}{dt}$ is positive. The opposite happens when the return in future is less than the cost associated with consumption foregone.

The phase diagram of the unique equilibrium case (Case I) is shown in Figure 5. In this case there exists a unique stable path (stable arm of the saddle point) which passes through the equilibrium point. This is the optimal path (the broken line in Figure 5). Along this path, starting from any initial point, the economy asymptotically approaches the non-trivial steady state point. This result is similar to the standard Cass-Koopmans result. Note that even when the equilibrium is not unique, the first equilibrium will be a saddle point so that over time the economy will approach *some* steady state.

(Figure 5 somewhere here)

The two equilibria case (Case II) is more interesting. Here the first equilibrium is an unstable one whereas the second one is a saddle point. The phase diagram is given in Figure 6. From the direction of movements of k and c as shown in the phase diagram, we observe that there can be two types of trajectories which satisfy all the necessary conditions for optimality (including Michel's condition). One of them over time asymptotically approaches the second equilibrium point (k^{**}, c^{**}), while the other one approaches the origin (0,0) as t goes to infinity. We call the trajectories of the first kind optimal trajectory I and trajectories belonging to the second set are called optimal trajectory II.

(Figure 6 somewhere here)

Optimal trajectory I may start from a close neighbourhood of the first equilibrium point itself (which is the case when the characteristic roots at (k^*, c^*) are real and positive) or it can untwist around the first equilibrium point (this happens when the characteristic roots at (k^*, c^*) are complex with positive real parts). Optimal trajectory II, on the other hand, either remains above the $dk/dt = 0$ curve for all values of k or intersects the $dk/dt = 0$ curve at some point (or points). Possible trajectories of the first kind are traced by continuous lines and trajectories of the second kind are traced by the broken lines in the c-k plane in Figure 7. If an

(Figure 7 somewhere here)

optimal path exists, it is represented either by trajectory I or by trajectory II. Since both the trajectories satisfy all the necessary conditions for optimality, we have to rank them in terms of total infinite-time utility to determine the global optima.[13] For this purpose we now state a theorem and its three corollaries which enable us to compare the two paths. This theorem is analogous to Skiba [25].[14]

Theorem I. *Along any optimal trajectory (i.e., trajectory which satisfies all the necessary conditions), value of the integral or the infinite-time utility for a given initial capital stock is equal to the current value of the Hamiltonian evaluated at that initial point on the trajectory.*

In other words, given k_0, if $\hat{c}(t)$ is an optimal consumption path, then

$$\int_0^\infty u(\hat{c}_t)\, e^{-\int_0^t \delta(\hat{c}_t)\, dv}\, dt = \int_0^\infty \frac{u(\hat{c})}{\delta(\hat{c})} e^{-\rho}\, d\rho = \overline{H}(\hat{c}_0, k_0) = \overline{H}_0 .$$

Therefore we can compare trajectory I and II by simply comparing the values of $\overline{H}_0$ for a given k_0. Also from our assumptions about $u(c)$, $\delta(c)$ and $f(k)$, we can derive the following three corollaries :

Corollary I(a). *For a given k_0, an optimal trajectory with higher $\hat{c}_0$ will give higher, same or lower infinite-time utility according as $\hat{c}_0$ is greater than, equal to or less than $f(k_0) - nk_0$.*

Corollary I(b). *Along any optimal trajectory, as we move to a higher k_0, infinite-time utility increases.*

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[13] Note that since these trajectories satisfy only the necessary conditions, it is possible that none of these represents the optimal path. But in that case, there is no optimal path at all. If however an optimal indeed exists, then it *has to be* either of the two.

[14] Proofs of the theorem and its corollaries are given in the appendix at the end of the paper.

Corollary I(c). *Along any optimal trajectory, as we move to a higher k_0, the rate at which infinite-time utility increases depends on $\hat{c}_0$. Given k_0, the lower is $\hat{c}_0$ the higher is the rate.*

Using theorem I and corollaries I(a), I(b), I(c), we can now rank optimal trajectory I and trajectory II for Case II. We discuss below three mutually exclusive (and exhaustive) possibilities.

