1. Introduction

A liability rule typically decides whether and how much damage (liability) payments are to be made by the injurer(s) to the victim(s) of an accident as a function of the level of care taken by the parties. The damage awards under a liability rule affect the efficiency of the rule, as is pointed out by the rich literature on efficient liability rules. Starting with Brown (1973), formal analyses of liability rules have been carried out and systematically advanced in Diamond (1974), Landes and Posner (1987), Shavell (1987), Posner (1992), Miceli (1997), Cooter and Ulen (1998), and Jain and Singh (2001), among others. Kaplow (1998) and Arlen (2000) provide a detailed discussion and summary of the literature on 'efficient' damage awards.[1] One of the major conclusions which have emerged from this work is that for liability rules to be efficient, it is important to take into account the full losses suffered by the victims, while deciding on the amount of damages to be paid by the injurers to the victims. While analyzing the efficiency characteristics of liability rules it is generally assumed that courts, when adjudicating accident cases, can calculate the harm suffered by the victims correctly and costlessly.[2] One crucial factor that could affect damage awards or the liability payments and therefore the efficiency characteristics of liability rules, is the error made by a court in estimating the harm suffered by the victims.[3] In the literature on the effects of court errors on the level of care taken by parties, the study is mainly confined to the rules of negligence and strict liability. This paper aims to provide an efficiency characterization of *all* liability rules, when courts make errors in the estimation of the harm suffered by victims.

The literature on the effects of errors made by courts in estimating the harm has been fairly extensive with important contributions made by Cooter (1984), Shavell (1987), Kaplow (1994), Kaplow and Shavell (1992, 1996), Miceli (1997), and Cooter and Ulen (1998). These contributions have been critically evaluated in Kaplow (1998) and Arlen (2000). In particular, both Cooter and Arlen have argued that the rule of negligence is superior to the rule of strict liability when courts make errors in the estimation of harm,

[1] Also see Shavell (1980) and Cooter (1984).

[2] In Jain & Singh (2001), we provide a complete characterization of efficient liability rules under the standard assumption of accurate adjudication by courts.

[3] Other factors could be court's uncertainty regarding the levels of care taken the parties, or the socially optimum levels of care, or causation of accident etc. See Kaplow (1998), Arlen (2000), and Schwartz (2000).

because under the rule of negligence the injurers' behaviour is less sensitive to errors than it would be under the rule of strict liability. Cooter and Ulen (pp. 284-286), Miceli (1997, pp. 34-35) and Arlen have also argued that as long as errors by courts are small, whether upper-biased or lower-biased, the rule of negligence is efficient in that it motivates the parties to take levels of care which minimize the total social costs of accident, which is not the case under the rule of strict liability. In this context, Kaplow and Shavell (1992, 1996) and Kaplow (1994, 1998) have shown that when the individuals have *ex-ante* knowledge of harm and courts make errors in estimating the harm, the injurers will not take efficient care under the rule of strict liability. In Shavell (1987, pp. 131-32, 151-53), it is proved that unbiased court errors will not affect the efficiency characteristics of liability rules.

This paper provides a characterization of efficient simple liability rules in the presence of court errors in estimation of the harm. The problem is considered in the standard framework of economic analysis of liability rules, i.e., we consider accidents that might result from interaction of two risk-neutral parties who are strangers to each other. It is taken that the social objective is to minimize the total social costs of accident - the sum of the costs of care plus expected accident loss. To start with only one party namely the victim bears the loss of accident. Care by both the parties can affect the expected loss of accident.[4] It is assumed that whenever a liability rule specifies the legally binding due level of care for a party, it is set at a level commensurate with the objective of minimizing the total social costs of accident.[5]

Retaining most of the assumptions of the standard framework, the problem, however, is considered in a somewhat broader framework. No assumptions are made on the costs of care and expected loss functions, apart from assuming the existence of a pair of levels of care which minimizes the total social costs. In particular, unlike the standard framework, we allow the possibility of the existence of more than one configuration of care levels at which total social costs are minimized. Among other things, this paper demonstrates that the standard assumption about the costs of care and expected loss functions is completely

[4]Kaplow and Shavell (1992, 1996) have studied the effects of court errors when harm varies across the injurers in the framework of unilateral care - where care only by the injurers can affect the probability of accident and, as a consequence, care by the victims is not an issue.

[5]For an analysis of effects of court errors in determination of efficient levels of due care and other related issues see Green (1976), Craswell and Calfee (1986), Kaplow and Shavell (1994), Kahan (1989), Miceli (1990), Tullock, (1994), Kaplow (1995), Rasmusen (1995), Schwartz etc.

irrelevant for the efficiency characterization of simple liability rules.

The main results of this paper establish that when courts make lower-biased errors, no simple liability rule can be efficient. On the other hand, when courts make upper-biased errors, then, irrespective of the magnitude of error, the necessary and sufficient condition for a simple liability rule to motivate both the parties to take efficient levels of care is that it satisfies the condition of *'negligent injurer's liability'*. The condition of negligent injurer's liability requires that a liability rule be such that (i) whenever the injurer is nonnegligent, i.e., he is taking at least the due level of care, the entire loss in the event of an accident is borne by the victim irrespective of the level of care taken by the victim, and (ii) when the injurer is negligent and the victim is nonnegligent, the entire loss in the event of an accident is borne by the injurer. In the presence of upper-biased errors, this common feature of the various rules of negligence allows us to frame and resolve efficiency concerns in a general fashion.

2. Framework of Analysis

Accidents resulting from interaction of two stranger parties are considered. To start with, the entire loss falls on one party to be called the victim; the other party being the injurer. We denote by $c \geq 0$ the cost of care taken by the victim and by $d \geq 0$ the cost of care taken by the injurer. Costs of care are assumed to be strictly increasing functions of indices of care, i.e., care levels. As a result, cost of care for a party will also represent the index of care for that party. Let $C = \{c \mid c \geq 0$ is the cost of some feasible level of care which the victim can take $\}$ and $D = \{d \mid d \geq 0$ is the cost of some feasible level of care which the injurer can take $\}$. Both C and D are assumed to be nonempty. And $0 \in C$ and $0 \in D$.

Let π be the probability of occurrence of accident and $H \geq 0$ the loss in case accident actually materializes. π and H are assumed to be functions of c and d; $\pi = \pi(c,d)$, $H = H(c,d)$. Let, L denote the expected loss due to accident. Thus, L is equal to πH and is a function of c and d; $L = L(c,d)$. Clearly, $L \geq 0$. We assume that L is a non-increasing function of care level of each party. That is, a larger care by either party, given the care level of the other party, results in lesser or equal expected accident loss. Formally we assume:

Assumption A 1 $(\forall c \in C)$ $(\forall d \in D)$ $[[c > c' \rightarrow L(c,d) \leq L(c',d)]$ *and* $[d > d' \rightarrow L(c,d) \leq L(c,d')]]$.

Decrease in L can take place because of decrease in H or π or both. Activity levels of both the parties are assumed to be given.

Total social costs (TSC) of the accident are the sum of costs of care by the two parties and the expected loss due to accident; TSC $= c+d+L(c,d)$. Social objective is to minimize the total social costs. Let M be the set of all costs of care configurations which are total TSC minimizing. That is, $M = \{(\acute{c},\acute{d}) \mid \acute{c}+\acute{d}+L(\acute{c},\acute{d})$ is minimum of $\{c+d+L(c,d) \mid c \in C, d \in D\}\}$. Further, we assume that:

Assumption A 2 *C, D, and L are such that* $\sharp M \geq 1$.

An accident-context is characterized by specification of C, D, L and M. In the standard economic analyses of liability rules generally it is assumed that if an accident takes place, courts while deciding the proportions of loss to be borne by the two parties can correctly estimate the harm H suffered by the victim. On the other hand, when courts make errors in calculation of harm, the assessed harm, for the purpose of awarding the damages, will, in general, be different from the actual harm. Let, $H+\epsilon$ denote the assessed harm when actual harm is H, where ϵ denotes the error term in the assessment of harm. We assume that $\epsilon = 0$ when $H = 0$. Further, errors by the courts may be unbiased, i.e., $E(\epsilon) = 0$, in that case expected assessed harm, $E(H+\epsilon) = H + E(\epsilon) = H$, the actual harm. Or, errors may be biased, i.e., $E(\epsilon) \neq 0$, then expected assessed harm, $E(H+\epsilon) = H + E(\epsilon) \neq H$. Let $H+\epsilon = \alpha H$, or $\alpha = 1+\epsilon/H$. $E(\alpha) = 1 + E(\epsilon)/H$, or $E(\alpha)H = H + E(\epsilon)$. Therefore, $E(\alpha)H$ also represents the expected assessed harm when actual harm is H. Let, $E(\alpha) = \bar{\alpha}$. Clearly, $\bar{\alpha} \geq 1$ iff $E(\epsilon) \geq 0$, and $\bar{\alpha} < 1$ iff $E(\epsilon) < 0$. We assume that $\bar{\alpha}$ is known to all the parties.

A liability rule uniquely determines the proportions in which the two parties will bear the loss H, in case accident actually occurs, as a function of the proportions of their non-negligence. Let I denote the closed unit interval $[0,1]$. Given C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha}$, we define functions p and q as follows:

$p : C \to I$ such that:
$p(c) = 1$ if c$\geq c^*$
$p(c) = c/c^*$ if $c < c^*$,

q: D $\to I$ such that:
$q(d) = 1$ if $d \geq d^*$
$q(d) = d/d^*$ if $d < d^*$.

A liability rule may specify the due care levels for both the parties, or for only one of them, or for none[6]. If a liability rule specifies the due care levels for both the parties, c^* and d^* used in the definitions of functions p and q will be taken to be identical with the legally specified due care levels for the victim and the injurer respectively. If the liability rule specifies the due care level for only the injurer, d^* used in the definition of function q will be taken to be identical with the legally specified due care level for the the injurer and c^* used in the definition of p will be taken as any element of $\{c \in C \mid (c, d^*) \in M\}$[7]. Similarly, if the liability rule specifies due care level for only the victim, c^* used in the definition of function p will be taken to be identical with the legally specified due care level and d^* used in the definition of q will be any element of $\{d \in D \mid (c^*, d) \in M\}$. If the liability rule does not specify due care level for any party then any element of M can be used in the definitions of p and q.

In other words, we are making the assumption that legal due care standard for a party, wherever applicable, is set at a level appropriate for the objective of minimization of TSC of the accident. This standard assumption is very crucial for the efficiency of a liability rule[8].

Given the definitions of p and q, $q(d) = 1$ would mean that the injurer is taking at least the due care and $q(d) < 1$ would mean that the injurer is taking less than the due care, i.e., he is negligent. $1 - q(d)$ will be his proportion of negligence. If $q(d) = 1$, the injurer would be called nonnegligent. Similarly for the victim.

[6]The rules of negligence with defense of contributory negligence, the rule of negligence, and the rule of strict liability, for example, are respectively the rules with legally specified due care standards for both the party, for only one party, and for none.

[7]As we are allowing the possibility that there may be more than one configuration of care levels which are TSC minimizing, $\{c \in C \mid (c, d^*) \in M\}$ may contain more than one element.

[8]It can be argued that if the courts make errors in estimation of the harm then they may do so in estimation of efficient levels of care as well. Here, apart from appealing to the expository simplicity, we argue that courts may rely on customs while determining due levels of care or may determine due levels of care through other methods, e.g., adopting the levels of care determined to be efficient by regulatory bodies as due levels of care etc. Therefore, errors in estimation of harm do not necessarily mean errors in estimation of efficient due levels of care. For arguments and discussion see Posner and Arlen.

A liability rule can be defined as a rule which specifies the proportions in which the victim and the injurer will bear the loss, in case accident actually materializes, as a function of proportions of two parties' nonnegligence. Formally, a liability rule is a function f, f: $[0,1]^2 \to [0,1]^2$, such that:

$$f(p,q) = (x,y) = (x[p(c),q(d)], y[p(c),q(d)])$$

where $x[p(c),q(d)] + y[p(c),q(d)] = 1$.

When courts make errors while assessing the harm, not only the proportion of the loss a party is required to bear but also the magnitude of errors in calculation of H will affect its expected costs and hence its behaviour, in general. And, an application of a liability rule involves specification of the accident-context as also the legal standards. Let C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha}$ be given. From parties point of view when court makes error, it will assess harm equal to $\bar{\alpha}H$. Again, from parties point of view, if accident takes place and loss of H materializes, then the court will require the injurer to bear $y[p(c),q(d)]\bar{\alpha}H(c,d)$ instead of $y[p(c),q(d)]H(c,d)$, when court made no error.

As, when accident takes place, the entire loss is suffered by the victim, $y[p(c),q(d)]\bar{\alpha}H(c,d)$ represents the liability payment to be made by the injurer to the victim. The expected costs of a party are the sum of the cost of care taken by it plus its expected liability. The injurer's expected costs, therefore, are:
$d + y[p(c),q(d)]\pi(c,d)\bar{\alpha}H(c,d)$ or $d + y[p(c),q(d)]\bar{\alpha}L(c,d)$;
and victim's expected costs are:
$c + L(c,d)$ - $y[p(c),q(d)]\bar{\alpha}L(c,d)$.
Both parties are assumed to be rational and risk-neutral.

In the terminology of this paper:
The rule of negligence is defined by:
$(q = 1 \to x = 1)$ and $(q < 1 \to x = 0)$.
The rule of negligence with the defense of contributory negligence is defined by:
$(q = 1 \to x = 1)$ and $(p = 1 \ \& q < 1 \to x = 0)$ and $(p < 1 \ \& \ q < 1 \to x = 1)$.
The rule of comparative negligence is defined by:
$(q = 1 \to x = 1)$ and $(p = 1 \ \& \ q < 1 \to x = 0)$ and $(p < 1 \ \& \ q < 1 \to (0 < x < 1),\ x/y \propto q/p)$.

The rule of strict liability is defined by:
$x = 0$, for all $p, q \in [0, 1]$.
The rule of strict liability with the defense of contributory negligence is defined by:
$(p < 1 \rightarrow x = 1)$ and $(p = 1 \rightarrow x = 0)$.
The rule of no liability is defined by:
$x = 1$, for all $p, q \in [0, 1]$.

Efficient Liability Rules:
A liability rule f is said to be efficient in a given accident-context, i.e., for given C, D, L, and $(c^*, d^*) \in M$, satisfying (A1) and (A2), iff (i) every Nash equilibrium (N.E.) is total social cost minimizing and (ii) there exists at least one Nash equilibrium.[9] Formally, a liability rule is efficient for given C, D, L, and $(c^*, d^*) \in M$ iff:
$(\forall \bar{c}, \bar{d} \in C \times D)$ $[(\bar{c}, \bar{d})$ is a Nash equilibrium $\rightarrow (\bar{c}, \bar{d}) \in M]$ & $(\exists (\bar{c}, \bar{d}) \in C \times D)[(\bar{c}, \bar{d})$ is a Nash equilibrium$]$.
A liability rule f is said to be ***efficient*** iff it is efficient in every possible accident-context, i.e., iff for every possible choice of C, D, L, and $(c^*, d^*) \in M$, satisfying (A1) and (A2), f is efficient.

Consider the following examples.

Example 1 *Let, $\bar{\alpha} = 0.9$. Consider the accident context characterized by the following specification:*
$C = \{0, 1\}$,
$D = \{0,\ 0.9,\ 1\}$,
$L(0,0) = 5,\ L(1,0) = 3 = L(0,1),\ L(0,\ 0.9) = 3.11,\ L(1,\ 0.9) = 0.11,\ L(1,1) = 0.$

From the specification in Example 1 it is clear that $(1, 1)$ is the unique configuration of care levels which is TSC minimizing. Let $(c^*, d^*) = (1, 1)$. Now, consider the application of the rule of strict liability with defense of contributory negligence in this accident context. Let $c^* = 1$ be opted by the victim. If the injurer opt for $d^* = 1$, his total expected costs are equal to 1. On the other hand, if he opts for 0.9 his total expected costs are $0.999 (= 0.9 + \bar{\alpha} L(1,\ 0.9) = 0.9 + 0.9 \times 0.11)$. This means given 1 opted by the victim, 1 is not a best response for the injurer. Thus, $(1, 1)$ which is the unique TSC minimizing

[9] We consider only the pure strategy Nash Equilibria. Also, as is the case with the standard assumption, if (c^*, d^*) is the unique TSC minimizing configuration then a liability rule will be efficient iff (c^*, d^*) is a unique N.E.

configuration of care levels is not a N.E. and hence the rule is not efficient in this accident context. Next, consider the application of the rule of negligence in this context. Exactly the same argument shows that with $\bar{\alpha} = 0.9$, under the rule of negligence $(1,1)$ is not a N.E. and hence the rule of negligence also is not efficient in this accident context.

Example 2 *Let,* $\bar{\alpha} = 1.2$. *Consider the accident context characterized by the following specification:*
$C = \{0, 1\}$,
$D = \{0,\ 1,\ 2.1\}$,
$L(0,0) = 5,\ L(1,0) = 3 = L(0,1),\ L(1,1) = 1,\ L(0,2.1) = 2,\ L(1,2.1) = 0.$

For the specification in Example 2, $(1,1)$ is the unique configuration of care levels which is TSC minimizing. Let $(c^*, d^*) = (1,1)$. Now, consider the application of the rule of strict liability with defense of contributory negligence in this accident context. Let $c^* = 1$ be opted by the victim. If the injurer opt for $d^* = 1$, his total expected costs are $1 + 1.2 \times 1 = 2.2$. On the other hand, if he opts for 2.1 his total expected costs are 2.1. That is, given 1 opted by the victim, 1 is not a best response for the injurer. Thus, $(1,1)$ which is the unique TSC minimizing is not a N.E. and hence the rule is not efficient in this accident context.
Next, consider the application of the rule of negligence in this context. Under this rule it is easy to see that given $c^* = 1$ by the victim, $d^* = 1$ is the unique best response for the injurer and vice-versa. So, $(1,1)$ is a N.E. Furthermore, there is no other N.E. Thus, the unique TSC minimizing configuration $(1,1)$ is the unique N.E. and hence the rule of negligence is efficient in this accident context.

3. Characterization of Efficient Liability Rules when Errors made by the Courts are Unbiased

Under the standard assumption that the courts can calculate the harm H, correctly and costlessly, we have the following result about the efficiency characteristics of liability rules.

Theorem (Jain & Singh): A liability rule f is efficient for every possible choice of C, D, L, and $(c^*, d^*) \in M$ satisfying $(A1)$ and $(A2)$ iff [10]:
$p < 1 \rightarrow [f(p,1) = (1,0)]$ and $q < 1 \rightarrow [f(1,q) = (0,1)]$.

[10] See Jain and Singh.