Subcase II(a): *Optimal trajectory II lies above the $dk/dt = 0$ curve for the entire feasible range of k:*

Trajectory I in this case may either start in a close neighbourhood of the first equilibrium point or it may untwist around the first equilibrium point. This case is shown in Figure 8. In this case, for $k_0 \geq k^{**}$, $\hat{c}_0 > f(k_0) - nk_0$ for both the

(Figure 8 somewhere here)

trajectories, but $(\hat{c}_0)_{II} > (\hat{c}_0)_I$. Therefore by corollary I(a), the trajectory II gives more infinite-time utility. Again by corollary I(c), as we move to a lower k_0 than k^{**}, $\overline{H}_0$ falls along both the trajectories, but the *rate* at which it is falling is greater along trajectory I than along trajectory II. This, coupled with the fact that $(\overline{H}_0)_{II} > (\overline{H}_0)_I$ for $k_0 \geq k^{**}$, enable us to conclude that *if optimal trajectory II lies* *above the $dk/dt = 0$ curve all along, then trajectory II gives more infinite-time utility, whatever be the initial per capita capital stock.*

Subcase II (b): *Optimal trajectories I and II both start in a close neighbourhood of the first equilibrium point itself:*

Here the first equilibrium point is an unstable *node*. The possible shapes of the two trajectories are shown in Figure 9. In this case, for $k_0 < k^*$ trajectory II is the only available optimal trajectory and similarly for $k_0 > k^*$ trajectory I is the only available trajectory. Therefore *if an economy starts with an initial capital stock $k_0 > k^*$, then it moves along trajectory I; if it starts with $k_0 < k^*$, then it follows trajectory II and if $k_0 = k^*$, then the economy remains stationary at the first equilibrium point.*

(Figure 9 somewhere here)

Subcase II(c): *Both optimal trajectories I and II untwist around the first equilibrium point:*

In this case the first equilibrium point is an unstable *focus*. Therefore in the neighbourhood of the first equilibrium point, for any k_0, there are more than one $\hat{c}_0$ along trajectory I as well as trajectory II. So our task here is three-fold: first, to determine the $\hat{c}_0$ which gives the maximum utility for a given k_0 along trajectory I; secondly, to determine a similar $\hat{c}_0$ value for a given k_0 along trajectory II and finally, rank these two $\hat{c}_0$ values in terms of the corresponding infinite-time utility.

Let us first consider optimal trajectory I which is shown in Figure 10. Along trajectory I, consider a k_0 in the neighbourhood of the first equilibrium point such that there are more than one $\hat{c}_0$ corresponding to that. For example, let us take $k_0 = k^*$. In this case the steady state per capita consumption, c^*, is also an admissible initial consumption level. However, from corollary I(a), given the initial capital stock, infinite-time utility along trajectory I is actually minimised at $\hat{c}_0 = c^*$. So if other optimal paths are available, then c^* will never be chosen; the economy will start with an initial consumption either greater or less than c^*. Now let us trace the

(Figure 10 somewhere here)

path from the second equilibrium point to the first one and denote the successive points with different c values corresponding to the same initial capital stock k^* respectively by M1, M2, M3, ... etc. Let us first consider M1 and M2. Along trajectory I, between M1 and M2 we can find a point on the trajectory such that the k value at that point is associated with a unique c and hence a unique value of $\overline{H}_0$. Let us denote this per capita capital stock by k_1 (see Figure 10). Now from k_1, as we move towards k^*, $\overline{H}_0$ rises (by corollary I(a)), but it rises at a faster rate as we move to M1 rather than M2 (by corollary I(c)). Hence ,

$$(\overline{H}_0)_{M1} > (\overline{H}_0)_{M2}.$$

Next consider M2 and M3. Once again on trajectory I between M2 and M3, we can find a point such that corresponding to that k, there is a unique c. Let us call this k

value k_2. Using corollary I(b) and I(c), it can be shown that as we move from k_2 towards k^*, $\overline{H}_0$ rises at a faster rate as we move to M2 rather than M3. So,

$$(\overline{H}_0)_{M2} > (\overline{H}_0)_{M3}.$$

Proceeding this way, we can show that

$$(\overline{H}_0)_{M1} > (\overline{H}_0)_{M2} > (\overline{H}_0)_{M3} > \ldots\ldots\ldots > (\overline{H}_0)_{Mn} > (\overline{H}_0)_{Mn+1} > \ldots\ldots .$$

Therefore, *for any* k_0*, along trajectory I, the path associated with the lowest initial value of per capita consumption gives the maximum infinite-time utility.*

Let us now consider trajectory II which is shown in Figure 11. Once again we choose a k in the neighbourhood of the first equilibrium point, say k^* itself, such that there are more than one c values along trajectory II associated with this k. Let us move along trajectory II from the origin towards the first equilibrium point and denote the successive points on trajectory II corresponding to k^* by N1, N2, N3,... etc (see Figure 11). Using similar arguments we can now show that

$$(\overline{H}_0)_{N1} > (\overline{H}_0)_{N2} > (\overline{H}_0)_{N3} > \ldots\ldots\ldots > (\overline{H}_0)_{Nn} > (\overline{H}_0)_{Nn+1} > \ldots\ldots .$$

Therefore, *for any* k_0*, along trajectory II the path associated with the highest initial value of per capita consumption gives the maximum infinite-time utility.*

(Figure 11and 12 here)

Finally, we have to compare trajectory I and II for all k values within the relevant range. For this purpose consider Figure 12. Let us move along trajectory II from the origin towards the first equilibrium point and denote the per capita capital stock associated with the first point of intersection between the $dk/dt = 0$ curve and trajectory II by k'. Similarly as we move from the second equilibrium point to the first one, let the k value associated with the first point of intersection between the $dk/dt = 0$ curve and trajectory I be denoted by k''. Then by corollary I(a), at $k_0 = k'$, $(\overline{H}_0)_{II} > (\overline{H}_0)_I$ and at $k_0 = k''$, $(\overline{H}_0)_I > (\overline{H}_0)_{II}$. As we move from k' to k'', $\overline{H}_0$ increases at a faster rate along trajectory I than trajectory II. Therefore, *there exists a* $\tilde{k} \in (k', k'')$ *such that if* $k_0 < \tilde{k}$, *trajectory II gives more infinite-time utility than trajectory I. On the other hand, if* $k_0 > \tilde{k}$, *then trajectory I gives more infinite-time utility.*

Let us now summarise the findings of this section. In Case I, where the equilibrium is unique and a saddle point, there is a stable optimal path which takes the economy to the non-trivial equilibrium. However, in the two equilibria case (Case II), where one equilibrium is unstable and the other one a saddle point, either the economy approaches the zero production-zero consumption point for the entire relevant range of k (Subcase II(a)), or there exists a certain k value such that if the economy starts to the left of this k, then it follows an optimal path such that it approaches the origin over time (Subcases II(b) and II(c)).

5. Conclusion

This paper establishes two points. Firstly, contrary to what is generally believed, the postulate that poor people are more impatient does not necessarily result in instability. Stability of the dynamic system depends on the relative magnitudes of the marginal product of capital at low income levels and the rate of discount that is associated with a constant stream of consumption, which can be maintained at that income level. The second point is that when rate of discount is higher than the marginal product of capital for low k values, the economy may *optimally* choose a consumption path such that over time it approaches the zero-production and zero consumption point. Thus something akin to Nurkse's [20] 'poverty trap' is possible whereby an economy starting with a per capita capital stock less that a certain critical minimum value follows a path of economic retrogression.

The possibility of this kind of a 'poverty trap' is well recognised in the development literature. However, In this paper we offer a different explanation for the existence such 'poverty traps': a poor economy gets stuck with a low or even decreasing per capita income not because the economy is technologically backward, but because the households, being poor, are more impatient and therefore dis-save (or over-consume). Here we recognise the fact that even leaving aside the purely biological subsistence considerations, there may exist a lower limit below which the households are not prepared to push back their current consumption level. This unwillingness may arise because the rate at which they discount future utilities vary inversely with their income level (or rather the level of consumption that they enjoy). The poorer are the households, the more unwilling they are to postpone today's consumption for sake of tomorrow. So households' time preference varies inversely with the past and present consumption stream. As a result, a poor economy may follow an optimal path which leads to economic retrogression. Also, unlike the Harrodian case, it is not a mismatch between expectations and reality that is giving rise to this instability. The households have perfect foresight here, still they choose a path which approaches the zero production-zero consumption point over time. This happens because for a poor household, the rate of time preference is so high that even though it maximises its dynastic utility over an infinite horizon, the future utility terms become insignificant and in effect it is looking at a much shorter time span. So present becomes more important than future and an optimal consumption stream which gives higher consumption today is preferred even if it leads to zero consumption at some distant future point. It is this preoccupation with the present on the part of the households that leads to this kind of an apparently "irrational" outcome.