That is a liability rule is efficient in every possible accident context iff its structure is such that (i) whenever the injurer is nonnegligent, i.e., he is taking at least the due care, and the victim is negligent the entire loss in case of occurrence of accident is borne by the victim, and (ii) when injurer is negligent and the victim is nonnegligent, the entire loss in case of occurrence of accident is borne by the injurer. Now, when errors made by courts are unbiased, i.e., $E(\epsilon) = 0$, $\bar{\alpha} = 1$.
The injurer's expected costs, therefore, are:
$d + y[p(c), q(d)]L(c, d)$, as $\bar{\alpha} = 1$;
and, similarly, victim's expected costs are:
$c + L(c, d) - y[p(c), q(d)]L(c, d)$. Therefore, with unbiased errors by the courts expected costs of the injurer and the victim are equal to their respective expected costs when courts made no errors. As both the parties are assumed to be rational and risk-neutral, unbiased errors by courts will not affect their choices of levels of care. In view of this observation and above cited result, the following claim and corollary are immediate.

Claim 1 *When errors made by courts are unbiased, a liability rule f is efficient in all accident contexts, i.e., for every possible choice of C, D, L, and $(c^*, d^*) \in M$, satisfying (A1) and (A2), iff:*
$p < 1 \rightarrow [f(p, 1) = (1, 0)]$ *and* $q < 1 \rightarrow [f(1, q) = (0, 1)]$.

Corollary 1 *When errors made by courts are unbiased, while the rules of negligence, negligence with the defense of contributory negligence, comparative negligence and strict liability with the defense of contributory negligence are efficient in every possible accident context, satisfying (A1) and (A2). On the other hand, the rules of no liability and strict liability are not.*

With above characterization of efficient liability rules the following remark is obvious.

Remark 1 *When courts make no errors or when they make unbiased errors, the proportion of loss which a party is required to bear when both parties are nonnegligent, i.e., value of f at $(1, 1)$ is irrelevant for the efficiency characteristics of a liability rule.*

4. Characterization of Efficient Liability Rules when Errors made by the Courts are Biased

Condition of Negligent Injurer's Liability (NIL):
A liability rule f is said to satisfy the condition of negligent injurer's liability (NIL) iff its

structure is such that (i) whenever the injurer is nonnegligent, i.e., he is taking at least the due care, the entire loss in case of occurrence of accident is borne by the victim irrespective of the level of care taken by the victim, and (ii) when injurer is negligent and the victim is not, the entire loss in case of occurrence of accident is borne by the injurer. Formally, a liability rule f satisfies the condition of negligent injurer's liability iff:

$$(\forall p \in [0,1])[f(p,1) = (1,0)] \text{ and } (\forall q \in [0,1))[f(1,q) = (0,1)].$$

When errors made by courts are biased we assume that the estimated harm will never be negative. That is, when $E(\epsilon) \neq 0$ or $\bar{\alpha} \neq 1$, we assume $\bar{\alpha} \geq 0$. When $\bar{\alpha} \neq 1$, errors by the courts may be systematically upper-biased, i.e., $E(\epsilon) > 0$, in that case we have $\bar{\alpha} > 1$, or errors may be lower-biased, i.e., $E(\epsilon) < 0$, in that case we have $\bar{\alpha} < 1$. Below, we show that when courts systematically under estimate the harm, i.e., when errors by the courts are lower-biased, irrespective of the magnitude of the bias, no liability rule can be efficient. Formally, with $E(\epsilon) < 0$, we have the following result.

Theorem 1 *A liability rule is efficient for every possible choice of C, D, L and $(c^*, d^*) \in M$, satisfying (A1) and (A2), only if $\bar{\alpha} \geq 1$.*

Proof: Suppose not. This implies that there exist a liability rule such that $0 \leq \bar{\alpha} < 1$ and the rule is efficient for every possible choice of C, D, L and $(c^*, d^*) \in M$ satisfying $(A1)$ and $(A2)$. Let f be the rule.
Take any $\bar{\alpha} \in [0,1)$. Let $f(p(c), q(d)) = (x[p(c), q(d)],\ y[p(c), q(d)])$, where $x + y = 1$.
Let t be a positive number. As $\bar{\alpha} \in [0,1)$, $\bar{\alpha}t < t$. Choose $r > 0$ such that $\bar{\alpha}t < r < t$. Now consider the following accident context, i.e., following specification of C, D and L:
$C = \{0, c_0\}$, $c_0 > 0$, $D = \{0, \bar{\alpha}d_0, d_0\}$, where $d_0 = r/(1-\bar{\alpha})$,
$L(0,0) = t + \bar{\alpha}d_0 + c_0 + \delta$, where $\delta > 0$,
$L(0, \bar{\alpha}d_0) = t + c_0 + \delta$, $L(0, d_0) = c_0 + \delta$, $L(c_0, 0) = t + \bar{\alpha}d_0$,
$L(c_0, \bar{\alpha}d_0) = t$ and $L(c_0, d_0) = 0$.
It is clear that $L(c,d)$ satisfies $(A1)$ and $(c_0, d_0) \in C \times D$ is the unique TSC minimizing configuration. Therefore, $(A2)$ is also satisfied. Let $(c^*, d^*) = (c_0, d_0)$.
Now, given c_0 opted by the victim, if injurer chooses d_0 his expected costs are d_0. And, if he chooses $\bar{\alpha}d_0$, his expected costs are $\bar{\alpha}d_0 + y[p(c_0), q(\bar{\alpha}d_0)]\bar{\alpha}t$. But, $d_0(1-\bar{\alpha}) > \bar{\alpha}t$ as $r > \bar{\alpha}t$. This implies that $d_0 > \bar{\alpha}d_0 + \bar{\alpha}t$. Therefore, $d_0 > \bar{\alpha}d_0 + y[p(c_0), q(\bar{\alpha}d_0)]\bar{\alpha}t$, as $y[p(c_0), q(\bar{\alpha}d_0)]\bar{\alpha}t \leq 1$.
Hence, the unique TSC minimizing pair of care levels (c_0, d_0) is not a N.E. Therefore, f is

not efficient for the above specification of C, D, L and $(c^*, d^*) \in M$, satisfying $(A1)$ and $(A2)$. This, in turn, implies that when $\bar{\alpha} < 1$, f is not *efficient*, as f is not efficient for every possible choice of C, D, L, and $(c^*, d^*) \in M$, satisfying $(A1)$ and $(A2)$.

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Intuitively the argument of Theorem 1 can be sketched as follows. Consider the accident contexts such that (i) total social cost minimizing configuration is unique, (ii) d^* is positive and when both the parties take efficient levels of care expected accident loss is zero, and (iii) $\bar{\alpha}d^*$ is an element of D. Now, suppose that the victim is choosing care level c^*. In such accident contexts if the injurer opts for d^* his total expected costs are d^* and expected accident loss is zero. If the injurer decides to switch over to $\bar{\alpha}d^*$ the resulting increase in the expected accident loss is greater than the decrease in the cost of care by the injurer, i.e., $L(c^*, \bar{\alpha}d^*)$ - $L(c^*, d^*) = L(c^*, \bar{\alpha}d^*) > (1 - \bar{\alpha})d^*$. With $\bar{\alpha}d^*$ care level by the injurer, from the injurer's point of view the court will require him to bear expected loss equal to $y'\bar{\alpha}L(c^*, \bar{\alpha}d^*)$, where $y' = y[p(c^*), q(\bar{\alpha}d^*)]$ and will depend upon the liability rule concerned. His total expected costs, therefore, will be $\bar{\alpha}d^* + y'\bar{\alpha}L(c^*, \bar{\alpha}d^*)$. But, $\bar{\alpha} < 1$ implies that $y'\bar{\alpha} < 1$, as $y' \leq 1$. Thus, at $(c^*, \bar{\alpha}d^*)$ even if the liability rule holds the injurer fully liable he will bear only a fraction of the increased expected accident loss. On the other hand, entire benefits of the decreased costs of care will accrue to him. In such accident contexts, it is easy to see that whenever the expected loss function is such that (iv) $L(c^*, \bar{\alpha}d^*) > (1 - \bar{\alpha})d^* > y'\bar{\alpha}L(c^*, \bar{\alpha}d^*)$ holds, expected costs of the injurer are less if he chooses $\bar{\alpha}d^*$ rather than d^*. So, (c^*, d^*) which is the unique total social costs minimizing configuration of care levels is not a N.E. Here it should be noted that we have not assumed any thing about the liability rule concerned apart from assuming that $y' \leq 1$, which is true for every liability rule. Thus, no liability rule will be efficient in such contexts. Moreover, as the proof of the theorem shows, such contexts can be specified irrespective of the magnitude of $\bar{\alpha}$ as long as it is less than one.

In particular, consider the application of the rule of negligence in the accident contexts satisfying (i)-(iv) with $(A1)$ and $(A2)$. For example, one such context is specified in the proof of Theorem 1. Let c^* be opted by the victim. Given this, when courts do not make any errors, under the rule the expected costs of the injurer will be:

$d + L(c^*, d)$ when $d < d^*$, and

d when $d \geq d^*$,

as shown by the curves AB and BC respectively in the Diagram 1.[11]

[11]For expository purpose, here, we assume that d is a continuous variable and $L(c^*, d)$ is a continuous and

Please insert the Diagram 1 here

In this case clearly d^* is the best response by the injurer for c^* by the victim. On the other hand, when courts make lower-biased errors the expected costs to the injurer are:

$d + \bar{\alpha}L(c^*, d)$ when $d < d^*$, and

d when $d \geq d^*$.

Irrespective of the smallness of errors by courts curve AB shifts downwards as is shown by the curve $A'B$. In the accident contexts satisfying (i)-(iv), $\bar{\alpha}d^*$ will lie in between d' and d^*. Expected costs of the injurer are less at $\bar{\alpha}d^*$ than at d^*. Therefore, injurer will do better by switching over to $\bar{\alpha}d^*$, which means that (c^*, d^*) is not a N.E. Furthermore, given c^* opted by the victim, expected costs of the injurer will exactly be the same under the rules of negligence with the defense of contributory negligence, and comparative negligence as they are under the rule of negligence. As a consequence these rule will also be inefficient in such contexts.