Appendix:

Proof of Theorem I : In our model the infinite time utility function of the representative household is given by :

$$W = \int_0^\infty u(c_t)\, e^{-\int_0^t \delta(c_v)dv} dt = \int_0^\infty \frac{u(c)}{\delta(c)} e^{-\rho} d\rho .$$

The current value Hamiltonian of the dynamic optimization problem (in terms of ρ) is :

$$\overline{H} = \frac{u(c)}{\delta(c)} + \mu(\rho)\left[\frac{f(k) - nk - c}{\delta(c)}\right].$$

An optimal trajectory satisfies the following four first order conditions:

$$\frac{\partial \overline{H}}{\partial c} = 0 , \quad \text{(A.1)}$$

$$\frac{\partial \overline{H}}{\partial k} = -\frac{d\mu}{d\rho} + \mu , \quad \text{(A.2)}$$

$$\frac{\partial \overline{H}}{\partial \mu} = \frac{dk}{d\rho} , \quad \text{(A.3)}$$

$$\lim_{\rho \to \infty} e^{-\rho} \overline{H} = 0 . \quad \text{(A.4)}$$

If $\hat{c}(\rho)$ and $\hat{k}(\rho)$ denote some optimal consumption and accumulation paths respectively (in term of ρ) then from condition (A.4),

$$\lim_{\rho \to \infty} e^{-\rho} \left[\overline{H}(\hat{c}, \hat{k}, \hat{\mu}) \right] = 0 . \quad \text{(A.5)}$$

Hence, $\int_0^\infty d\left[e^{-\rho}\, \overline{H}(\hat{c},\hat{k},\hat{\mu})\right] = \left[e^{-\rho}\, \overline{H}(\hat{c},\hat{k},\hat{\mu})\right]_0^\infty = -\overline{H}(\hat{c}_0,\hat{k}_0,\hat{\mu}_0)$.

Therefore, $\overline{H}(\hat{c}_0, \hat{k}_0, \hat{\mu}_0) = -\int_0^{\infty} d\left[e^{-\rho}\, \overline{H}(\hat{c}, \hat{k}, \hat{\mu})\right]$

$$= \int_0^{\infty} \overline{H}\, e^{-\rho} d\rho - \int_0^{\infty} e^{-\rho} d\overline{H}$$

$$= \int_0^{\infty} \frac{u(\hat{c})}{\delta(\hat{c})} e^{-\rho} d\rho$$

$$+ \int_0^{\infty} e^{-\rho} \left[\frac{\hat{\mu}\,(f(\hat{k}) - n\hat{k} - \hat{c}}{\delta(\hat{c})} - \left\{ \frac{\partial \overline{H}}{\partial \hat{c}} \frac{d\hat{c}}{d\rho} + \frac{\partial \overline{H}}{\partial \hat{k}} \frac{d\hat{k}}{d\rho} + \frac{\partial H}{\partial \hat{\mu}} \frac{d\hat{\mu}}{d\rho} \right\} \right] d\rho .$$

Now from conditions (A.1), (A.2) and (A.3), it can be shown that the second term of the above expression is zero. Q. E. D.

Note that in the above proof we have not assumed any specific form of the utility function of the instantaneous time preference function. The three corollaries of Theorem I that have been stated in the paper are proved below with respect to a linear δ function of the following form:

$$\delta(c) = A - Bc;\; c < \frac{A}{B},\; A, B > 0 .$$

Proof of Corollary I(a) : Using (A.1), (A.2) and (A.3), along any optimal path we can write the current-value Hamiltonian in terms of the initial values of c and k as :

$$\overline{H}(\hat{c}_0, k_0) = \frac{u(\hat{c}_0) + (f(k_0) - nk_0 - \hat{c}_0)\, u'(\hat{c}_0)}{A - B\,(f(k_0) - nk_0)} .$$

Therefore, $\dfrac{\partial \overline{H}(\hat{c}_0, k_0)}{\partial \hat{c}_0} = \dfrac{-u''(\hat{c}_0)\,[\hat{c}_0 - f(k_0) - nk_0]}{A - B\,(f(k_0) - nk_0)}$.

Now, $f(k) - nk < \dfrac{A}{B}$ (by assumption) .

So, $\frac{\partial \overline{\overline{H}}_0}{\partial \hat{c}_0}$ >, = or < 0 according as $\hat{c}_0$ >, = or < $f(k_0) - nk_0$. Hence the corollary follows from Theorem I. Q. E. D.

Proof of Corollary I(b) : Total differentiating $\overline{\overline{H}}(\hat{c}_0, k_0)$ with respect to k_0, we get

$$\frac{d\overline{\overline{H}}_0}{dk_0} = \frac{(A - B\,\hat{c}_0)\,u'(\hat{c}_0)}{A - B(f(k_o - nk_0)}.$$

Now by assumption, $u'(c) > 0$, $c < \frac{A}{B}$ and $f(k) - nk < \frac{A}{B}$. Therefore, whatever be the value of k_0, $\frac{d\overline{\overline{H}}_0}{dk_0}$ is always greater than zero. Q.E.D.

Proof of Corollary I(c) : From Corollary I(b),

$$\frac{1}{\overline{\overline{H}}_0}\frac{d\overline{\overline{H}}_0}{dk_0} = \frac{(A - B\,\hat{c}_0)\,u'(\hat{c}_0) + B\,u(\hat{c}_0)}{u(\hat{c}_0) + (f(k_0) - nk_0 - \hat{c}_0)\,u'(\hat{c}_0)}.$$

Therefore, $\frac{\partial}{\partial \hat{c}_0}\left[\frac{1}{\overline{\overline{H}}_0}\frac{d\overline{\overline{H}}_0}{dk_0}\right] = \frac{u(\hat{c}_0)\,u''(\hat{c}_0)\,[A - B(f(k_0) - nk_0)]}{[u(\hat{c}_0) + (f(k_0) - nk_0 - \hat{c}_0)\,u'(\hat{c}_0)]^2} < 0.$ Q. E. D.

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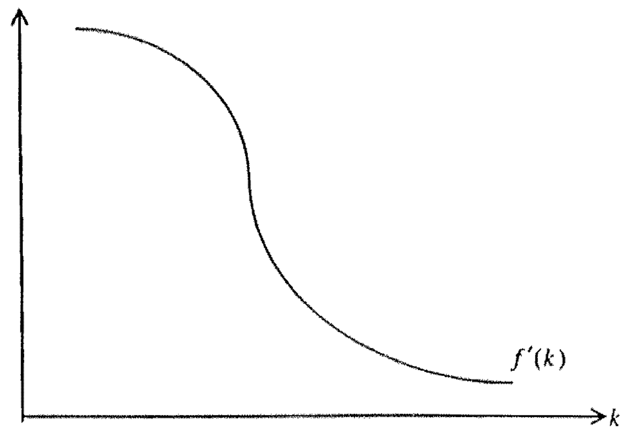
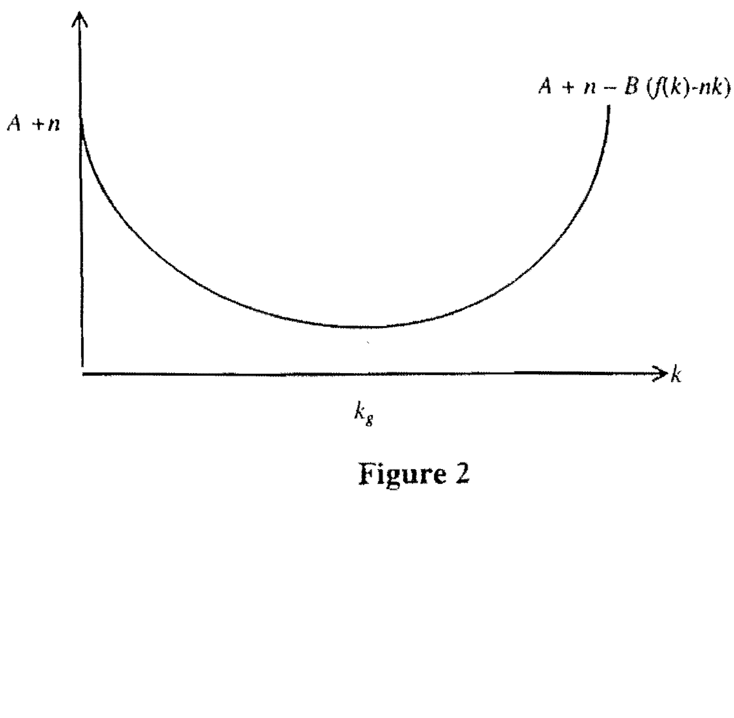