Below we demonstrate that when errors by the courts are upper-biased there do exist liability rules which are efficient in every possible accident contexts. Formally, with $E(\epsilon) > 0$ we have the following results about the efficiency of liability rules.

Proposition 1 *If a liability rule satisfies condition NIL then for every possible choice of C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying (A1) and (A2), (c^*, d^*) constitutes a Nash equilibrium.*

Proof: Let the liability rule f satisfy condition NIL. Take any arbitrary C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$. Let c^* be opted by the victim. For all $d \geq d^*$ expected costs of the injurer are $d + y[p(c^*), q(d)]\bar{\alpha}L(c^*, d) = d$, as $y[p(c^*), q(d \geq d^*)] = 0$ by NIL. So, if the injurer chooses d^* his expected costs are only d^*. Now, consider a choice of $d' \neq d^*$ by the injurer. First, consider the case $d' > d^*$. In this case his expected costs clearly are d'. Therefore, the injurer will be strictly worse-off choosing $d' > d^*$ rather than

decreasing function of d and that (c^*, d^*) is the unique TSC minimizing configuration.

choosing d^*.

Next consider the case $d' < d^*$. For $d' < d^*$ expected costs of the injurer are $d' + y[p(c^*), q(d')]\bar{\alpha}L(c^*, d') = d' + \bar{\alpha}L(c^*, d')$, as $y[p(c^*), q(d' < d^*)] = 1$ by NIL.
But, $d' < d^*$ can be advantageous to the injurer only if
$d' + \bar{\alpha}L(c^*, d') < d^*$, or only if $c^* + d' + \bar{\alpha}L(c^*, d') < c^* + d^*$.
But, $L(c^*, d') < \bar{\alpha}L(c^*, d')$ as $\bar{\alpha} > 1$ and $L(c^*, d' < d^*) > 0$.[12]
This implies that $c^* + d' + L(c^*, d') < c^* + d^*$ or,
$c^* + d' + L(c^*, d') < c^* + d^* + L(c^*, d^*)$.
That is TSC at (c^*, d') are less than TSC at (c^*, d^*). Which is a contradiction as (c^*, d^*) is TSC minimizing. Therefore, switching over to $d'(< d^*)$ can not be advantageous to the injurer. Thus, given c^* by the victim, d^* is a best response by the injurer. Similarly, it can easily be demonstrated that given d^* opted by the injurer, c^* is a best response by the victim. Which establishes that (c^*, d^*) is a N.E.

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Proposition 2 *If a liability rule satisfies condition NIL then for every possible choice of C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying (A1) and (A2),*
$(\forall(\bar{c}, \bar{d}) \in C \times D)[(\bar{c}, \bar{d})$ *is a N.E.*$\rightarrow (\bar{c}, \bar{d}) \in M]$.

For a formal and complete proof see Appendix. Intuitive outlines of the proof are as follows. Let liability rule f satisfy condition NIL. Take any arbitrary C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$. Suppose $(\bar{c}, \bar{d})$ is a N.E. With $(\bar{c}, \bar{d})$ as a N.E., there are two possible cases, $\bar{d} \geq d^*$ or $\bar{d} < d^*$. First consider the case $\bar{d} \geq d^*$. In this case NIL implies that the injurer can avoid the liability for the accident loss merely by taking care equal to d^*. So, $(\bar{c}, \bar{d} > d^*)$ can not be a N.E. Thus, $(\bar{c}, \bar{d})$ is a N.E. and $\bar{d} \geq d^*$ imply that $\bar{d} = d^*$. Further, $(\bar{c}, \bar{d})$ is a N.E. means that (given $\bar{d} = d^*$) expected costs to the victim of choosing $\bar{c}$ are less than or equal to that of choosing c^*, i.e.,
$\bar{c} + L(\bar{c}, \bar{d} = d^*) \leq c^* + L(c^*, \bar{d} = d^*)$, as $\bar{d} = d^*$ and NIL imply that the victim will get no compensation in this case. Or,
$\bar{c} + \bar{d}(= d^*) + L(\bar{c}, \bar{d} = d^*) \leq c^* + d^* + L(c^*, d^*)$.
That is total social costs at $(\bar{c}, \bar{d})$ are less than equal to the total social costs at (c^*, d^*). But, total social costs are minimum at (c^*, d^*), therefore it must be the case that total social

[12] $L(c^*, d' < d^*) > 0$ is easy to see, as $L(c^*, d' < d^*) \geq 0$ and $L(c^*, d' < d^*) = 0$ would imply that (c^*, d^*) is not TSC minimizing, which is a contradiction.

costs at $(\bar{c}, \bar{d})$ are at least as large as total social costs at (c^*, d^*). This implies that total social costs at $(\bar{c}, \bar{d})$ are equal to the total social costs at (c^*, d^*). That is $(\bar{c}, \bar{d}) \in M$ in this case.
When $\bar{d} < d^*$, through a series of steps (as is shown in the proof) it can be demonstrated that in this case $(\bar{c}, \bar{d})$ can not be a N.E. This establishes that whenever $(\bar{c}, \bar{d})$ is a N.E. it is total social costs minimizing.

Claim 2 *For a liability rule f if $f(1,1) \neq (1,0)$ holds, then there exists a specification of C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying (A1) and (A2), for which f is not efficient.*

For proof see Appendix. For intuitive simplicity let $f(1,1) = (0,1)$. That is injurer is fully liable when both the parties are taking efficient levels of care. Consider the accident context, satisfying $(A1)$ and $(A2)$, such that (i) (c^*, d^*) is the unique total social costs minimizing configuration and when both the parties take efficient levels of care expected accident loss is positive, say t, i.e., $L(c^*, d^*) = t > 0$, and (ii) $\bar{\alpha} d^*$ is an element of D, and when the victim opts for c^* and the injurer opts for $\bar{\alpha} d^*$ expected loss is zero. Now, suppose that the victim is choosing care level c^*. In such an accident context if the injurer opts for d^* his total expected costs are $d^* + \bar{\alpha} t$, and if he opts for $\bar{\alpha} d^*$ his total expected costs are only $\bar{\alpha} d^*$. If we assume that apart from satisfying (i) and (ii) the expected loss function is such that $\bar{\alpha} L(c^*, d^*) > (\bar{\alpha} - 1) d^*$, then $d^* + \bar{\alpha} t > \bar{\alpha} d^*$. That is, given c^* by victim, injurer is better-off choosing $\bar{\alpha} d^*$. So, the unique total social costs minimizing configuration, (c^*, d^*), is not a N.E. and hence the rule is not efficient in this context. When $f(1,1) = (x_1, y_1)$, where $0 < y_1 < 1$, if we assume that $\bar{\alpha} > 1/y_1$ same argument will hold.

In other words if a liability rule is such that when both the parties take efficient levels of care an injurer is required to bear a positive fraction of loss then, under certain accident contexts and sufficiently large errors by the courts, the parties will not take efficient levels of care under the rule. In particular, the injurer will find it advantageous to take more than efficient level of care given the optimum care by the victim.

Claim 3 *For a liability rule f if (i) $\exists p \in [0,1)[f(p,1) \neq (1,0)]$ or (ii) $q \in [0,1)[f(1,q) \neq (0,1)]$ holds, then there exists a specification of C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying (A1) and (A2), for which f is not efficient.*

For proof see Appendix. Informal explanation of the claim is similar to that of Claim 2. Claim 3 says that if (i) holds, that is when the injurer is nonnegligent and, for some level of negligence, a negligent victim is not required to bear all the loss then under certain accident contexts f will not be efficient. In particular, under certain accident contexts given efficient care by the injurer, it will be advantageous to the victim to take a level of care lower than efficient care. Furthermore, it is intuitively easy to see that the claim holds even if $\bar{\alpha} = 1$ (see the proof). If (ii) holds, that is, when the victim is nonnegligent, a negligent injurer is not required to bear all the loss for some level of negligence then, for sufficiently small errors by the courts, under certain accident contexts the injurer will find it advantageous to take less than efficient level of care given the optimum care by the victim. the following proposition shows that the necessary condition for a liability rule to be efficient in all accident contexts is that it satisfies the condition NIL.

Proposition 3 *If a liability rule is efficient for every possible choice of C, D, L,* $(c^*, d^*) \in M$ *and* $\bar{\alpha} > 1$, *satisfying (A1) and (A2), then it satisfies the condition NIL*

Proof: Suppose, the liability rule f is efficient for every possible choice of C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$, and f violates NIL. Now, f violates NIL $\rightarrow$

$$(\exists p \in [0,1])[f(p,1) \neq (1,0)]$$

or

$$(\exists q \in [0,1))[f(1,q) \neq (0,1)].$$

Case 1: Suppose, $(\exists p \in [0,1])[f(p,1) \neq (1,0)]$ holds. Here we have two possible cases, $p = 1$ or $p < 1$.

$p = 1$ implies that $f(1,1) \neq (1,0)$ holds. And, $p < 1$ implies that $\exists p \in [0,1)[f(p,1) \neq (1,0)]$ holds. But, under both the cases from Claim 2 and Claim 3 (Case 1), respectively, it follows that there exists at least one specification of C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$, for which f is not efficient.

Case 2: $(\exists q \in [0,1))[f(1,q) \neq (0,1)]$ holds. In this case also from Claim 3 (Case 2) it follows that there exists at least one specification of C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$, for which f is not efficient. Thus, it can not be the case that a liability rule f violates NIL and is still efficient for every possible choice of C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$.

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Theorem 2 *A liability rule is efficient for every possible choice of* C, D, L, $(c^*, d^*) \in M$ *and* $\bar{\alpha} > 1$, *satisfying (A1) and (A2), iff it satisfies the condition of negligent injurer's liability.*

Proof: If a liability rule f satisfies the condition NIL then by Propositions 1 and 2 it is efficient for every possible choice of C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$.
And, if f is efficient for every possible choice of C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$, then Proposition 3 establishes that f satisfies the condition NIL.

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The following important corollary is immediate from Theorem 2.

Corollary 2 *The rules of negligence, negligence with the defense of contributory negligence, and comparative negligence are efficient for every possible choice of* C, D, L, $(c^*, d^*) \in M$ *and* $\bar{\alpha} > 1$, *satisfying (A1) and (A2). The rules of no liability, strict liability and strict liability with the defense of contributory negligence, on the other hand, are not.*

Having established that all the liability rules which satisfy the condition NIL are efficient in every possible accident context, satisfying $(A1)$ and $(A2)$, and for every upper-biased error, in the remaining part of the paper we study in detail the efficiency characteristics of the liability rules when the condition is violated. It should be noted from the proof of Proposition 3 that if a liability rule f violates the condition NIL, then the specifications of C, D, L,and $(c^*, d^*) \in M$, satisfying $(A1)$ and $(A2)$, for which f is inefficient depend upon the $\bar{\alpha}$, i.e., the magnitude of the errors made by the courts, in general. The following examples make this point more clear.

Example 3 *Let* f *be a liability rule such that:*
$(\forall p \in [0,1))[f(p,1) = (1,0)]$ *and*$[(\forall q \in [0,1))[f(1,q) = (0,1)]]$ *and* $[f(1,1) = (x_1, y_1)$, *where* $0 < y_1 < 1]$ *and* $[f(p,q) = (x[p(c), q(d)], y[p(c), q(d)])]$, *otherwise.*

Though the liability rule f in Example 3 violates the condition NIL, it is efficient for small upper-biased errors by the courts. Formally speaking, we have the following claim about the rule.

Claim 4 f *in Example 3 is efficient for every possible choice of* C, D, L, *and* $(c^*, d^*) \in M$, *satisfying (A1) and (A2), iff* $1 < \bar{\alpha} \leq 1/y_1$.

For proof see Appendix.

Example 4 *Let f be a liability rule such that:*
$(\forall p \in [0,1])[f(p,1) = (1,0)]$, *and* $[(\forall q \in [0,1))[f(1,q) = (x_q, y_q)]$, *such that* $y_q > 0]$, *and* $[(\exists q \in [0,1))[f(1,q) = (x_q, y_q)]$, *where* $0 < y_q < 1]$, *and* $f(p,q) = (x[p(c), q(d)], y[p(c), q(d)])$, *otherwise.*

Let $y_{\bar{q}} = min\{y_q \mid (x_q, y_q) = f(1,q) \ \ and\ q \in [0,1)\}$. In view of the Claim 1, when courts do not make errors f in Example 4 is not efficient. Further this liability rule violates the condition NIL. But, interestingly, it is efficient for large upper-biased errors by the courts. To put formally, we have the following claim about the rule.

Claim 5 *f in Example 4 is efficient for every possible choice of C, D, L, and* $(c^*, d^*) \in M$, *satisfying (A1) and (A2), iff* $\bar{\alpha} \geq 1/y_{\bar{q}}$.

For proof see Appendix.

Based upon the analysis done so far we can make the following observations about liability rules.

Remark 2 *Examples 3 and 4 with the Theorem 1 show that, in general, both the direction and the magnitude of courts errors affect the efficiency characteristics of liability rules.*

Remark 3 *Example 3 shows that unlike the cases when courts made no errors, or made unbiased errors (Remark 1), with positively biased errors by courts, i.e., with* $\bar{\alpha} > 1$, *the value of a liability rule at* $(p = 1, q = 1)$ *has not only the distributional but also the efficiency implications, in general.*

Theorem 2 establishes that any liability rule satisfying NIL is efficient for every possible choice of C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying (A1) and (A2). The above examples, however, show that when $\bar{\alpha} > 1$, depending upon the magnitude of the error made by the court, i.e., magnitude of $\bar{\alpha}$, a particular liability rule violating the condition NIL may or may not be efficient for every possible choice of C, D, L, and $(c^*, d^*) \in M$, satisfying (A1) and (A2). The theorem, of course, demonstrates that a liability rule violating NIL can not be efficient for every possible choice of C, D, L, $(c^*, d^*) \in M$ satisfying (A1) and (A2) and for every possible choice of $\bar{\alpha} > 1$. In other words, $\bar{\alpha} > 1$ per say does not imply that a liability rule violating condition NIL will necessarily be inefficient. Simple liability rules, on the contrary, are different in regards to their efficiency characteristics when errors by courts are upper-biased.

In the case of accidents involving two parties, a simple liability rule can be defined as a rule which specifies the party, the victim or the injurer, which will bear the loss fully in case of occurrence of accident, as a function of proportions of two parties' nonnegligence. In other words, simple liability rules have one additional requirement that liability will never be shared between the parties. Formally, a simple liability rule can be defined as follows.

Simple Liability Rules: A liability rule f is defined to be a simple liability rule iff $f : [0,1]^2 \rightarrow \{0,1\}^2$ such that:

$$(\forall p, q \in [0,1])[f(p,q) = (0,1) \ \ or \ \ (1,0)].$$

Most of the rules discussed in the literature on economic analysis of the liability rules, like the rules of negligence, negligence with the defense of contributory negligence, strict liability with the defense of contributory negligence, also the rules of no liability and strict liability are simple liability rules in the sense that these rules do not require sharing of liability between the parties. Rule of comparative negligence, on the other hand, is not a simple liability rule in this sense.

When $\bar{\alpha} > 1$, irrespective of its magnitude, a simple liability rule which violates condition NIL can not be efficient for every possible choice of C, D, L, and $(c^*, d^*) \in M$, satisfying (A1) and (A2). That is, in the case of a simple liability rule violating condition NIL, $\bar{\alpha} > 1$ per say implies that the rule can not be efficient in all accident contexts. For simple liability rules, we prove the following proposition.

Proposition 4 $\bar{\alpha} > 1 \rightarrow$ *[If a simple liability rule is efficient for every possible choice of C, D, L and $(c^*, d^*) \in M$, satisfying (A1) and (A2), then it satisfies the condition NIL]*

For proof see Appendix[13]. Informally but more concisely argument can be put as follows. Let f be a simple liability rule violating NIL. From the definition of NIL and also the proof of Proposition 3 it is obvious that violation of NIL by f means (i) injurer bears a non-zero (positive) fraction of loss when both parties are nonnegligent, i.e., $f(1,1) \neq (1,0)$, or (ii) injurer bears a non-zero fraction of loss when he is nonnegligent and the victim is not, i.e.,

[13] Examples 3 and 4 make it very clear that this claim can not be made for a general liability rule.

$f(p,1) \neq (1,0)$ for some $p \in [0,1)$, or (iii) victim bears a non-zero fraction of loss when he is nonnegligent and the injurer is not, i.e., $f(1,q) \neq (0,1)$ for some $q \in [0,1)$. As f is a simple liability rule, under f a party bearing non-zero (positive) fraction of loss means it will bear the loss fully. In view of this fact (i) above means $f(1,1) = (0,1)$, (ii) means $f(p,1) = (0,1)$ for some $p \in [0,1)$, and (iii) means $f(1,q) = (1,0)$ for some $q \in [0,1)$. Assume that (c^*, d^*) is the unique total social cost minimizing combination of care levels. Now, if (i) holds same informal argument as provided for Claim 2 establishes the existence of accident contexts in which f is inefficient. If (ii) holds then it is obvious that whenever $pc^* \in C$, (c^*, d^*) is not a N.E. Similarly, if (iii) holds. This establishes that if f is to be efficient in all accident contexts then it can not violate NIL. Moreover, from the argument (and proof) it should be noted that the claim of the proposition is independent of the magnitude of $\bar{\alpha}$ as long as $\bar{\alpha} > 1$.

Theorem 3 $\bar{\alpha} > 1 \rightarrow$ [*A simple liability rule is efficient for every possible choice of* C, D, L *and* $(c^*, d^*) \in M$, *satisfying (A1) and (A2), iff it satisfies the condition of negligent injurer's liability.*]

Proof: In this case sufficiency of NIL follows from Propositions 1 and 2 . Proposition 4 establishes the necessity of NIL.

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It immediately follows from Theorem 3 that when $\bar{\alpha} > 1$, while the simple liability rules of negligence, negligence with the defense of contributory negligence are efficient for every possible choice of C, D, L and $(c^*, d^*) \in M$, satisfying (A1) and (A2), the rules of no liability, strict liability, and strict liability with the defense of contributory negligence are not. In view of this conclusion and Theorem 1 we have the following observation about the rule of strict liability with the defense of contributory negligence.

Corollary 3 *When, courts make biased errors, whether positive or negative, i.e., when* $\bar{\alpha} \neq 1$, *the rule of strict liability with the defense of contributory negligence is not efficient in every possible accident context, satisfying (A1) and (A2), irrespective of the magnitude of errors.*

4. Concluding Remarks

Theorem 1 shows that when courts make lower-biased errors no liability rule can be efficient. Theorem 2 establishes that in the setting of upper-biased errors by courts, i.e., with $\bar{\alpha} > 1$; (I) if a liability rule f satisfies NIL then in every accident context, i.e., for every possible choice of C, D, L, and $(c^*, d^*) \in M$, satisfying $(A1)$ and $(A2)$, it is efficient, (II) if f violates NIL then in some accident contexts and for some $\bar{\alpha} > 1$, it will not motivate both the parties to take efficient levels of care (From the proof of Proposition 3 it should be noted that in principle one can construct infinitely many such contexts.). On the other hand, from Claim 1 we know that when courts make no errors or make unbiased errors, the necessary and sufficient condition for a liability to be efficient is that the liability rule be such that whenever one party is negligent and other is not then the negligent party should bear all the loss. Therefore, biased errors by courts affect the efficiency characterization of liability rules. In particular, the rule of strict liability with the defense of contributory negligence which otherwise is efficient is not so when courts make biased errors. Further, Theorem 3 shows that that, in the setting of upper-biased errors, if a liability rule is such that it does not allow sharing of accident loss and it violates NIL then the liability rule can not be efficient in all accident contexts irrespective of the smallness of errors.