Figure 1



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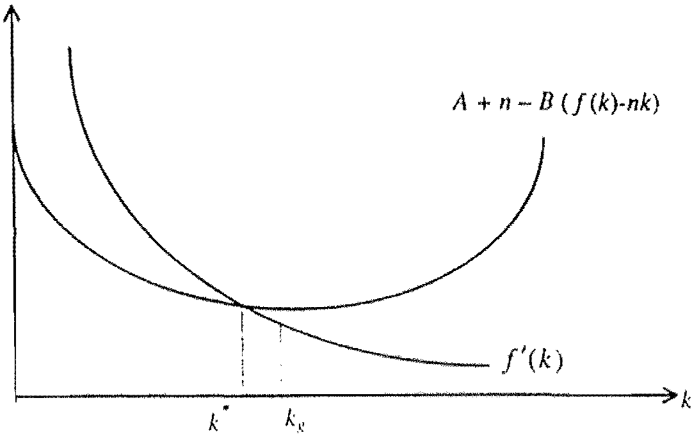

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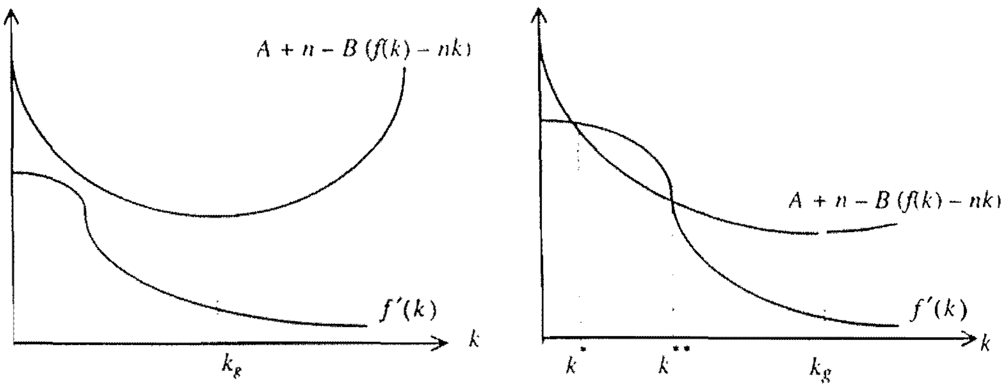
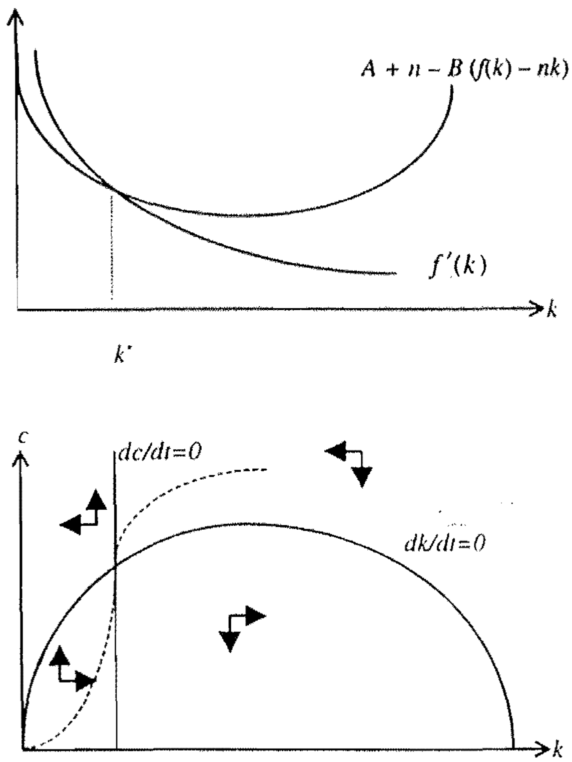


(a) (b)