Finally, from the proofs of theorems in the paper it should be noted that the claims of Theorems 1-3 will not change even if in stead of $(A2)$, i.e, C, D and L are such that $M \geq 1$ we make the standard assumption that C, D and L are such that $M = 1$. In the latter case sufficiency results follow immediately. Necessity claims will follow from noting that in all the necessity proofs C, D and L in addition to being consistent with $(A2)$ are also such that $M = 1$.

Appendix

Proof of Proposition 2:

Let liability rule f satisfy condition NIL. Take any arbitrary C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying (A1) and (A2). Suppose $(\bar{c}, \bar{d}) \in C \times D$ is a N.E. $(\bar{c}, \bar{d})$ is a N.E$\rightarrow$

$$(\forall c \in C)[\bar{c} + L(\bar{c}, \bar{d}) - y[p(\bar{c}), q(\bar{d})]\bar{\alpha}L(\bar{c}, \bar{d}) \leq c + L(c, \bar{d}) - y[p(c), q(\bar{d})]\bar{\alpha}L(c, \bar{d})] \quad (1)$$

and

$$(\forall d \in D)[\bar{d} + y[p(\bar{c}), q(\bar{d})]\bar{\alpha}L(\bar{c}, \bar{d}) \leq d + y[p(\bar{c}), q(d)]\bar{\alpha}L(\bar{c}, d)] \quad (2)$$

Now, (1), in particular, $\rightarrow$

$$\bar{c} + L(\bar{c}, \bar{d}) - y[p(\bar{c}), q(\bar{d})]\bar{\alpha}L(\bar{c}, \bar{d}) \leq c^* + L(c^*, \bar{d}) - y[p(c^*), q(\bar{d})]\bar{\alpha}L(c^*, \bar{d}) \quad (3)$$

and (2)$\rightarrow$

$$\bar{d} + y[p(\bar{c}), q(\bar{d})]\bar{\alpha}L(\bar{c}, \bar{d}) \leq d^* \tag{4}$$

as condition NIL implies $y[p(\bar{c}), q(d^*)] = 0$.
Adding (3) and (4)

$$\bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + L(c^*, \bar{d}) - y[p(c^*), q(\bar{d})]\bar{\alpha}L(c^*, \bar{d}) \tag{5}$$

Consider the case: $\bar{d} \geq d^*$:
When $\bar{d} \geq d^*$ from (5) we get
$\bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + L(c^*, \bar{d})$, because $\bar{d} \geq d^*$ and condition NIL imply that $y[p(c^*), q(\bar{d})] = 0$. But, $L(c^*, \bar{d}) \leq L(c^*, d^*)$ as $\bar{d} \geq d^*$. So, $\bar{c}+\bar{d}+L(\bar{c}, \bar{d}) \leq c^*+d^*+L(c^*, d^*)$.

But, as $(c^*, d^*) \in M$, $\bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \geq c^* + d^* + L(c^*, d^*)$. Therefore, $\bar{c} + \bar{d} + L(\bar{c}, \bar{d}) = c^* + d^* + L(c^*, d^*)$. Which means $(\bar{c}, \bar{d}) \in M$. Thus,

$$(\bar{c}, \bar{d}) \ \textit{is a N.E. and} \ \bar{d} \geq d^* \rightarrow (\bar{c}, \bar{d}) \in M. \tag{6}$$

Next, consider the case: $\bar{d} < d^*$:
$\bar{d} < d^*$ and $(\bar{c}, \bar{d})$ is a N.E., through (5), $\rightarrow$
$\bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + L(c^*, \bar{d}) - \bar{\alpha}L(c^*, \bar{d})$, as $\bar{d} < d^*$ and condition NIL imply that $y[p(c^*), q(\bar{d})] = 1$. Thus $\bar{d} < d^*$ and $(\bar{c}, \bar{d})$ is a N.E. $\rightarrow$
$\bar{c} + \bar{d} + L(\bar{c}, \bar{d}) < c^* + d^*$,
as $\bar{\alpha} > 1$ and $L(c^*, \bar{d}) > 0$, when $\bar{d} < d^*$. So,
$\bar{c} + \bar{d} + L(\bar{c}, \bar{d}) < c^* + d^* + L(c^*, d^*)$.
That is TSC at $(\bar{c}, \bar{d})$ are less than TSC at (c^*, d^*), a contradiction as $(c^*, d^*) \in M$. Therefore,

$$\bar{d} < d^* \rightarrow (\bar{c}, \bar{d}) \ \textit{is not a N.E.} \tag{7}$$

Finally,

$$(6) \ \& \ (7) \rightarrow [(\bar{c}, \bar{d}) \ \textit{is a N.E.} \rightarrow (\bar{c}, \bar{d}) \in M].$$

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Proof of Proposition 4

Given $\bar{\alpha} > 1$, suppose simple liability rule f violates NIL and is efficient for every possible choice of C, D, L, and $(c^*, d^*) \in M$, satisfying (A1) and (A2). f violates NIL$\rightarrow$ $(\exists p \in [0,1])[f(p,1) \neq (1,0)]$ or $(\exists q \in [0,1))[f(1,q) \neq (0,1)]$. As f is a simple liability rule, this implies

$$(\exists p \in [0,1])[f(p,1) = (0,1)] \tag{8}$$

or

$$(\exists q \in [0,1))[f(1,q) = (1,0)] \tag{9}$$

Case 1: Suppose, (8), i.e., $(\exists p \in [0,1])[f(p,1) = (0,1)]$, holds.
Subcase 1: p=1:
$p = 1\&$ (8)$\rightarrow$ $f(1,1) = (0,1)$.
Let $t > 0$. Clearly, $\bar{\alpha}t > t$. Let $r > 0$, be such that $\bar{\alpha}t > r > t$.
Now consider the following C, D and L:
$C = \{0, c_0\}, c_0 > 0,\quad D = \{0, d_0, \bar{\alpha}d_0\}$, where $d_0 = r/(\bar{\alpha} - 1)$,
$L(0,0) = t + \bar{\alpha}d_0 + c_0 + \delta$, where $\delta > 0$,
$L(c_0, 0) = t + \bar{\alpha}d_0$, $\quad$L(0, d$_0$) $= t + c_0 + \delta$,
$L(0, \bar{\alpha}d_0) = c_0 + \delta$, $\quad L(c_0, d_0) = t$, $\quad$L($c_0, \bar{\alpha}d_0$) $= 0$.
For this specification clearly (c_0, d_0) is the unique TSC minimizing configuration and $(A1)$ and $(A2)$ are satisfied. Let $(c^*, d^*) = (c_0, d_0)$. Given c_0 opted by victim, if injurer chooses $\bar{\alpha}d_0$ his expected costs, $\bar{\alpha}d_0$ are strictly less than his costs when he opts for d_0, $d_0 + \bar{\alpha}t$. Therefore, there exist C, D, L and $(c^*, d^*) \in M$, satisfying $(A1)$ and $(A2)$, such that

$$(8)\ \textit{and}\ p = 1 \rightarrow (c^*, d^*)\ \textit{is not a N.E.} \tag{10}$$

Subcase 2: $p < 1$:
$p < 1\&(8) \rightarrow f(p,1) = (0,1)$.
Let $t > 0$. So, $pt < t$. Choose r such that $pt < r < t$. Consider the following C, D, and L:
$C = \{0, pc_0, c_0\}$, where $c_0 = r/(1-p)$, $\quad D = \{0, d_0\}$, where $d_0 > 0$,
$L(0,0) = t + pc_0 + d_0 + \delta$, where $\delta > 0$,
$L(pc_0, 0) = t + d_0 + \delta$, $\quad L(c_0, 0) = d_0 + \delta$, $\quad L(0, d_0) = t + pc_0$,
$L(pc_0, d_0) = t$, $\ L(c_0, d_0) = 0$.
Clearly, (c_0, d_0) is the unique TSC minimizing configuration. Let, $(c^*, d^*) = (c_0, d_0)$. Given d_0 by injurer, if victim opts for c_0 his costs are c_0. And, if he opts for pc_0 his costs are

$pc_0 + t - \bar{\alpha}t < pc_0 < c_0$. Again, (c_0, d_0), is not a N.E. Therefore, there exist C, D, L and $(c^*, d^*) \in M$, satisfying $(A1)$ and $(A2)$, such that

$$p < 1\&(8) \to (c^*, d^*) \textit{ is not a N.E.} \tag{11}$$

Case 2: Suppose (9), i.e., $(\exists q \in [0,1))[f(1,q) = (1,0)]$ holds.
Let $t > 0$. Clearly, $qt < t$. Let, $r > 0$ be such that $qt < r < t$.
Now consider the following C, D, and L:
$C = \{0, c_0\}, c_0 > 0, \quad D = \{0, qd_0, d_0\}$, where $d_0 = r/(1-q)$,
$L(0,0) = t + qd_0 + c_0 + \delta$, where $\delta > 0$
$L(c_0, 0) = t + qd_0$, $\mathrm{L}(0, \mathrm{qd}_0) = t + c_0 + \delta$, $L(0, d_0) = c_0 + \delta$,
$L(c_0, qd_0) = t$, $\mathrm{L}(\mathrm{c}_0, d_0) = 0$.
Again, (c_0, d_0) is the unique TSC minimizing configuration and $(A1)$ and $(A2)$ are satisfied. Let, $(c_0, d_0) = (c^*, d^*)$. Further, (c_0, d_0) is not a N.E.
Therefore, there exist C, D, L and $(c^*, d^*) \in M$, satisfying $(A1)$ and $(A2)$, such that

$$(9) \to (c^*, d^*) \textit{ is not a N.E.} \tag{12}$$

Now, $(10) - (12)$, in view of the fact that in all the contexts considered above (c^*, d^*) is the unique TSC minimizing configuration, imply that if f violates the condition NIL then there exist a specification of C, D, L and $(c^*, d^*) \in M$, satisfying $(A1)$ and $(A2)$, for which f is not efficient[14]. Therefore, if for every $\bar{\alpha} > 1$, f is to be efficient for every possible choice of C, D, L and $(c^*, d^*) \in M$, $(A1)$ and $(A2)$, it must satisfy NIL.