Figure 4



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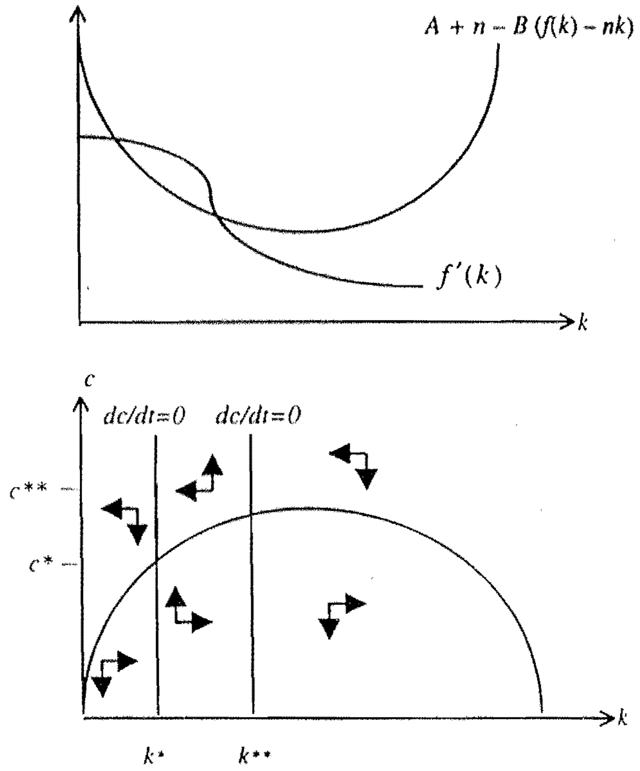

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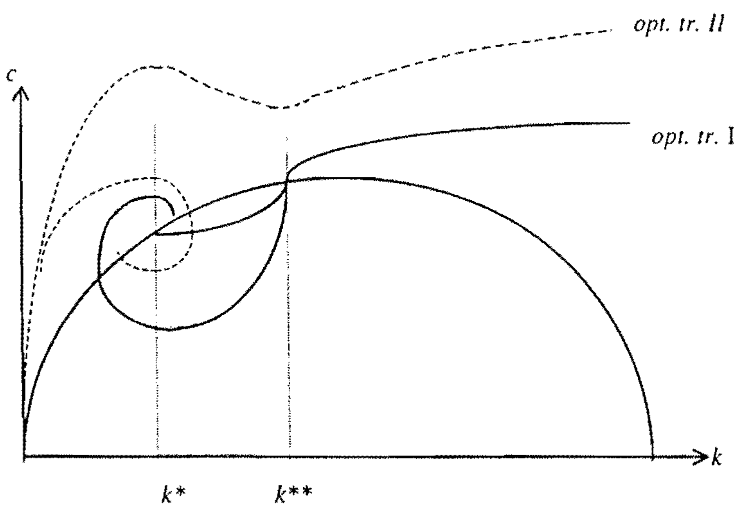

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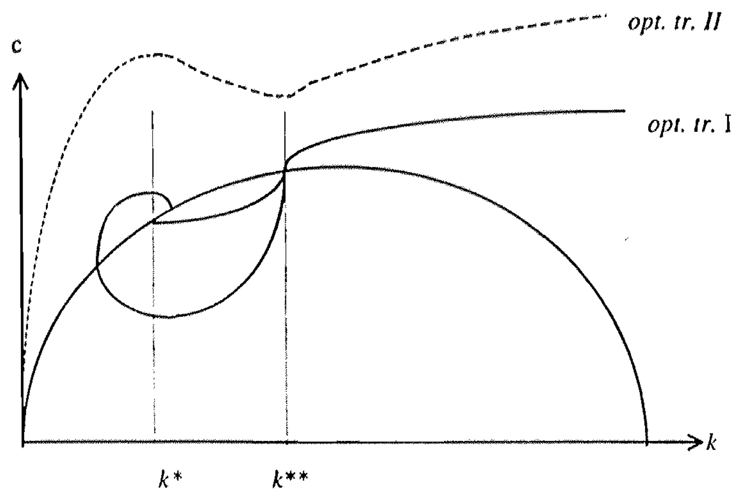