•

Proof of Claim 2

Suppose, for a liability rule f, $f(1,1) \neq (1,0)$ holds. Let, $f(1,1) = (x_1, y_1)$ where $x_1 + y_1 = 1$ and $0 < y_1 \leq 1$. Let, $t > 0$ and $\bar{\alpha} > 1/y_1$.
For $\bar{\alpha} > 1/y_1$, $y_1\bar{\alpha}t > t$. Choose r such that $y_1\bar{\alpha}t > r > t$.
Consider the following C, D, and L:
$C = \{0, c_0\}$, $c_0 > 0$, $\quad D = \{0, d_0, \bar{\alpha}d_0\}$, where $d_0 = r/(\bar{\alpha} - 1)$,
$L(0,0) = t + \bar{\alpha}d_0 + c_0 + \delta$, where $\delta > 0$,
$L(0, d_0) = t + c_0 + \delta$, $L(0, \bar{\alpha}d_0) = c_0 + \delta$,

[14]Note that we have not assumed any thing about the magnitude of $\bar{\alpha}$ apart from assuming that it is greater than one.

$L(c_0, 0) = t + \bar{\alpha}d_0, \qquad L(c_0, d_0) = t, \qquad L(c_0, \bar{\alpha}d_0) = 0.$
For the above specification (c_0, d_0) is the unique TSC minimizing configuration. Also, $(A1)$ and $(A2)$ are satisfied. Let $(c_0, d_0) = (c^*, d^*)$.
Now, given c_0 opted victim, if injurer opts for d_0 his expected costs are $d_0 + y_1\bar{\alpha}t$. And, if he chooses $\bar{\alpha}d_0$ his expected costs are $\bar{\alpha}d_0$. But, $d_0 + y_1\bar{\alpha}t > \bar{\alpha}d_0$, as $y_1\bar{\alpha}t > r = \bar{\alpha}d_0 - d_0$. As a consequence, the unique TSC minimizing configuration (c_0, d_0) is not a N.E. Therefore, there exist C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying (A1) and (A2), for which f is not efficient.

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Proof of Claim 3

Suppose for a liability rule f, $\exists p \in [0,1)[f(p,1) \neq (1,0)]$ or $\exists q \in [0,1)[f(1,q) \neq (0,1)]$ holds.
Case 1: Suppose $\exists p \in [0,1)[f(p,1) \neq (1,0)]$ holds.
Let, $f(p,1) = (x_p, y_p)$, where $x_p < 1$ and $x_p + y_p = 1$. Take $t > 0$. As $x_p < 1$, $x_pt < t$. Choose $r > 0$ such that $x_pt < r < t$. Now consider the following C, D, L and $\bar{\alpha}$:
Let $\bar{\alpha} > 1$, $C = \{0, pc_0, c_0\}$, where $c_0 = r/(1-p)$,
$D = \{0, d_0\}$, where $d_0 > 0$,
$L(0,0) = t + pc_0 + d_0 + \delta$, where $\delta > 0$,
$L(pc_0, 0) = t + d_0 + \delta, \qquad L(c_0, 0) = d_0 + \delta, \qquad L(0, d_0) = t + pc_0,$
$L(pc_0, d_0) = t, \qquad L(c_0, d_0) = 0.$
Clearly, (c_0, d_0) is the unique TSC minimizing configuration and the specification satisfies $(A1)$ and $(A2)$. Let $(c^*, d^*) = (c_0, d_0)$. Given d_0 opted by the injurer, if the victim opts for c_0 his expected costs are c_0. And, if he chooses pc_0 his expected costs are $pc_0 + t - y_p\bar{\alpha}t$. But, $1 - y_p\bar{\alpha} < x_p$, as $\bar{\alpha} > 1$ and $y_p > 0$. Therefore, $(1 - y_p\bar{\alpha})t < x_pt$, or $pc_0 + t - y_p\bar{\alpha}t < pc_0 + x_pt$. Also, $pc_0 + x_pt < c_0$, by construction. That is, expected cost to the victim of choosing pc_0 are strictly less than that of c_0. Thus, the unique TSC minimizing configuration (c_0, d_0) is not a N.E. Therefore, there exist C, D, L $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$, for which f is not efficient.

Case 2: Suppose $\exists q \in [0,1)[f(1,q) \neq (0,1)]$ holds.
Let $f(1,q) = (x_q, y_q)$, where $y_q < 1$. There are two possible cases.
Subcase 1: $y_q > 0$.
In this case let $1 < \bar{\alpha} < 1/y_q$. Let $t > 0$. As $\bar{\alpha}y_q < 1$, $y_q\bar{\alpha}t < t$. Choose $r > 0$ such that $y_q\bar{\alpha}t < r < t$.

Consider the following C, D, L:
$C = \{0,\ c_0\}$, $c_0 > 0$, $\quad D = \{0,\ qd_0,\ d_0\}$, where $d_0 = r/(1-q)$,
$L(0,0) = t + qd_0 + c_0 + \delta$, where $\delta > 0$,
$L(0, qd_0) = t + c_0 + \delta$, $L(0, d_0) = c_0 + \delta$,
$L(c_0, 0) = t + qd_0$, $\ L(c_0, qd_0) = t$, $\ L(c_0, d_0) = 0$.
Again, (c_0, d_0) is the unique TSC minimizing configuration. Also, $(A1)$ and $(A2)$ are satisfied. Let $(c_0, d_0) = (c^*, d^*)$. Now, given c_0 opted by victim, if injurer opts d_0 his expected costs are d_0. And, if he chooses qd_0 his expected costs are $qd_0 + y_q\bar{\alpha}t$. But, $qd_0 + y_q\bar{\alpha}t < d_0$, by construction. So, the unique TSC minimizing configuration, (c_0, d_0) is not a N.E.
Subcase 2: $y_q = 0$.
In this case it can easily be demonstrated that for any C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$ whenever (c^*, d^*) is the unique TSC minimizing configuration and $qd^* \in D$, (c^*, d^*) is not a N.E.
Therefore, in the Case 2 also we have shown that there exist C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$, for which f is not efficient.

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Proof of Claim 4

First, we prove that if $1 < \bar{\alpha} \leq 1/y_1$, then (c^*, d^*) **is a N.E.**
Take any arbitrary C, D, L, $(c^*, d^*) \in M$ and $\bar{\alpha} > 1$, satisfying $(A1)$ and $(A2)$. Suppose (c^*, d^*) is not a N.E.$(c^*, d^*)$is not a N.E.$\rightarrow$

$$(\exists d' \in D)[d' + y[p(c^*), q(d')]\bar{\alpha}L(c^*, d') < d^* + y_1\bar{\alpha}L(c^*, d^*)], \tag{13}$$

or

$$(\exists c' \in C)[c' + L(c', d^*) - y[p(c'), q(d^*)]\bar{\alpha}L(c', d^*) < c^* + L(c^*, d^*) - y_1\bar{\alpha}L(c^*, d^*)], \tag{14}$$

as $f(1,1) = (x_1, y_1)$ implies $y[p(c^*), q(d^*)] = y_1$.
Suppose, (13) holds.
First, consider the case: $d' > d^*$:
$d' > d^* \& (13) \rightarrow$
$d' + y_1\bar{\alpha}L(c^*, d') < d^* + y_1\bar{\alpha}L(c^*, d^*)$ or
$c^* + d' + y_1\bar{\alpha}L(c^*, d') < c^* + d^* + y_1\bar{\alpha}L(c^*, d^*)$ or
$y_1\bar{\alpha}[c^* + d' + L(c^*, d')] + (1 - y_1\bar{\alpha})[c^* + d'] < y_1\bar{\alpha}[c^* + d^* + L(c^*, d^*)] + (1 - y_1\bar{\alpha})[c^* + d^*]$.