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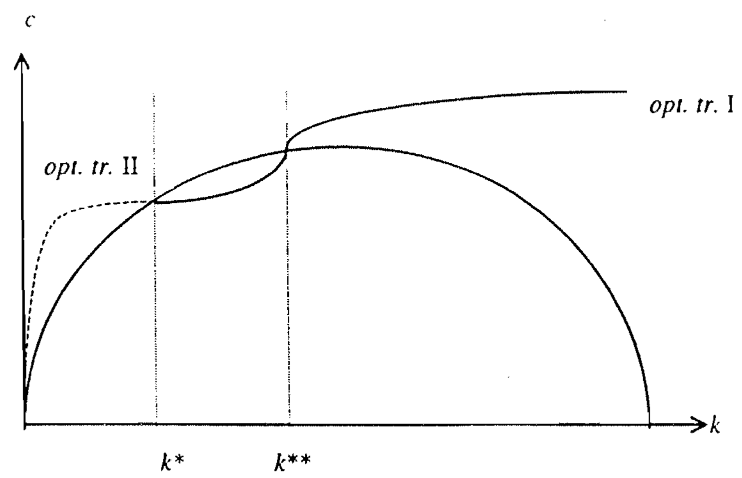

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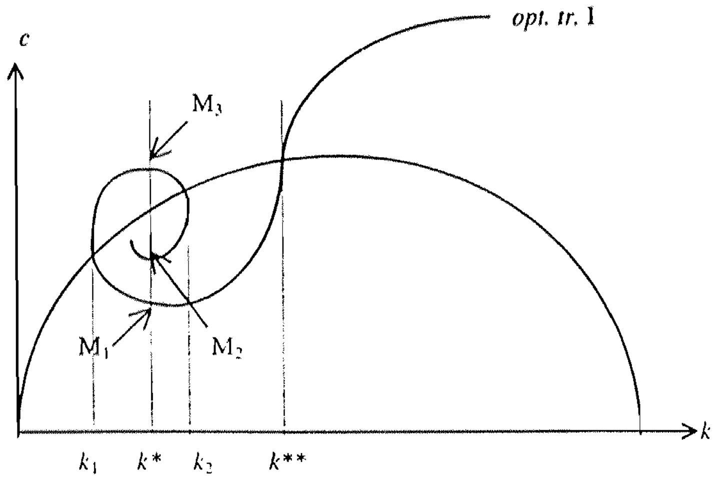

Figure 10

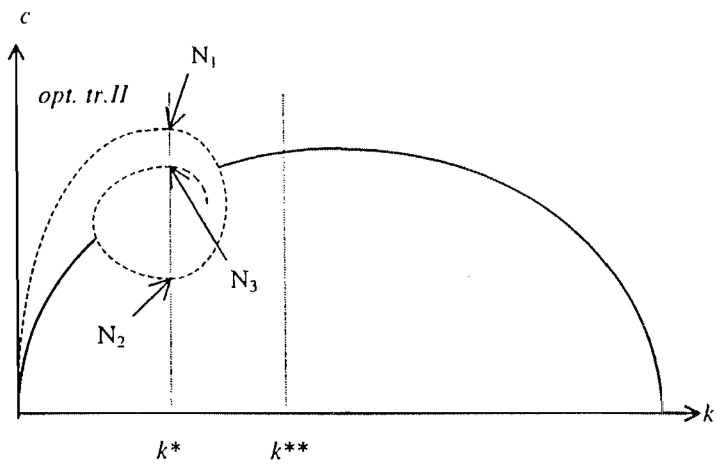

Figure 11

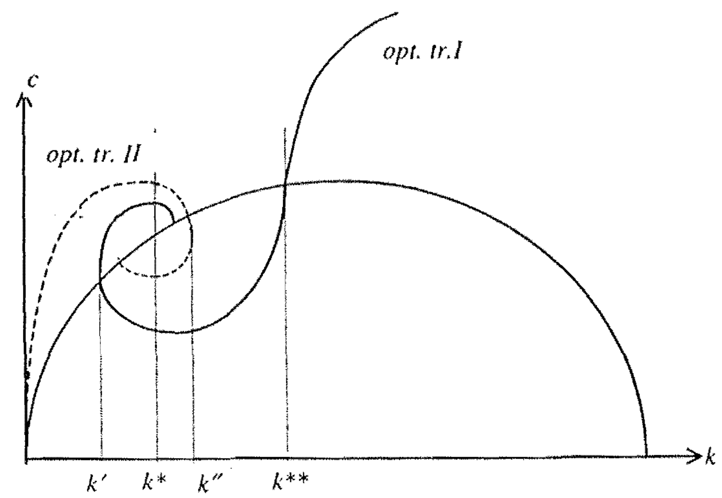

Figure 12

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