Which implies,
$(1-y_1\bar{\alpha})[c^*+d'] < (1-y_1\bar{\alpha})[c^*+d^*]$, as $y_1\bar{\alpha} > 0$ and $c^*+d'+L(c^*,d') \geq c^*+d^*+L(c^*,d^*)$.
Therefore, (13) $\rightarrow$
$c^*+d' < c^*+d^*$ or $d' < d^*$, if $1-y_1\bar{\alpha} > 0$, contradicting the hypothesis that $d' > d^*$. And $0 < 0$, if $1-y_1\bar{\alpha} = 0$, again a contradiction.
Now, consider the case: $d' < d^*$:
$d' < d^* \& (13) \rightarrow$
$d'+\bar{\alpha}L(c^*,d') < d^*+y_1\bar{\alpha}L(c^*,d^*)$, as $y[p(c^*),q(d' < d^*)] = 1$, by construction of the rule.
Or, (13) $\rightarrow$
$c^*+d'+\bar{\alpha}L(c^*,d') < c^*+d^*+y_1\bar{\alpha}L(c^*,d^*)$ or
$c^*+d'+\bar{\alpha}L(c^*,d') < c^*+d^*+L(c^*,d^*)$ because $y_1\bar{\alpha} \leq 1$, a contradiction as $\bar{\alpha} > 1$ and $(c^*,d^*) \in M$.
Therefore, (13) can not hold. Similarly, we can show that (14) can not hold. Thus, (c^*,d^*) is a N.E.
Now, we show that: $(\bar{c},\bar{d})$ **is a N.E** $\rightarrow (\bar{c},\bar{d}) \in M$.
$(\bar{c},\bar{d})$ is a N.E , in particular, $\rightarrow$

$$\bar{c}+L(\bar{c},\bar{d}) - y[p(\bar{c}),q(\bar{d})]\bar{\alpha}L(\bar{c},\bar{d}) \leq c^*+L(c^*,\bar{d}) - y[p(c^*),q(\bar{d})]\bar{\alpha}L(c^*,\bar{d}) \quad (15)$$

and

$$\bar{d}+y[p(\bar{c}),q(\bar{d})]\bar{\alpha}L(\bar{c},\bar{d}) \leq d^*+y[p(\bar{c}),q(d^*)]\bar{\alpha}L(\bar{c},d^*) \quad (16)$$

Adding (15) and (16)

$$\bar{c}+\bar{d}+L(\bar{c},\bar{d}) \leq c^*+d^*+L(c^*,\bar{d}) - y[p(c^*),q(\bar{d})]\bar{\alpha}L(c^*,\bar{d}) + y[p(\bar{c}),q(d^*)]\bar{\alpha}L(\bar{c},d^*) \quad (17)$$

Consider the case: $\bar{c} \geq c^* \&\ \bar{d} \geq d^*$:
Here, from (17) we get
$\bar{c}+\bar{d}+L(\bar{c},\bar{d}) \leq c^*+d^*+(1-y_1\bar{\alpha})L(c^*,\bar{d})+y_1\bar{\alpha}L(\bar{c},d^*)$, or
$\bar{c}+\bar{d}+L(\bar{c},\bar{d}) \leq c^*+d^*+(1-y_1\bar{\alpha})L(c^*,d^*)+y_1\bar{\alpha}L(c^*,d^*)$, as $(1-y_1\bar{\alpha}) \geq 0$, $y_1\bar{\alpha} > 0$, and $\bar{d} \geq d^* \rightarrow L(c^*,\bar{d}) \leq L(c^*,d^*)$, and $\bar{c} \geq c^* \rightarrow L(\bar{c},d^*) \leq L(c^*,d^*)$.
Thus (17) $\rightarrow \bar{c}+\bar{d}+L(\bar{c},\bar{d}) \leq c^*+d^*+L(c^*,d^*)$.
Given $(c^*,d^*) \in M$ this implies that $\bar{c}+\bar{d}+L(\bar{c},\bar{d}) = c^*+d^*+L(c^*,d^*)$.
Therefore,

$(\bar{c},\bar{d})$ is a N.E., $\bar{c} \geq c^*$ and $\bar{d} \geq d^*$ $\rightarrow(\bar{c},\bar{d}) \in M$.

Next, consider the case: $(\bar{c},\bar{d})$ is a N.E., $\bar{c} \geq c^*$ and $\bar{d} < d^*$:
In this case (17) $\rightarrow$
$\bar{c}+\bar{d}+L(\bar{c},\bar{d}) \leq c^*+d^*+(1-\bar{\alpha})L(c^*,\bar{d})+y_1\bar{\alpha}L(\bar{c},d^*)$, as $\bar{d} < d^*$ implies $y[p(c^*),q(\bar{d})] = 1$.
Or
$\bar{c}+\bar{d}+L(\bar{c},\bar{d}) < c^*+d^*+y_1\bar{\alpha}L(\bar{c},d^*)$, as $\bar{d} < d^*$ implies $L(c^*,\bar{d}) > 0$, which means $(1-\bar{\alpha})L(c^*,\bar{d}) < 0$.
Therefore, we have $\bar{c}+\bar{d}+L(\bar{c},\bar{d}) < c^*+d^*+L(\bar{c},d^*)$, as $y_1\bar{\alpha} \leq 1$, or
$\bar{c}+\bar{d}+L(\bar{c},\bar{d}) < c^*+d^*+L(c^*,d^*)$, as $\bar{c} \geq c^*$ implies $L(\bar{c},d^*) \leq L(c^*,d^*)$. Which is a contradiction.
Therefore, $\bar{c} \geq c^*$ and $\bar{d} < d^*$$\rightarrow$$(\bar{c},\bar{d})$ is not a N.E.

Similarly, when $\bar{c} < c^*$ and $\bar{d} \geq d^*$ we can show that:
if $(\bar{c},\bar{d})$ is a N.E. then $(\bar{c},\bar{d}) \in M$, or $(\bar{c},\bar{d})$ is not a N.E.
And, $\bar{c} < c^*$ and $\bar{d} < d^*$$\rightarrow$$(\bar{c},\bar{d})$ is not a N.E.
Thus, we have established that:
$(\bar{c},\bar{d})$ is a N.E.$\rightarrow$$(\bar{c},\bar{d}) \in M$.
Therefore, if $1 < \bar{\alpha} \leq 1/y_1$ $\quad f$ is efficient for every possible choice of C, D, L and $(c^*,d^*) \in M$, satisfying $(A1)$ and $(A2)$.

When, $\bar{\alpha} \nleq 1/y_1$, i.e., $\bar{\alpha} > 1/y_1$, an argument analogous to the argument in Claim 2 shows that f is not efficient for every possible choice of C, D, L and $(c^*,d^*) \in M$, satisfying, $(A1)$ and $(A2)$.

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Proof of Claim 5

First, we prove that **if** $\bar{\alpha} \geq 1/y_{\bar{q}}$**, then** (c^*,d^*) **is a N.E.**
Let $\bar{\alpha} \geq 1/y_{\bar{q}}$. Therefore, $(\forall y_q \in \{y_q \mid (x_q,y_q) = f(1,q) \;\; and \; q \in [0,1)\}) \;\; [y_q\bar{\alpha} \geq 1]$. Take any arbitrary C, D, L and $(c^*,d^*) \in M$, satisfying $(A1)$ and $(A2)$. Suppose, (c^*,d^*) is not a N.E.
(c^*,d^*) is not a N.E.$\rightarrow$

$$(\exists d' \in D)[d' + y[p(c^*),q(d')]\bar{\alpha}L(c^*,d') < d^*], \tag{18}$$

or

$$(\exists c' \in C)[c' + L(c', d^*) < c^* + L(c^*, d^*)], \tag{19}$$

as $y[p(c), q(d^*)] = 0$, by construction of f.
Suppose, (18) holds.
First, consider the case: $d' > d^*$:
$d' > d^*$ & (18) $\rightarrow d' < d$, which is a contradiction. Therefore, $d' > d^* \rightarrow$ (18) can not hold.
Now, consider the case: $d' < d^*$:
$d' < d^*$ & (18)$\rightarrow$
$d' + y[p(c^*), q(d')]\bar{\alpha}L(c^*, d') < d^*$, or
$d' + L(c^*, d') < d^*$, as $y[p(c^*), q(d' < d^*)]\bar{\alpha} \geq 1$, or
$c^* + d' + L(c^*, d') < c^* + d^* + L(c^*, d^*)$, a contradiction as $(c^*, d^*) \in M$.
Therefore, (18) can not hold. Similarly, (19) can not hold. Thus, (c^*, d^*) is a N.E.

Now, we show that: $(\bar{c}, \bar{d})$ **is a N.E** $\rightarrow (\bar{c}, \bar{d}) \in M$**.**
$(\bar{c}, \bar{d})$ is a N.E , in particular, $\rightarrow$

$$\bar{c} + L(\bar{c}, \bar{d}) - y[p(\bar{c}), q(\bar{d})]\bar{\alpha}L(\bar{c}, \bar{d}) \leq c^* + L(c^*, \bar{d}) - y[p(c^*), q(\bar{d})]\bar{\alpha}L(c^*, \bar{d}) \tag{20}$$

and

$$\bar{d} + y[p(\bar{c}), q(\bar{d})]\bar{\alpha}L(\bar{c}, \bar{d}) \leq d^* \tag{21}$$

Adding (20) and (21)

$$\bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + L(c^*, \bar{d}) - y[p(c^*), q(\bar{d})]\bar{\alpha}L(c^*, \bar{d}) \tag{22}$$

Consider the case: $\bar{d} \geq d^*$:
Here, $\bar{d} \geq d^*$ & (22) $\rightarrow$
$\bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + L(c^*, \bar{d})$, as $y[p(c), q(\bar{d} \geq d^*)] = 0$ by construction, or
$\bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + L(c^*, d^*)$, as $\bar{d} \geq d^*$ implies $L(c^*, \bar{d}) \leq L(c^*, d^*)$.
Therefore, $(\bar{c}, \bar{d})$ is a N.E and $\bar{d} \geq d^* \rightarrow (\bar{c}, \bar{d}) \in M$.
Similarly, we can show that
If $\bar{d} < d^*$ and $(\bar{c}, \bar{d})$ is a N.E then $(\bar{c}, \bar{d}) \in M$.
Hence, if $\bar{\alpha} \geq 1/y_{\bar{q}}$, f is efficient.
When $\bar{\alpha} < 1/y_{\bar{q}}$, an argument analogous to the argument in the Subcase 1 of Case 2 of

Claim 3 shows that f is not efficient for every possible choice of C, D, L and $(c^*, d^*) \in M$, satisfying $(A1)$ and $(A2)$.

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